Ogre et Pythia: An invariance proof method for weak consistency models

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Objective

Example

```
0:{ w F1 false; w F2 false; w T 0; }
```

```
1:  w[] F1 true
2:  w[] T 2
3:  do
5:      r[] R1 F2
6:      r[] R2 T
7:  while R1 ∧ R2 ≠ 1
8:  ¬at 28
9:  w[] F1 false
10:
```

∥

```
21:w[] F2 true;
22:w[] T 1;
23:do
25:    r[] R3 F1;
26:    r[] R4 T;
27:while R3 ∧ R4 ≠ 2;
28:¬at 8
29:w[] F2 false;
39:
```

critical section

An invariance proof method for WCMs

- Extend Lamport's invariance proof method for parallel programs from sequentially consistent to weak consistency models so that
  - The weak consistency model is a *parameter* of the proof
  - We don't have to redo the whole proof when *changing the consistency model*

Note: Owicki & Gries is Lamport with auxiliary variables instead of programs counters

Separating invariance from WCM

- The invariance proof (that a specification S_{inv} is invariant for a program):
  - Done for a program consistency hypothesis S_{com}:
    - Sufficient for the program to be correct
    - Or better, also necessary for correctness (weakest consistency model)
  - This program consistency hypothesis S_{com} is expressed as an invariant
  - Sound and (relatively) complete

Separating invariance from WCM

- Consistency proof:

  a. The program consistency hypothesis S_{com} is strengthen into H_{com} written in a consistency specification language (e.g. `cat`)

  b. A `cat` architecture consistency model M is shown to imply the `cat` program consistency model H_{com}

- only b. to be redone when changing the architecture
- sound but possibly incomplete

Methodology

$$\text{invariants}\ \left\{ \quad \overbrace{S_{com} \overset{1}{\Longrightarrow} S_{inv}}^{\text{invariance proof}} \right.$$

$$\Uparrow 2$$

$$\text{cat}\ \left\{ \quad \underbrace{M \overset{3}{\Longrightarrow} H_{com}}_{\text{consistency proof}} \right.$$

The invariance proof method is designed by abstract interpretation of an *analytic* semantics

