Integrating Physical Systems in the Static Analysis of Embedded Control Software

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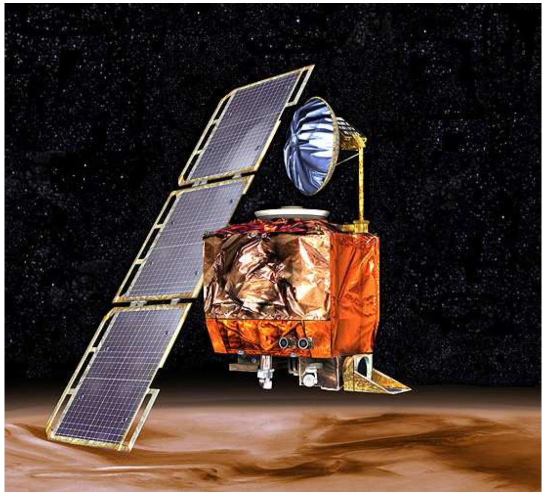
Talk Outline

Motivation

All Computer Scientists Have Experienced Bugs

Ariane 5.01 failure
(overflow)

Patriot failure
(float rounding)



Mars orbiter loss
(unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.

Static Analysis by Abstract Interpretation

Static analysis: analyze the program at compile-time to verify a program runtime property

Undecidability $\longrightarrow$

Abstract interpretation: effectively compute an abstraction/ sound approximation of the program semantics,

- which is precise enough to imply the desired property, and
- coarse enough to be efficiently computable.

Abstract Interpretation, Reminder using a simple example

Reference

[POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4th ACM POPL*.

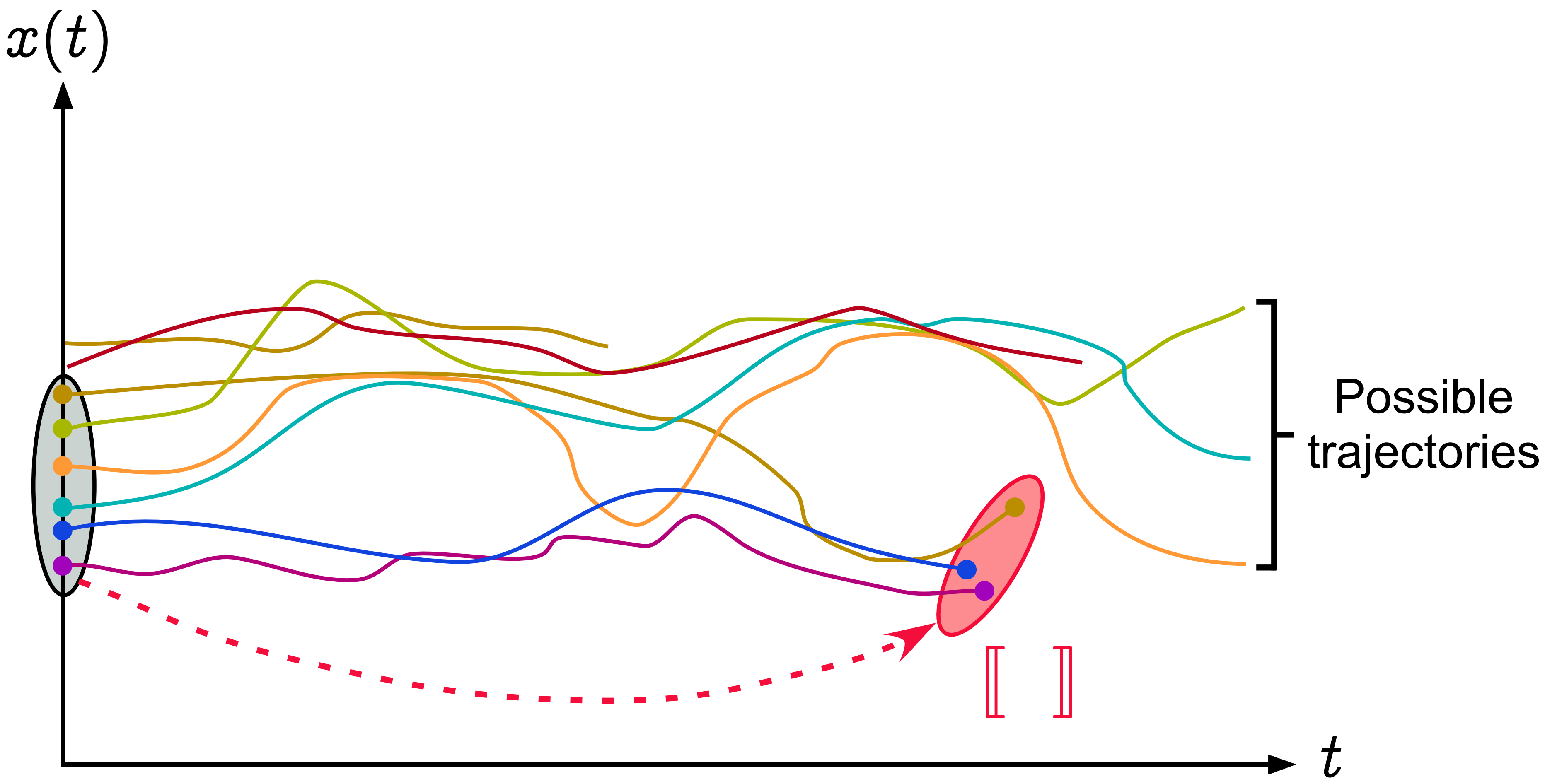
[Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In *6th ACM POPL*.

Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T\ X\ ;$	
$\quad \mid\ T\ X\ ;\ D'$	
$C ::= X$ = E;	commands $C \in \mathbb{C}$
$\quad \mid$ `while` $B\ C'$	
$\quad \mid$ `if` $B\ C'$ `else` C''	
$\quad \mid$ { $C_1 \ldots C_n$ }, $(n \geq 0)$	
$P ::= D\ C$	program $P \in \mathbb{P}$

Postcondition semantics



States

Values of given type:

$$\mathcal{V}[\![T]\!] \quad : \quad \text{values of type } T \in \mathbb{T}$$

$$\mathcal{V}[\![\mathtt{int}]\!] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \mathtt{min\_int} \leq z \leq \mathtt{max\_int}\}$$

Program states $\Sigma[\![P]\!]$ [1]:

$$\Sigma[\![D\ C]\!] \stackrel{\text{def}}{=} \Sigma[\![D]\!]$$

$$\Sigma[\![T\ X\,;]\!] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[\![T]\!]$$

$$\Sigma[\![T\ X\,;\ D]\!] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[\![T]\!]) \cup \Sigma[\![D]\!]$$

---

1 States $\rho \in \Sigma[\![P]\!]$ of a program P map program variables X to their values $\rho(X)$

Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[\![P]\!] \stackrel{\text{def}}{=} \wp(\Sigma[\![P]\!]) \qquad \text{sets of states}$$

i.e. program properties where $\subseteq$ is implication, $\emptyset$ is false, $\cup$ is disjunction.

Concrete Reachability Semantics of Programs

$$\mathcal{S}[\![X = E;]\!]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \qquad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$\mathcal{S}[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{B}[\![\neg B]\!]R$$

$$\mathcal{B}[\![B]\!]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$

$$\mathcal{S}[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{S}[\![C'']\!](\mathcal{B}[\![\neg B]\!]R)$$

$$\mathcal{S}[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \textsf{lfp}_{\emptyset}^{\subseteq} \lambda \mathcal{X} \cdot R \cup \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C_n]\!] \circ \ldots \circ \mathcal{S}[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C]\!](\Sigma[\![D]\!]) \quad \text{(uninitialized variables)}$$

Not computable (undecidability).

Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup\rangle$$

such that:

$$\langle \mathcal{D}[\![P]\!],\ \subseteq\rangle \xleftarrow[\alpha]{\gamma} \langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq\rangle$$

i.e.

$$\forall X \in \mathcal{D}[\![P]\!], Y \in \mathcal{D}^\sharp[\![P]\!] : \alpha(X) \sqsubseteq Y \iff X \subseteq \gamma(Y)$$

hence $\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup\rangle$ is a complete lattice such that $\bot = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup\, \gamma(X))$

Example 1 of Abstraction

Set of traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\stackrel{\alpha}{\rightarrow}$ Strongest liberal postcondition: final states s reachable from a given precondition P

$$\alpha(X) = \lambda P \cdot \{s \mid \exists \sigma_0 \sigma_1 \ldots \sigma_n \in X : \sigma_0 \in P \wedge s = \sigma_n\}$$

We have (Σ: set of states, $\dot{\subseteq}$ pointwise):

$$\langle \wp(\Sigma^\infty), \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma} \langle \wp(\Sigma) \stackrel{\cup}{\longmapsto} \wp(\Sigma), \dot{\subseteq} \rangle$$

Example 2 of Abstraction

Set of traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\stackrel{\alpha_0}{\rightarrow}$ Trace of sets of states: sequence of set of states appearing at a given time along at least one of these traces

$$\alpha_0(X) = \lambda i \cdot \{\sigma_i \mid \sigma \in X \wedge 0 \leq i < |\sigma|\}$$

$\stackrel{\alpha_1}{\rightarrow}$ Set of reachable states: set of states appearing at least once along one of these traces (global invariant)

$$\alpha_1(\Sigma) = \bigcup\{\Sigma_i \mid 0 \leq i < |\Sigma|\}$$

$\stackrel{\alpha_2}{\rightarrow}$ Partitionned set of reachable states: project along each control point (local invariant)

$$\alpha_2(\{\langle c_i,\ \rho_i\rangle \mid i \in \Delta\}) = \lambda c \cdot \{\rho_i \mid i \in \Delta \wedge c = c_i\}$$

$\xrightarrow{\alpha_3}$ Partitionned cartesian set of reachable states: project along each program variable (relationships between variables are now lost)

$$\alpha_3(\lambda c \cdot \{\rho_i \mid i \in \Delta_c\}) = \lambda c \cdot \lambda \mathtt{X} \cdot \{\rho_i(\mathtt{X}) \mid i \in \Delta_c\}$$

$\xrightarrow{\alpha_4}$ Partitionned cartesian interval of reachable states: take min and max of the values of the variables [2]

$$\alpha_4(\lambda c \cdot \lambda \mathtt{X} \cdot \{v_i \mid i \in \Delta_{c,\mathtt{X}}\} = \lambda c \cdot \lambda \mathtt{X} \cdot \langle \min\{v_i \mid i \in \Delta_{c,\mathtt{X}}\}, \max\{v_i \mid i \in \Delta_{c,\mathtt{X}}\}\rangle$$

α_0, α_1, α_2, α_3 and α_4, whence $\alpha_4 \circ \alpha_3 \circ \alpha_2 \circ \alpha_1 \circ \alpha_0$ are lower-adjoints of Galois connections

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[2] assuming these values to be totally ordered.