Analytic semantics

=

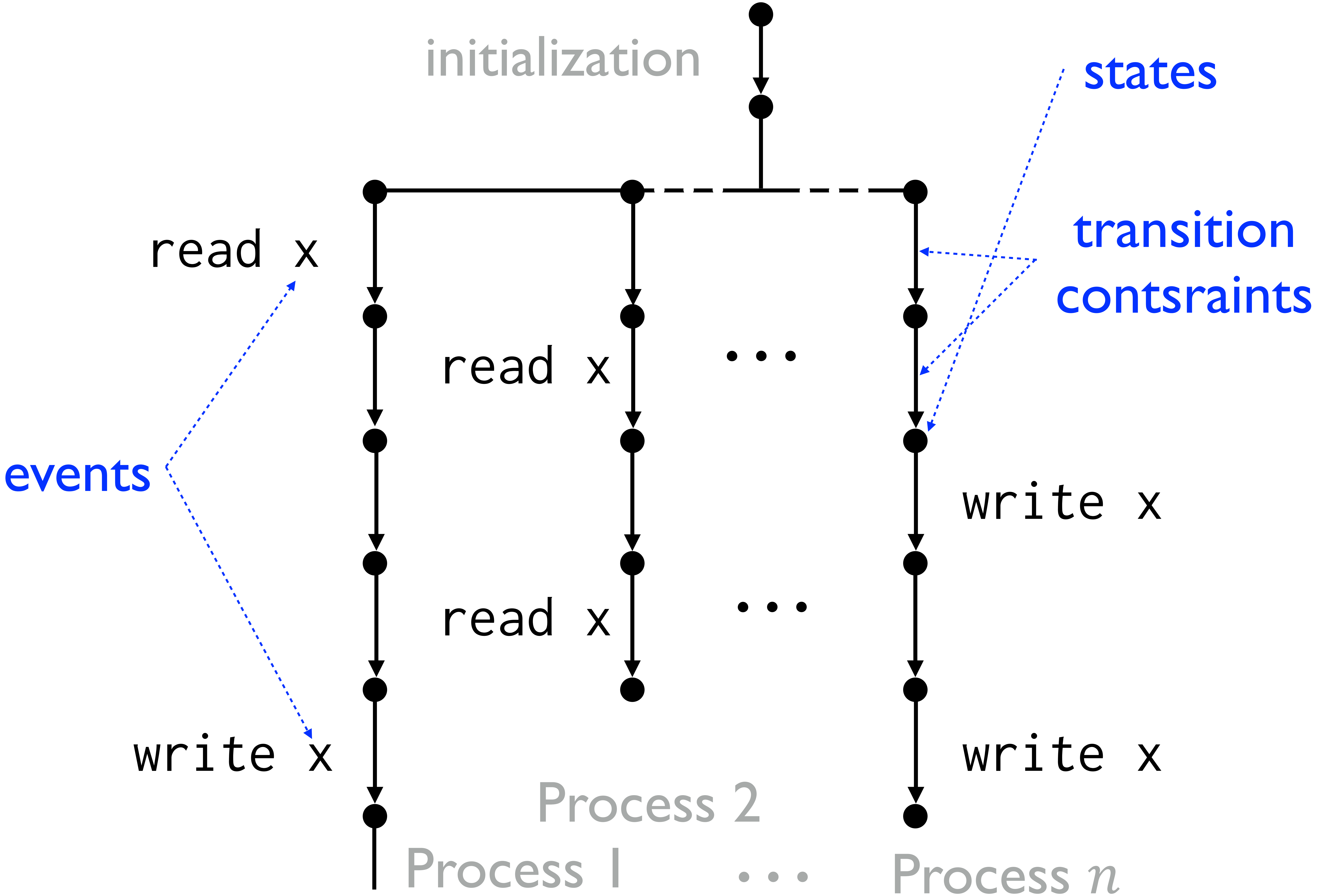
Anarchic semantics

$\sqcap$

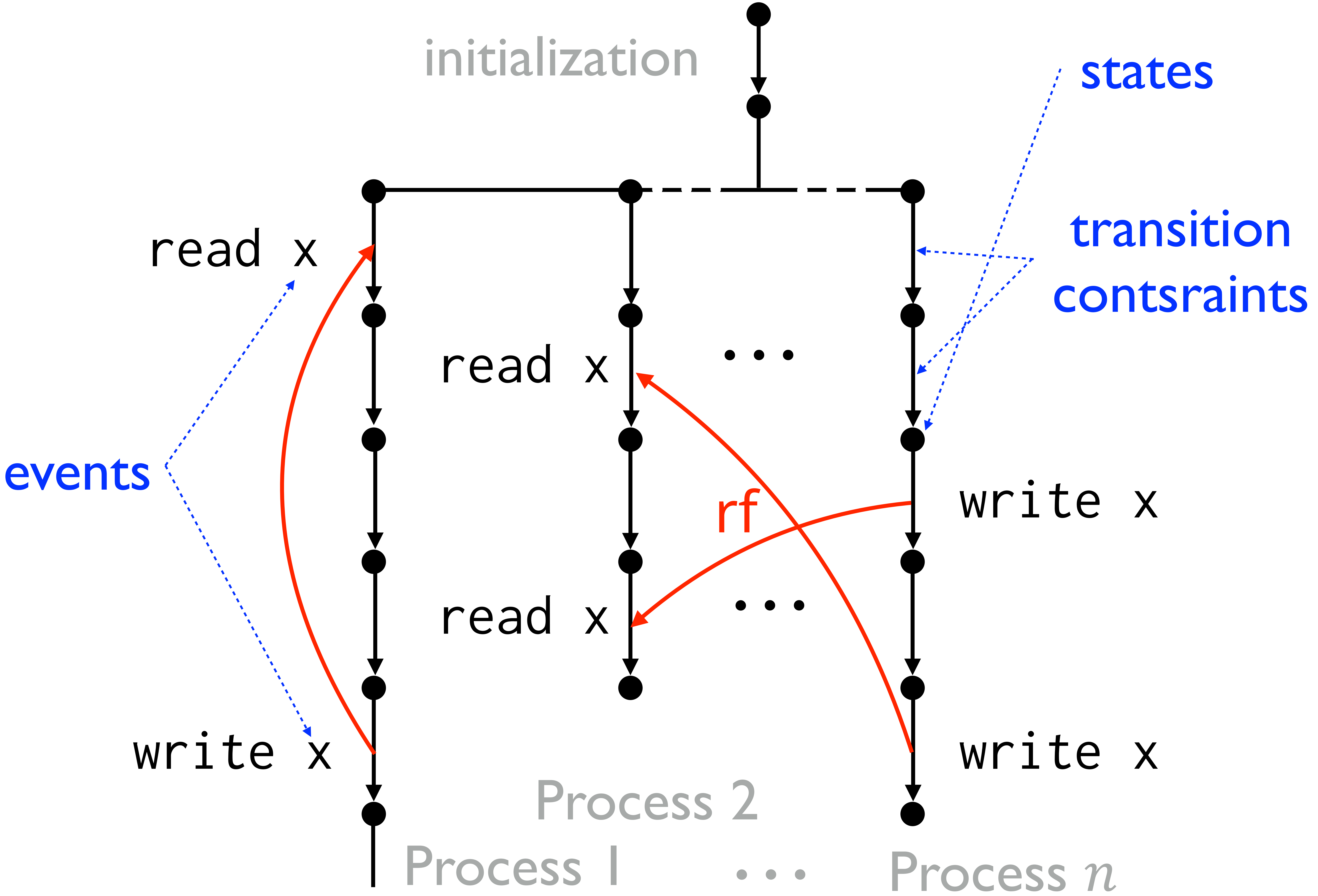
Weak consistency model

The anarchic semantics

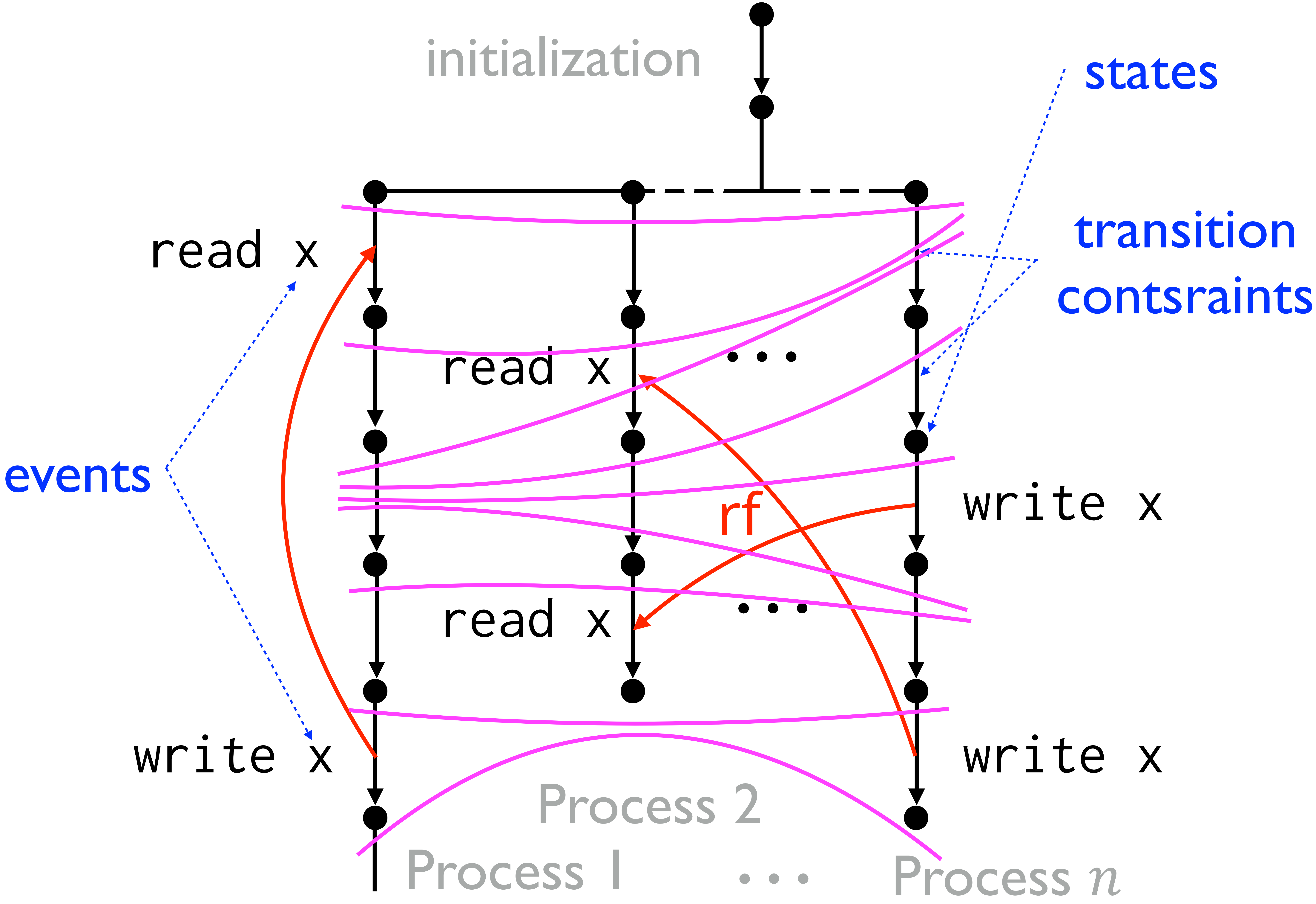
The anarchic semantics



The read-from relation rf



Cuts


initialization
states
transition
contsraints
read x
read x
...
events
write x
rf
read x
...
write x
write x
Process 2
Process 1
...
Process n

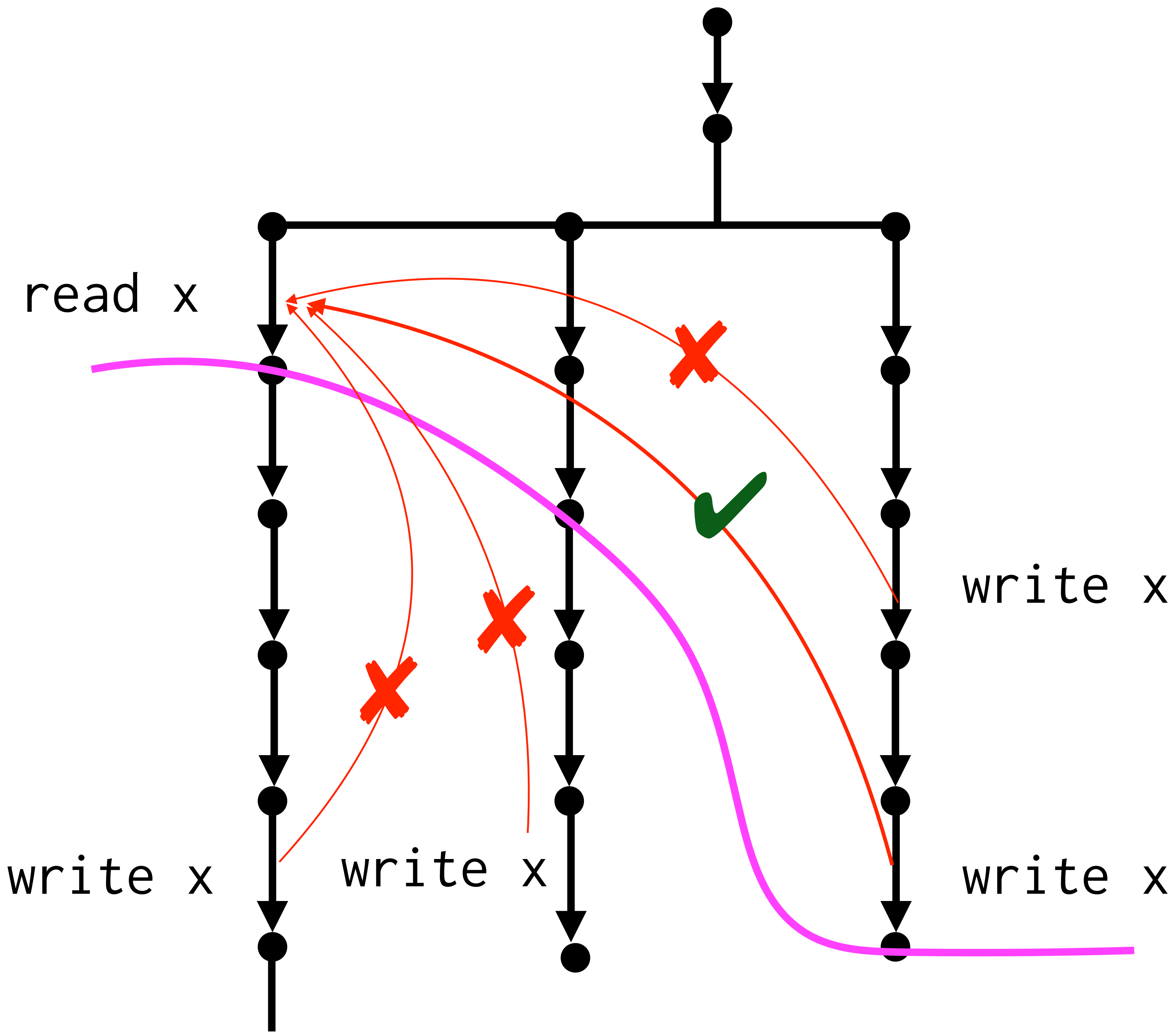
Anarchic semantics of fences

- The anarchic semantics of (localized) fences is `skip` (the state is unmodified)
- Fences are static marker events used by the WCM in `cat` to restrict the read-from relation rf

The weak consistency model

Weak consistency models

- Put restrictions on the read-from relation rf
- e.g. sequential consistency: a read at a cut reads from that last write in a process before that cut



Difficulties

Naming entities

- Invariants are logical formulæ
- can only describe entities that they name
- L/O-G use the name of shared variables to designate their current value in invariants

Naming entities

- Invariants are logical formulæ
- can only describe entities that they name
- L/O-G use the name of shared variables to designate their current value in invariants

Difficulty

- Meaningless with WCMs since there is no notion of ``the current value of a shared variable”

What is known on communications?

- Each process only knows the value of the shared variables from its last read
- Need to be named $\longrightarrow$ Pythia Variables

What we know on communications?

- Each process only knows the value of the shared variables from its last read
- Need to be named $\longrightarrow$ Pythia Variables

Difficulty

- Its dynamic, not static!
- A program read action can read from a different write each time it is executed $\longrightarrow$ Stamps (abstraction of local time)

Back to the anarchic semantics

State

- Per process:
  - A stamp (local time, no global time)
  - A program counter
  - The value of the local variables (registers) of the process
  - The stamped pythia variables (uniquely identifying all reads along a trace)
  - The value of the pythia variables (what was read)
- The read-from relation (rf)

Example (Peterson)

`0:{ w F1 false; w F2 false; w T 0; }`

P0:	P1:
`1:w[] F1 true`	`10:w[] F2 true;`
`2:w[] T 2`	`11:w[] T 1;`
`3:do` $\{i\}$	`12:do` $\{j\}$
`4:    r[] R1 F2` $\{\rightsquigarrow \mathtt{F2}^{i}_{4}\}$	`13:    r[] R3 F1;` $\{\rightsquigarrow \mathtt{F1}^{j}_{13}\}$
`5:    r[] R2 T` $\{\rightsquigarrow \mathtt{T}^{i}_{5}\}$	`14:    r[] R4 T;` $\{\rightsquigarrow \mathtt{T}^{j}_{14}\}$
`6:while R1` $\wedge$ `R2` $\neq$ `1` $\{i_{\mathrm{end}}\}$	`15:while R3` $\wedge$ `R4` $\neq$ `2;` $\{j_{\mathrm{end}}\}$
`7:skip (* CS1 *)`	`16:skip (* CS2 *)`
`8:w[] F1 false`	`17:w[] F2 false;`

Stamps (loop counters)

Stamps (on loop exit)

Example (Peterson)

0:{ w F1 false; w F2 false; w T 0; }

P0:
1:w[] F1 true
2:w[] T 2
3:do $\{i\}$
4:   r[] R1 F2 $\{\leadsto \mathtt{F2}^{i}_{4}\}$
5:   r[] R2 T $\{\leadsto \mathtt{T}^{i}_{5}\}$
6:while R1 $\wedge$ R2 $\neq$ 1 $\{i_{\mathrm{end}}\}$
7:skip (* CS1 *)
8:w[] F1 false

P1:
10:w[] F2 true;
11:w[] T 1;
12:do $\{j\}$
13:   r[] R3 F1; $\{\leadsto \mathtt{F1}^{j}_{13}\}$
14:   r[] R4 T; $\{\leadsto \mathtt{T}^{j}_{14}\}$
15:while R3 $\wedge$ R4 $\neq$ 2; $\{j_{\mathrm{end}}\}$
16:skip (* CS2 *)
17:w[] F2 false;

Pythia variables

Stamps (loop counters)

Stamps (on loop exit)

The abstraction

The invariance abstraction

- For each process

The invariance abstraction

- For each process
  - For each program point of that process

The invariance abstraction

- For each process
  - For each program point of that process
    - For each execution of the program

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        - The states of all processes, and

The invariance abstraction

- For each process
  - For each program point of that process
    - For each execution of the program
      - For each cut of that execution going through the program point of that process

        collect:

        - The states of all processes, and
        - The read-from relation rf

Example: Peterson

0: `{ w F1 false; w F2 false; w T 0; }`
$\{\mathtt{F1}=\mathtt{false} \;\land\; \mathtt{F2}=\mathtt{false} \;\land\; \mathtt{T}=0\}$ `}`

1: $\{\mathtt{R1}=0 \;\land\; \mathtt{R2}=0\}$
`w[] F1 true`
2: $\{\mathtt{R1}=0 \;\land\; \mathtt{R2}=0\}$
`w[] T 2`
3: $\{\mathtt{R1}=0 \;\land\; \mathtt{R2}=0\}$
`do` $\{i\}$
4: $\{(i=0 \;\land\; \mathtt{R1}=0 \;\land\; \mathtt{R2}=0) \;\lor\; (i>0 \;\land\; \mathtt{R1}=\mathtt{F2}_4^{i-1} \;\land\; \mathtt{R2}=\mathtt{T}_5^{i-1})\}$
`r[] R1 F2` $\{\leadsto \mathtt{F2}_4^i\}$
5: $\{\mathtt{R1}=\mathtt{F2}_4^i \land (i=0 \;\land\; \mathtt{R2}=0) \;\lor\; (i>0 \;\land\; \mathtt{R2}=\mathtt{T}_5^{i-1})\}$
`r[] R2 T` $\{\leadsto \mathtt{T}_5^i\}$
6: $\{\mathtt{R1}=\mathtt{F2}_4^i \land\; \mathtt{R2}=\mathtt{T}_5^i\}$
`while R1` $\land$ `R2`$\neq$`1` $\{i_{\mathtt{end}}\}$
7: $\{\neg\mathtt{F2}_4^{i_{\mathtt{end}}} \lor \mathtt{T}_5^{i_{\mathtt{end}}}=1\}$
`skip (* CS1 *)`
8: $\{\neg\mathtt{F2}_4^{i_{\mathtt{end}}} \lor \mathtt{T}_5^{i_{\mathtt{end}}}=1\}$
`w[] F1 false`
9: $\{\neg\mathtt{F2}_4^{i_{\mathtt{end}}} \lor \mathtt{T}_5^{i_{\mathtt{end}}}=1\}$