Example 3: Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \subseteq \rangle \xleftarrow[\alpha_1]{\gamma_1} \langle \mathcal{D}^\sharp_1, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \subseteq \rangle \xleftarrow[\alpha_2]{\gamma_2} \langle \mathcal{D}^\sharp_2, \sqsubseteq_2 \rangle$$

the reduced product is

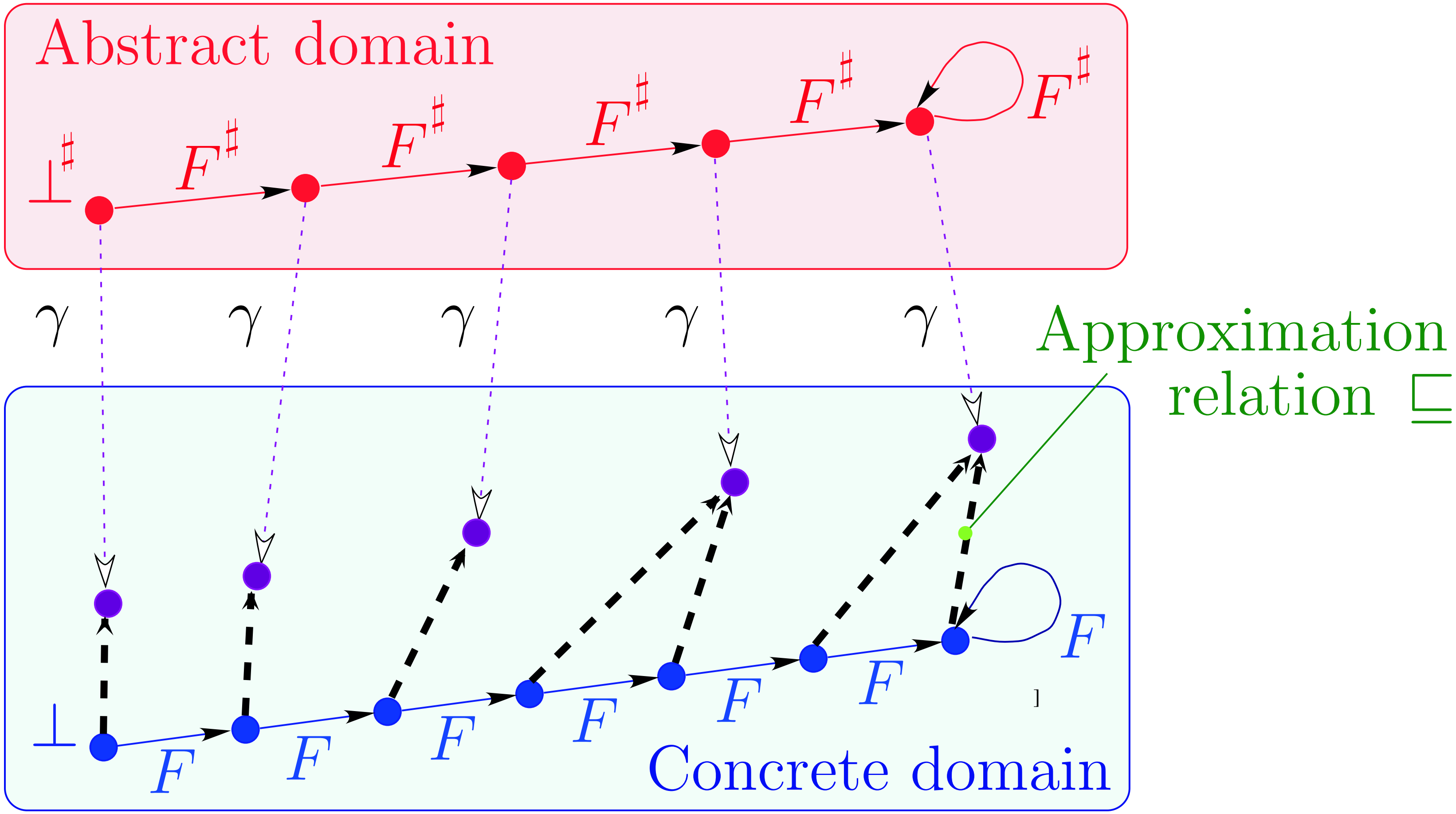
$$\alpha(X) \stackrel{\text{def}}{=} \sqcap\{\langle x, y\rangle \mid X \subseteq \gamma_1(x) \wedge X \subseteq \gamma_2(y)\}$$

such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \subseteq \rangle \xleftarrow[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $\mathtt{x} \in [1, 9] \wedge \mathtt{x} \bmod 2 = 0$ reduces to $\mathtt{x} \in [2, 8] \wedge \mathtt{x} \bmod 2 = 0$

Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\sharp \Rightarrow \mathsf{lfp}\, F \sqsubseteq \gamma(\mathsf{lfp}\, F^\sharp)$$

Abstract Reachability Semantics of Programs

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

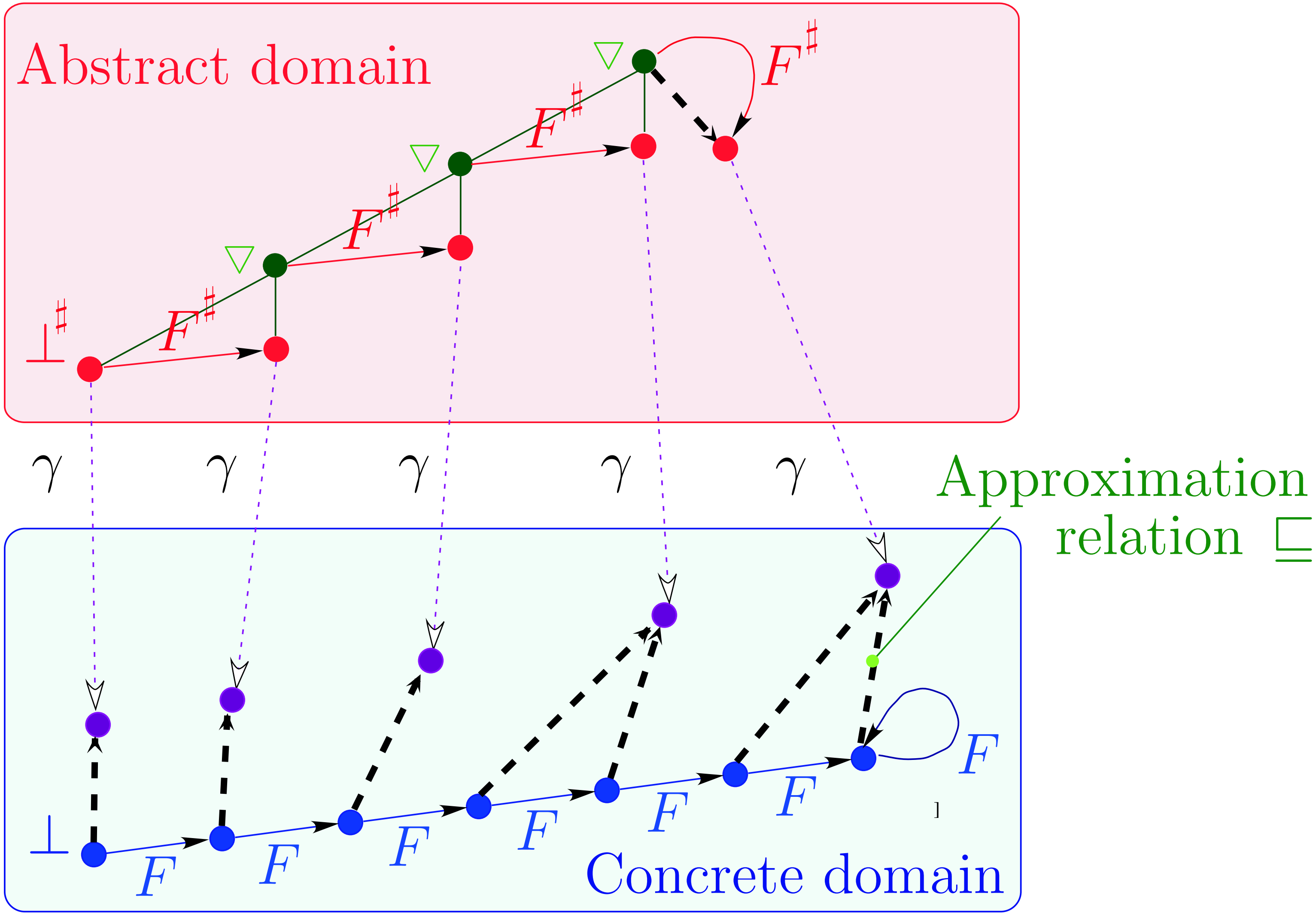
$$\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \textsf{lfp}^{\sqsubseteq}_{\bot}\ \lambda \mathcal{X} \cdot R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

Convergence Acceleration with Widening



Abstract Semantics with Convergence Acceleration [3]

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

$$\begin{aligned}\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} {} & \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X} \cdot \text{let } \mathcal{Y} = R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \\ & \qquad \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \mathbin{\nabla} \mathcal{Y} \\ & \text{and } \mathcal{W} = \textsf{lfp}_{\bot}^{\sqsubseteq} \mathcal{F}^\sharp \qquad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})\end{aligned}$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!]R \quad n > 0$$

$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

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[3] Note: $\mathcal{F}^\sharp$ <u>not</u> monotonic!

Applications of Abstract Interpretation

A few applications of Abstract Interpretation

- Static Program Analysis [POPL '77], [POPL '78], [POPL '79] including a.o. Dataflow Analysis [POPL '79], [POPL '00], Set-based Analysis [FPCA '95], Predicate Abstraction [Manna's festschrift '03], ...
- Syntax Analysis [TCS 290(1) 2002]
- Hierarchies of Semantics (including Proofs) [POPL '92], [TCS 277(1–2) 2002]
- Typing & Type Inference [POPL '97]

A few applications of Abstract Interpretation (Cont'd)

– **(Abstract) Model Checking** [POPL '00]

– **Program Transformation** [POPL '02]

– **Software Watermarking** [POPL '04]

– **Bisimulations** [RT-ESOP '04]

– ...

All these techniques involve sound approximations that can be formalized by abstract interpretation

A Practical Application of Abstract Interpretation to the ASTRÉE Static Analyzer

Reference

[1] http://www.astree.ens.fr/ P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, X. Rival

Programs analysed by ASTRÉE

– **Application Domain:** large safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.