||

10: $\{\mathtt{R3}=0 \;\land\; \mathtt{R4}=0\}$
`w[] F2 true;`
11: $\{\mathtt{R3}=0 \;\land\; \mathtt{R4}=0\}$
`w[] T 1;`
12: $\{\mathtt{R3}=0 \;\land\; \mathtt{R4}=0\}$
`do` $\{j\}$
13: $\{(j=0 \;\land\; \mathtt{R3}=0 \;\land\; \mathtt{R4}=0) \;\lor\; (j>0 \;\land\; \mathtt{R3}=\mathtt{F1}_{13}^{j-1} \;\land\; \mathtt{R4}=\mathtt{T}_{14}^{j-1})\}$
`r[] R3 F1` $\{\leadsto \mathtt{F1}_{13}^j\}$`;`
14: $\{\mathtt{R3}=\mathtt{F1}_{13}^j \land (j=0 \;\land\; \mathtt{R4}=0) \;\lor\; (j>0 \;\land\; \mathtt{R4}=\mathtt{T}_{14}^{j-1})\}$
`r[] R4 T;` $\{\leadsto \mathtt{T}_{14}^j\}$
15: $\{\mathtt{R3}=\mathtt{F1}_{13}^j \;\land\; \mathtt{R4}=\mathtt{T}_{14}^j)\}$
`while R3` $\land$ `R4`$\neq$`2` $\{j_{\mathtt{end}}\}$ `;`
16: $\{\neg\mathtt{F1}_{13}^{j_{\mathtt{end}}} \lor \mathtt{T}_{14}^{j_{\mathtt{end}}}=2\}$
`skip (* CS2 *)`
17: $\{\neg\mathtt{F1}_{13}^{j_{\mathtt{end}}} \lor \mathtt{T}_{14}^{j_{\mathtt{end}}}=2\}$
`w[] F2 false;`
18: $\{\neg\mathtt{F1}_{13}^{j_{\mathtt{end}}} \lor \mathtt{T}_{14}^{j_{\mathtt{end}}}=2\}$

Example: Peterson

```
0: { w F1 false; w F2 false; w T 0; }
{F1=false ∧ F2=false ∧ T=0} }
1:  {R1=0 ∧ R2=0}                      10: {R3=0 ∧ R4=0}
    w[] F1 true                             w[] F2 true;
2:  {R1=0 ∧ R2=0}                      11: {R3=0 ∧ R4=0}
    w[] T 2                                 w[] T 1;
3:
4:
```

4: $\{(\mathtt{i=0} \wedge \mathtt{R1=0} \wedge \mathtt{R2=0}) \vee (\mathtt{i>0} \wedge \mathtt{R1=F2}_4^{i-1} \wedge \mathtt{R2=T}_5^{i-1})\}$

5: $\{\mathtt{R1=F2}_4^{i} \wedge (\mathtt{i=0} \wedge \mathtt{R2=0}) \vee (\mathtt{i>0} \wedge \mathtt{R2=T}_5^{i-1})\}$

  r[] R2 T $\{\leadsto \mathtt{T}_5^{i}\}$

6: $\{\mathtt{R1=F2}_4^{i} \wedge \mathtt{R2=T}_5^{i}\}$

while R1 $\wedge$ R2$\neq$1 $\{i\mathtt{end}\}$

7: $\{\neg\mathtt{F2}_4^{i\mathtt{end}} \vee \mathtt{T}_5^{i\mathtt{end}}\mathtt{=1}\}$

skip (* CS1 *)

8: $\{\neg\mathtt{F2}_4^{i\mathtt{end}} \vee \mathtt{T}_5^{i\mathtt{end}}\mathtt{=1}\}$

w[] F1 false

9: $\{\neg\mathtt{F2}_4^{i\mathtt{end}} \vee \mathtt{T}_5^{i\mathtt{end}}\mathtt{=1}\}$

||

14: $\{\mathtt{R3=F1}_{13}^{j} \wedge (\mathtt{j=0} \wedge \mathtt{R4=0}) \vee (\mathtt{j>0} \wedge \mathtt{R4=T}_{14}^{j-1})\}$

  r[] R4 T; $\{\leadsto \mathtt{T}_{14}^{j}\}$

15: $\{\mathtt{R3=F1}_{13}^{j} \wedge \mathtt{R4=T}_{14}^{j})\}$

while R3 $\wedge$ R4$\neq$2 $\{j\mathtt{end}\}$;

16: $\{\neg\mathtt{F1}_{13}^{j\mathtt{end}} \vee \mathtt{T}_{14}^{j\mathtt{end}}\mathtt{=2}\}$

skip (* CS2 *)

17: $\{\neg\mathtt{F1}_{13}^{j\mathtt{end}} \vee \mathtt{T}_{14}^{j\mathtt{end}}\mathtt{=2}\}$

w[] F2 false;

18: $\{\neg\mathtt{F1}_{13}^{j\mathtt{end}} \vee \mathtt{T}_{14}^{j\mathtt{end}}\mathtt{=2}\}$

The calculational design of the verification conditions by abstract interpretation

The induction principle

- Given an invariance specification S_{inv} find a stronger inductive invariant S_{ind}
- Prove that S_{ind} satisfy verification conditions
  - Holds after initialization
  - Remains true after a computation step
  - Remains true after a communication
- Assuming S_{com} / H_{com}

The induction principle

- Given an invariance specification S_{inv} find a stronger inductive invariant S_{ind}
- Prove that S_{ind} satisfy verification conditions
  - Holds after initialization
  - Remains true after a computation step
  - Remains true after a communication
- Assuming S_{com} / H_{com}

Verification conditions = abstraction of the concrete transformer for one computation step

Calculational design of the verification conditions

$\alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \subseteq S_{inv}$

$\Leftrightarrow \alpha_{\mathsf{inv}}(\{\xi \in S^{\mathsf{a}}[\![\mathrm{P}]\!] \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv}$ (def. $\alpha_{\mathsf{ana}}[\![H_{com}]\!]$)

$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!] \cap \{\xi \in S^{\mathsf{a}}[\![\mathrm{P}]\!] \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv}$ (def. $\cap$)

$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \cap \alpha_{\mathsf{inv}}(\{\xi \in \Xi \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv}$

(since α_{inv} preserves intersections)

$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} \alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \mathrel{\dot\subseteq} S_{inv}$ (def. $\alpha_{\mathsf{ana}}[\![H_{com}]\!]$)

$\Leftrightarrow \exists S_{com} \,.\, \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} S_{com} \mathrel{\dot\subseteq} S_{inv} \wedge \alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \mathrel{\dot\subseteq} S_{com}$

(($\Leftarrow$) For soundness, we have $\alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} \alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \mathrel{\dot\subseteq} \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} S_{com} \mathrel{\dot\subseteq} S_{inv}$;

($\Rightarrow$) For completeness, we choose to describe exactly the communications that is $S_{com} = \alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!]))$.)

$\Leftrightarrow \exists S_{com} \,.\, (S_{com} \Rightarrow S_{inv}) \wedge (H_{com} \Rightarrow S_{com})$

by defining the conditional invariance proof $S_{com} \Rightarrow S_{inv}$ to be $\alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} S_{com} \mathrel{\dot\subseteq} S_{inv}$ and the inclusion proof $H_{com} \Rightarrow S_{com}$ to be $\alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \mathrel{\dot\subseteq} S_{com}$.

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Calculational design of the verification conditions

$$\alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \subseteq S_{inv}$$

$$\Leftrightarrow \alpha_{\mathsf{inv}}(\{\xi \in S^{\mathsf{a}}[\![\mathrm{P}]\!] \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv} \quad \text{(def. } \alpha_{\mathsf{ana}}[\![H_{com}]\!]\text{)}$$

$$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!] \cap \{\xi \in S^{\mathsf{a}}[\![\mathrm{P}]\!] \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv} \quad \text{(def. } \cap\text{)}$$

$$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \cap \alpha_{\mathsf{inv}}(\{\xi \in \Xi \mid S[\![H_{com}]\!]\xi = \mathtt{allowed}\}) \mathrel{\dot\subseteq} S_{inv}$$

$$\boxed{\Leftrightarrow \exists S_{com} \,.\, (S_{com} \Rightarrow S_{inv}) \wedge (H_{com} \Rightarrow S_{com})}$$

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by defining the conditional invariance proof $S_{com} \Rightarrow S_{inv}$ to be $\alpha_{\mathsf{inv}}(S^{\mathsf{a}}[\![\mathrm{P}]\!]) \mathrel{\dot\cap} S_{com} \mathrel{\dot\subseteq} S_{inv}$ and the inclusion proof $H_{com} \Rightarrow S_{com}$ to be $\alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathrm{P}]\!])) \mathrel{\dot\subseteq} S_{com}$.

...

...

...

Verification conditions

- Sequential proof
- Non-interference proof
  (like L/O-G but for different kind of invariants)
- Communication proof
  - a read event reading from a write event must be in rf
  - the value read for a variable is the one written
  - reading is fair in rf (cannot be delayed indefinitely)
  - …

  (useless in L/O-G since rf is fixed)

The program consistency hypothesis S_{com}

Communication hypothesis S_{com}

- A sufficient communication hypothesis can be discovered by calculational design:

$$S_{com} \Longrightarrow (S_{ind} \Longrightarrow S_{inv})$$

Communication hypothesis

Program inductive anarchic invariant

Program invariance specification

- i.e. (Sind ∧ ¬Sinv) ⟹ ¬Scom
- Necessary: by counter examples

Proving Consistency

$$H_{com} \Longrightarrow S_{com}$$

cat

invariant

Proof method

- Obtained by calculational design:

$$\alpha_{\mathsf{inv}}(\alpha_{\mathsf{ana}}[\![H_{com}]\!](S^{\mathsf{a}}[\![\mathtt{P}]\!])) \mathrel{\dot{\subseteq}} S_{com}$$

$$\Leftrightarrow \alpha_{\mathsf{inv}}(S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P}) \mathrel{\dot{\subseteq}} S_{com} \qquad \wr\text{def. } S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P}\wr$$

$$\Leftrightarrow \forall \xi \in S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P} \,.\, \alpha_{\mathsf{inv}}(\{\xi\}) \mathrel{\dot{\subseteq}} S_{com} \qquad \wr\alpha_{\mathsf{inv}} \text{ preserves } \cup\wr$$

$$\Leftrightarrow \forall \xi \in S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P} \,.\, \dot{\bigcup_{p=1}^{n}} \dot{\bigcup_{\mathtt{L}\in\mathtt{P}_p}} \{\alpha_{\mathsf{inv}}(\xi')_p(\mathtt{L}) \mid \xi' \in \{\xi\}\} \mathrel{\dot{\subseteq}} S_{com}$$
$$\wr\text{def. (19) of } \alpha_{\mathsf{inv}}\wr$$

$$\Leftrightarrow \forall(\tau_{\mathsf{start}} \times \prod_{p=0}^{n-1} \tau_p \times \pi \times \mathsf{rf}) \in S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P} \,.\, \forall p \in [1, n] \,.\, \forall \mathtt{L} \in \mathtt{P}_p \,.$$
$$\alpha_{\mathsf{inv}}(\tau_{\mathsf{start}} \times \prod_{p=0}^{n-1} \tau_p \times \pi \times \mathsf{rf})_p(\mathtt{L}) \subseteq S_{com\,p}(\mathtt{L})$$