– **C programs:**

– <u>with</u>

- basic numeric datatypes, structures and arrays
- pointers (including on functions),
- floating point computations
- tests, loops and function calls
- limited branching (forward `goto`, `break`, `continue`)

- without
  - `union` (new memory model in progress[4])
  - dynamic memory allocation
  - recursive function calls
  - backward branching
  - conflicting side effects
  - C libraries, system calls (parallelism)

4 Thanks A. Miné

Concrete Operational Semantics

- International norm of C (ISO/IEC 9899:1999)
- *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. encoding of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific user requirements (e.g. volatile environment specified by a *trusted configuration file*, assert, execution stops on first runtime error [5],)

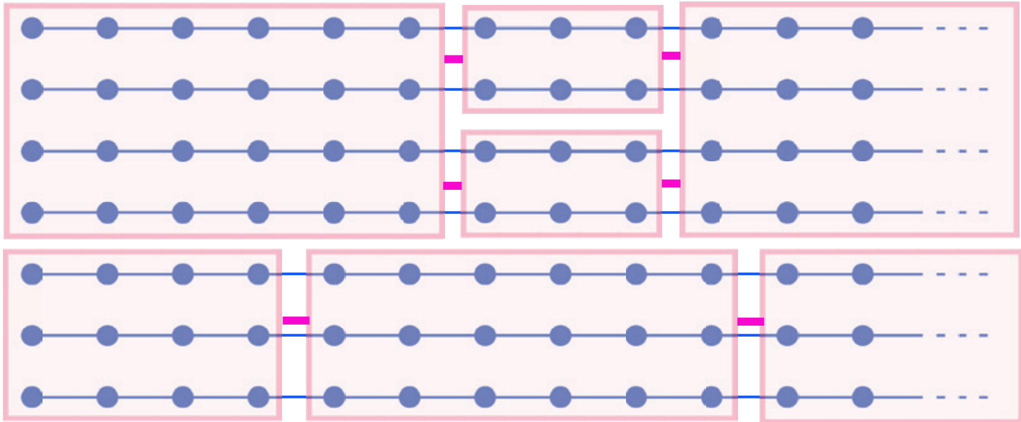
[5] semantics of C unclear after an error, equivalent if no alarm

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds, division by zero)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767, no float NaN)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Abstraction

– Set of traces of relational state abstractions of subtraces for the concrete trace operational semantics



Requirements on the Abstract Semantics

- Soundness: absolutely essential for verification
- Precision: few or no false alarm [6] (full certification)
- Efficiency: rapid analyses and fixes during development

6 Potential runtime error signaled by the analyzer due to overapproximation but impossible in any actual program run compatible with the configuration file.

Example of Industrial applications

– Primary flight control software of the Airbus A340 family/A380 fly-by-wire system

– C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)

– A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays

– A380: $\times$ 3/7 (up to 1.000.000 LOCs)

Characteristics of the ASTRÉE Analyzer

Static: compile time analysis ($\neq$ run time analysis Rational Purify, Parasoft Insure++)

Program Analyzer: analyzes programs not micromodels of programs ($\neq$ PROMELA in SPIN or Alloy in the Alloy Analyzer)

Automatic: no end-user intervention needed ($\neq$ ESC Java, ESC Java 2)

Sound: covers the whole state space ($\neq$ MAGIC, CBMC) so never omit potential errors ($\neq$ UNO, CMC from coverity.com) or sort most probable ones ($\neq$ Splint)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains ($\neq$ symbolic constraints in Bane or the canonical abstraction of TVLA)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing ($\neq$ model checking based analyzers such as VeriSoft, Bandera, Java PathFinder)

Efficient: always terminate ($\neq$ counterexample-driven automatic abstraction refinement BLAST, SLAM)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) ($\neq$ general-purpose analyzers PolySpace Verifier)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in C Global Surveyor)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

Characteristics of the ASTRÉE Analyzer (Cont'd)

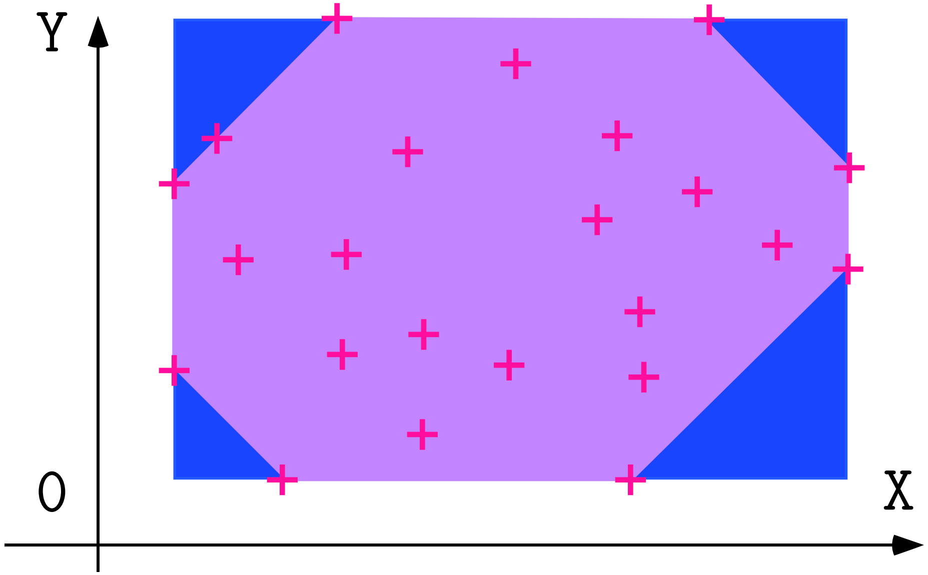
Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of O-CAML modules from a collection, each module implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain ⟶ it is a VERIFIER!