$\wr$def. $\in$, $\dot{\bigcup}$, $\dot{\subseteq}$, and $S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P}$ so that ξ has the form $\xi = \tau_{\mathsf{start}} \times \prod_{p=0}^{n-1} \tau_p \times \pi \times \mathsf{rf}$. By def. (19) of α_{inv} and $\subseteq$, we get$\wr$

$$\Leftrightarrow \forall(\tau_{\mathsf{start}} \times \prod_{p=0}^{n-1} \tau_p \times \pi \times \mathsf{rf}) \in S^{\mathsf{ana}}[\![H_{com}]\!]\mathtt{P} \,.\, \forall i \in [1, n] \,.\, \forall \mathtt{L} \in \mathtt{P}_p \,.\, \forall q \in [0, n[\,.\, \forall k_q < |\tau_q| \,.\, (\tau_{q_{k_q}} = \mathfrak{s}\langle \kappa_{q,k_q}, \theta_{q,k_q}, \rho_{q,k_q}, \nu_{q,k_q}\rangle \wedge \kappa_{p,k_p} = \mathtt{L}) \Rightarrow \langle \kappa_{0,k_0}, \theta_{0,k_0}, \rho_{0,k_0}, \nu_{0,k_0}, \ldots, \nu_{p-1,k_{p-1}}, \theta_{p,k_p}, \rho_{p,k_p}, \nu_{p,k_p}, \kappa_{p+1,k_{p+1}}, \ldots, \kappa_{n-1,k_{n-1}}, \theta_{n-1,k_{n-1}}, \rho_{n-1,k_{n-1}}, \nu_{n-1,k_{n-1}}, \mathsf{rf}\rangle \in S_{com\,i}(\mathtt{L}) \tag{20}$$

Proof method

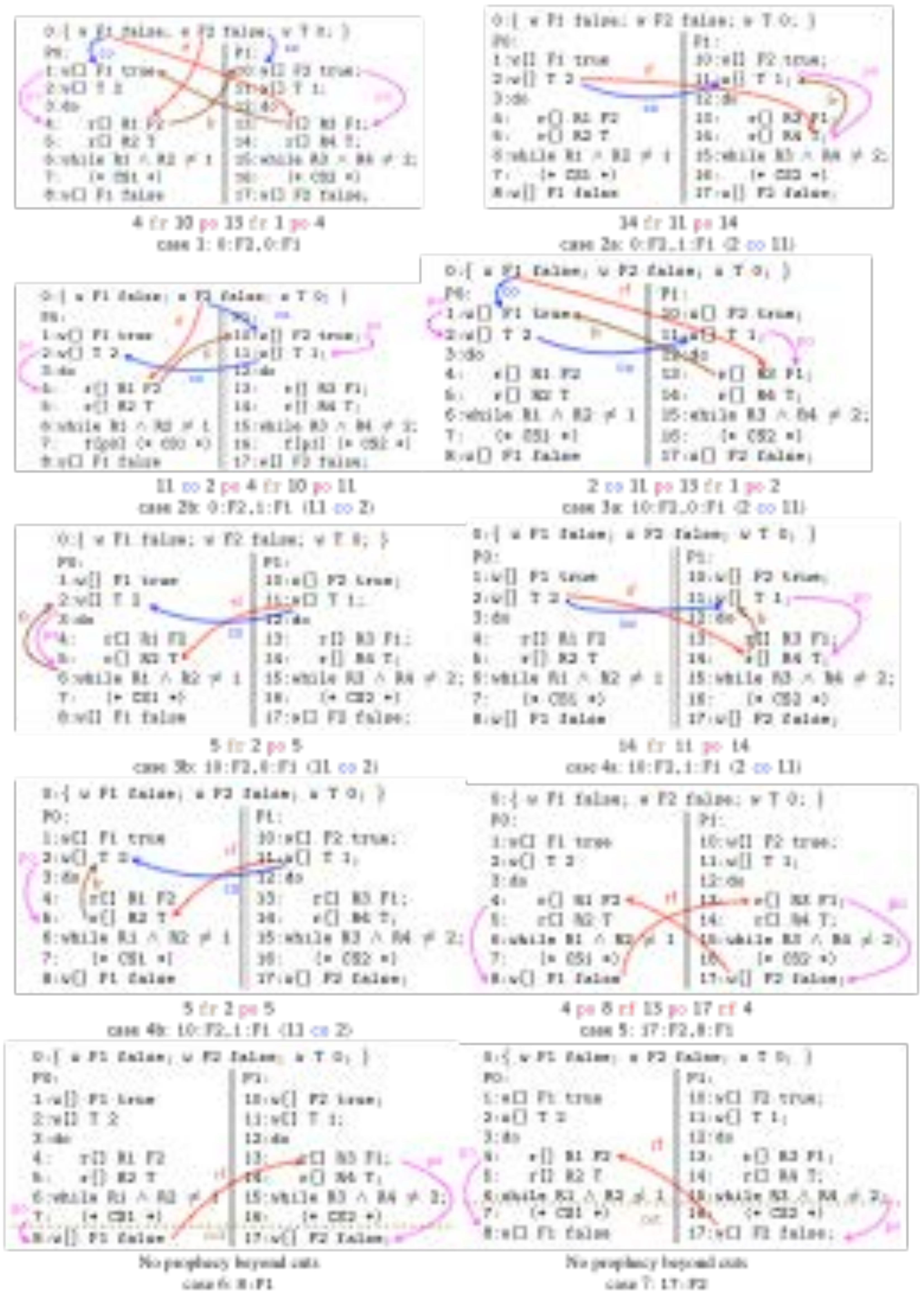
- The anarchic invariants can be used to calculate all communication scenarios violating S_{com}
- These scenarios must be forbidden by the cat specification H_{com}

(no need to reason at the level of traces of the anarchic semantics)

Example (Peterson)

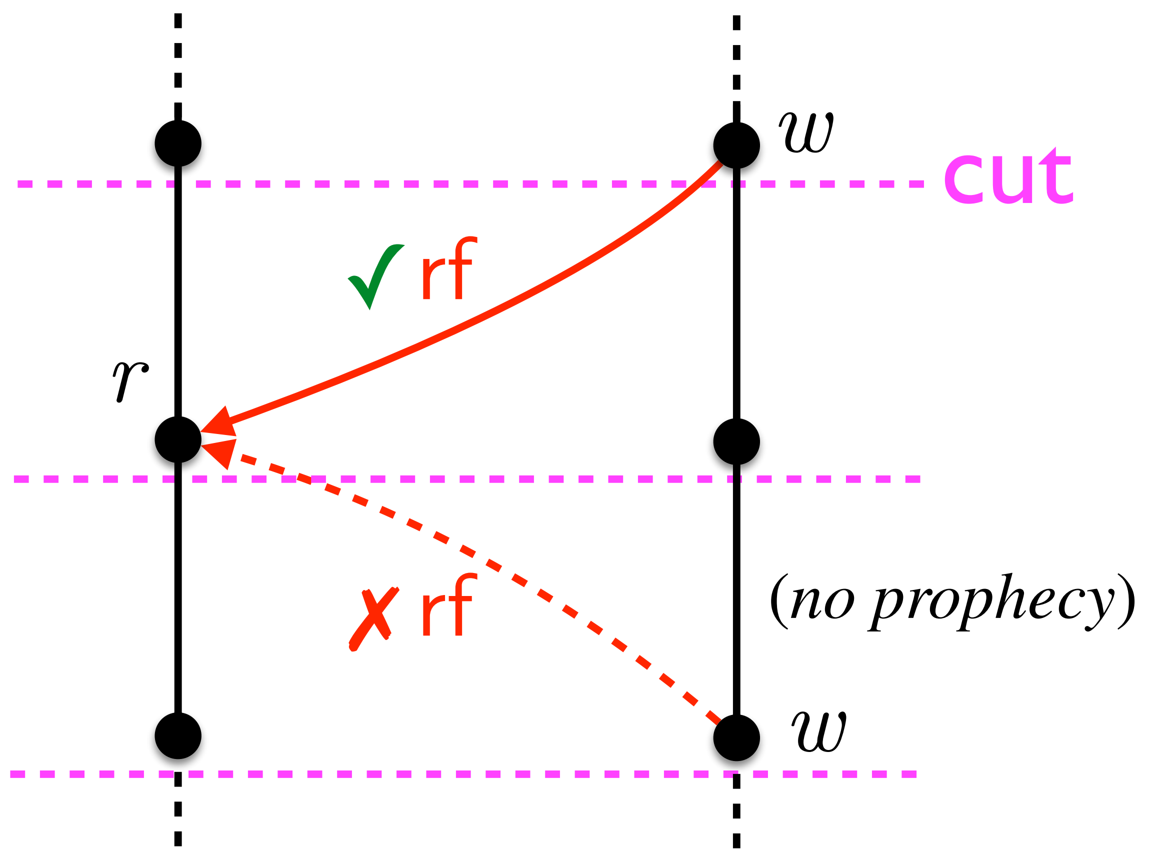
Communication scenarios violating S_{com} for Peterson

$$S_{com} \triangleq \neg[\exists i, j.[\mathit{rf}\langle \mathtt{F2}_4^i,\ \mathtt{0},\ \mathsf{false}\rangle \vee \mathit{rf}\langle \mathtt{F2}_4^i,\ \mathtt{17},\ \mathsf{false}\rangle \vee \mathit{rf}\langle \mathtt{T}_5^i,\ \mathtt{11},\ 1\rangle] \wedge [\mathit{rf}\langle \mathtt{F1}_{13}^j,\ \mathtt{0},\ \mathsf{false}\rangle \vee \mathit{rf}\langle \mathtt{F1}_{13}^j,\ \mathtt{8},\ \mathsf{false}\rangle \vee \mathit{rf}\langle \mathtt{T}_{14}^j,\ \mathtt{2},\ 2\rangle]]$$



Incompleteness

- In general you have to add fences for H_{com} (do not change the invariants, S_{inv}, S_{ind}, and S_{com} remain valid)
- S_{com} can refer to communicated values not H_{com} in cat (redesign your algorithm without assuming that the hardware does know about communicated values)
- cat may not be expressive enough:



No read beyond cut

Proving Architectural Consistency

$$M \Longrightarrow H_{com}$$

cat cat

$M \Longrightarrow H_{com}$ *in* `cat`

- sound and complete proof method
- unpublished paper of JA and PC with Luc Maranget

Beyond L/O-G: non-starvation

Reasoning on one execution only

- A particular execution can be uniquely characterized by its read-from relation rf
- We can reason on one execution only (S_{com} for this execution + S_{ind})
- Not directly possible with L/O-G
- Can be used to prove non-starvation

Non-starvation (e.g. PostgrSQL)

- Consider all traces that may starve (for an appropriate S'_{com} for each trace)
- Prove each of them to be infeasible:
  - the inductive invariant S_{ind} under the program communication hypothesis S_{com} is unsatisfied
  - or, by strengthening the program communications S_{com} (maybe implemented by adding fences in H_{com})
  - or, by a fairness hypothesis.

Communication fairness hypothesis(*)

- All writes eventually hit the memory:
  - If, at a cut of the execution, all the processes infinitely often write the same value v to a shared variable x and only that value v
  - and from a later cut point of that execution, a process infinitely often repeats reads to that variable x
  - then the reads will end up reading that value v

(*) The SPARC Architecture Manual, Version 8, Section K2, p. 283: ``if one processor does an S, and another processor repeatedly does L's to the same location, then there is an L that will be after the S''.

Conclusion

Conclusion

- To design a correct parallel algorithm, specify:
  - the algorithm
  - the invariance specification S_{inv}
  - the program-specific consistency model S_{com}
- Find an anarchic inductive invariant S_{ind} satisfying the verification conditions such that $(S_{com} \wedge S_{ind}) \Longrightarrow S_{inv}$

Conclusion

- To implement a parallel algorithm correctly:
  - Implement the program consistency model on an architecture consistency model M (possibly adding fences)
  - Prove $M \Longrightarrow S_{com}$
- Or better
  - Find a minimal/weakest H_{com} such that $H_{com} \Longrightarrow S_{com}$
  - $M \Longrightarrow H_{com}$

More work needed

- Specification of parallel/distributed program consistency models (more refined than architecture consistency models, e.g. cuts needed)
- Liveness (beyond non-starvation)
- Collection of certified algorithms for WCM (e.g. transactional memory, databases, etc)
- Static analysis (by abstract interpretation of the analytic semantics parameterized by a WCM)

The End, Thank You