Examples of Abstractions

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [11]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [POPL '77, 11, 12]

Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

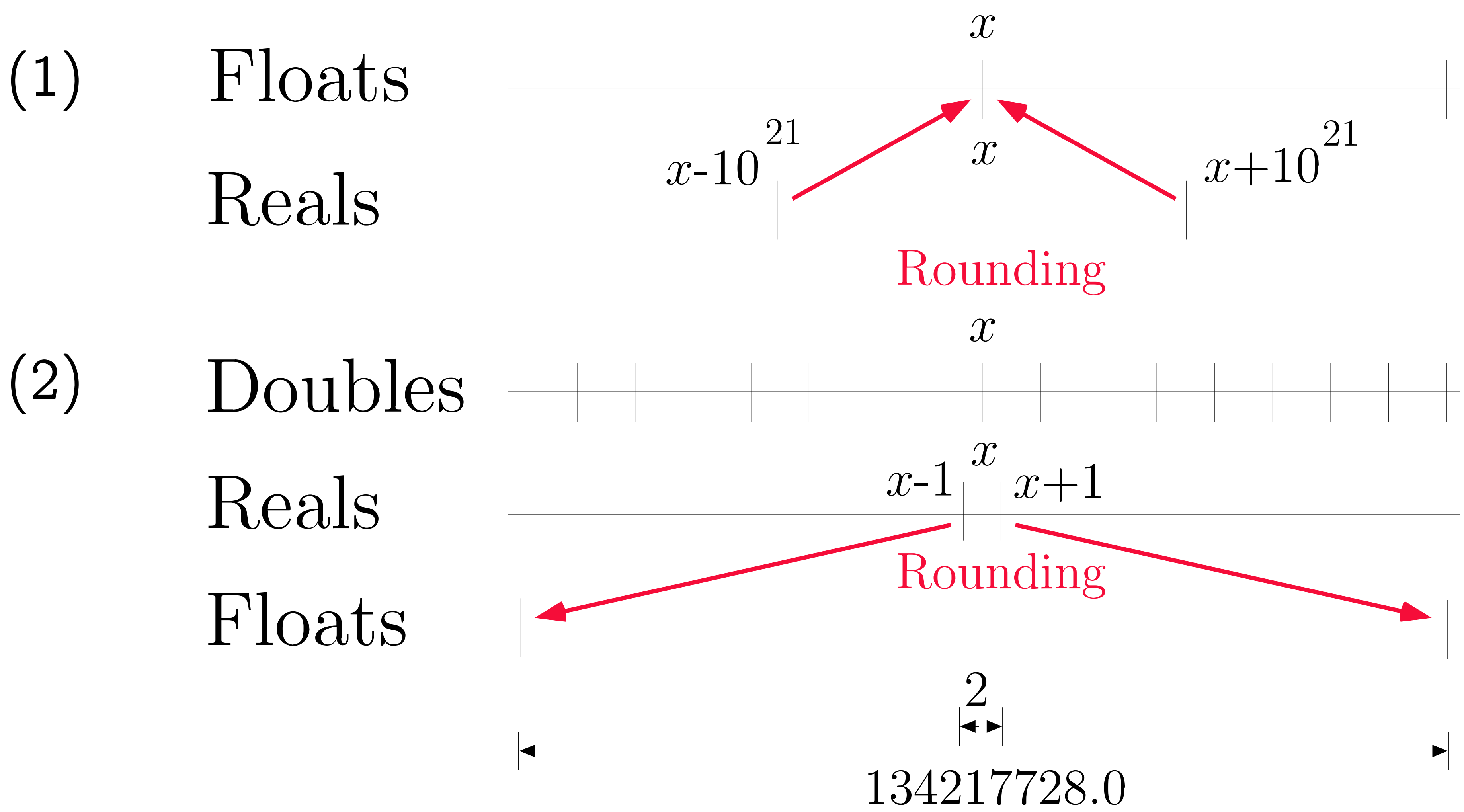
Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951487.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

Explanation of the huge rounding error



Floating-point linearization [12, 13]

- Approximate arbitrary expressions in the form
  $[a_0, b_0] + \sum_k([a_k, b_k] \times V_k)$
- Example:
  `Z = X - (0.25 * X)` is linearized as
  $\mathtt{Z} = ([0.749\cdots, 0.750\cdots] \times \mathtt{X}) + (2.35\cdots 10^{-38} \times [-1, 1])$
- Allows simplification even in the interval domain
  if $\mathtt{X} \in [\text{-}1,1]$, we get $|\mathtt{Z}| \leq 0.750\cdots$ instead of $|\mathtt{Z}| \leq 1.25\cdots$
- Allows using a relational abstract domain (octagons)
- Example of good compromize between cost and precision

Symbolic abstract domain [12, 13]

- Interval analysis: if $x \in [a, b]$ and $y \in [c, d]$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
     1  void main () { int X, Y;
     2          __ASTREE_known_fact(((0 <= X) && (X <= 100)));
     3          Y = (X - X);
     4          __ASTREE_log_vars((Y));
     5  }
```

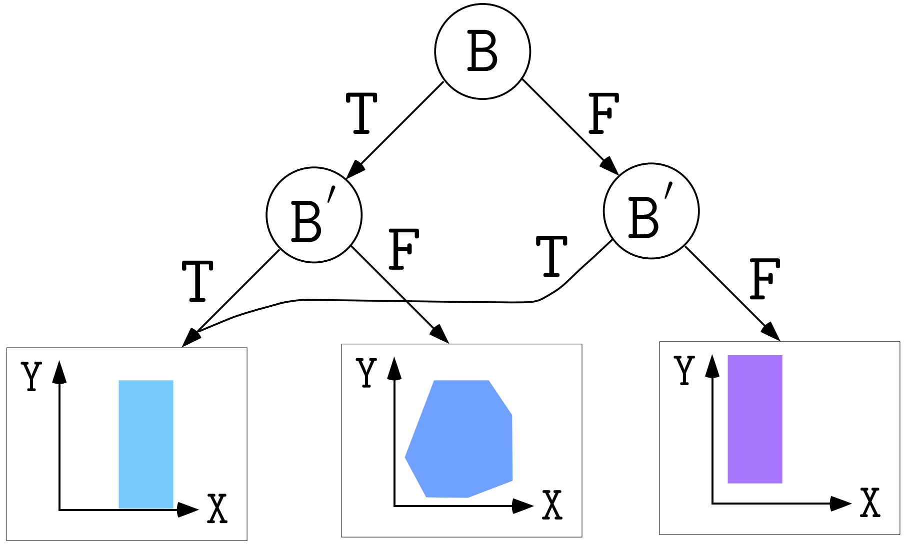
```
astree -exec-fn main -no-relational x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```

Boolean Relations for Boolean Control

– Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



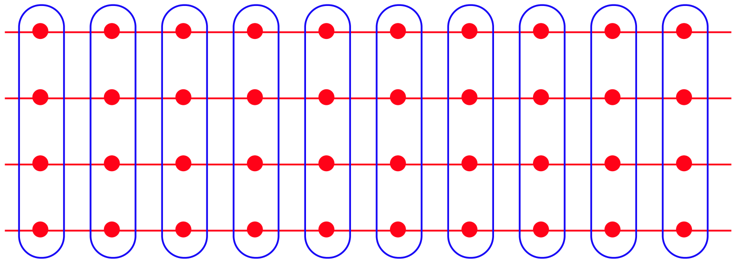
The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Control Partitionning for Case Analysis

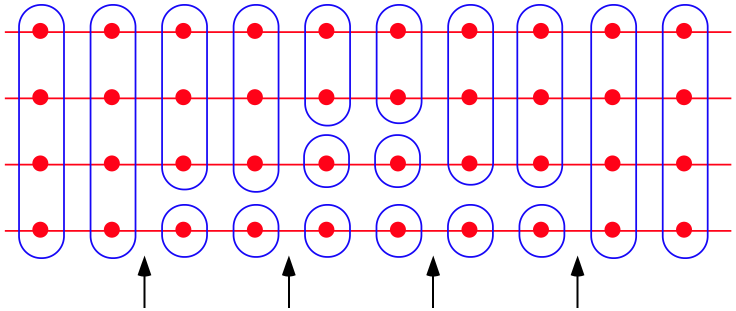
—Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant −100 ≤ x ≤ 100 ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:



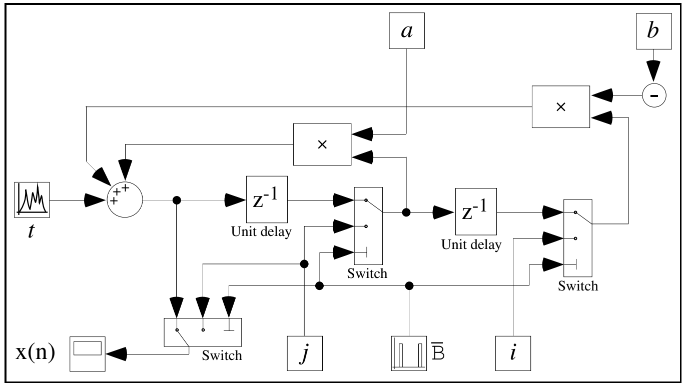
Trace partitionning:



Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

Ellipsoid Abstract Domain for Filters

2[d] Order Digital Filter:

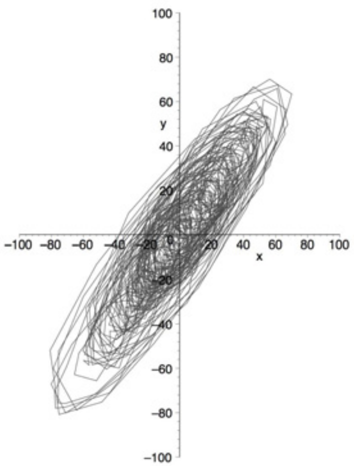


– Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$

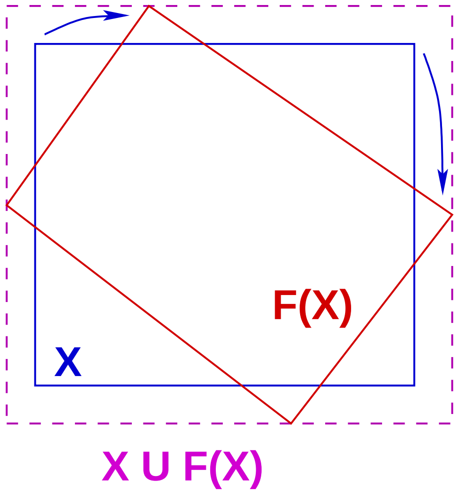
– The concrete computation is bounded, which must be proved in the abstract.

– There is no stable interval or octagon.

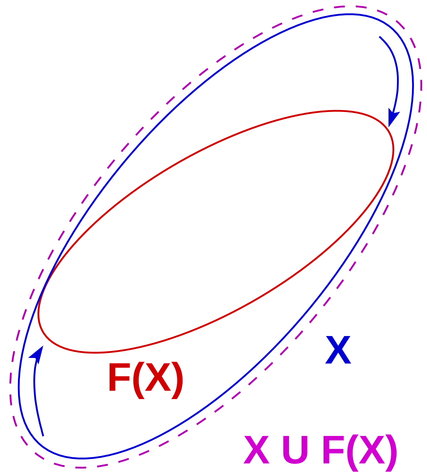
– The simplest stable surface is an ellipsoid.



execution trace



unstable interval



stable ellipsoid

Filter Example [8]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
              + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```

Arithmetic-geometric progressions [7] [9]

– Abstract domain: $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \longmapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') =$$
$$\{f \mid \forall k \in \mathbb{N} : |f(k)| \leq \left(\lambda x \cdot ax + b \circ (\lambda x \cdot a'x + b')^k\right)(M)\}$$

i.e. any function bounded by the arithmetic-geometric progression.

[7] here in $\mathbb{R}$

Arithmetic-Geometric Progressions (Example 1)

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
  R = 0;
  while (TRUE) {
    __ASTREE_log_vars((R));
    if (I) { R = R + 1; }          ← potential overflow!
    else { R = 0; }
    T = (R >= 100);
    __ASTREE_wait_for_clock(());
  }}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

Arithmetic-geometric progressions (Example 2)

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P =  (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
```

```
|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <=
23.0393526881
```

Integrating Physical Systems in Static Analysis

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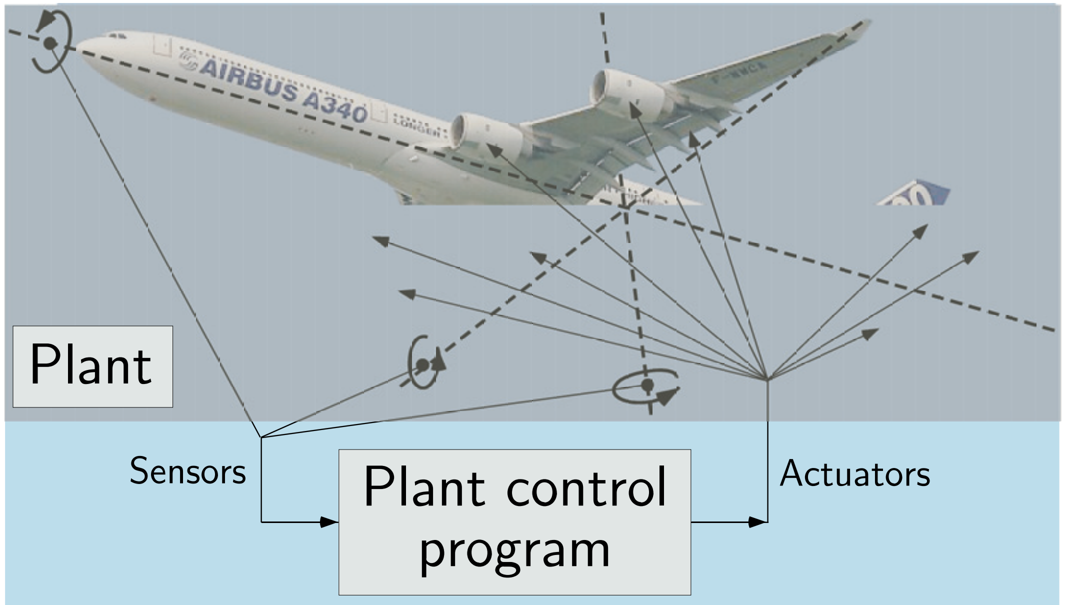
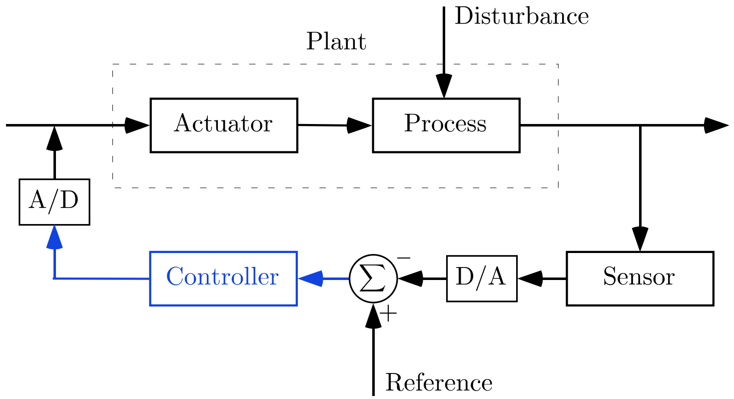
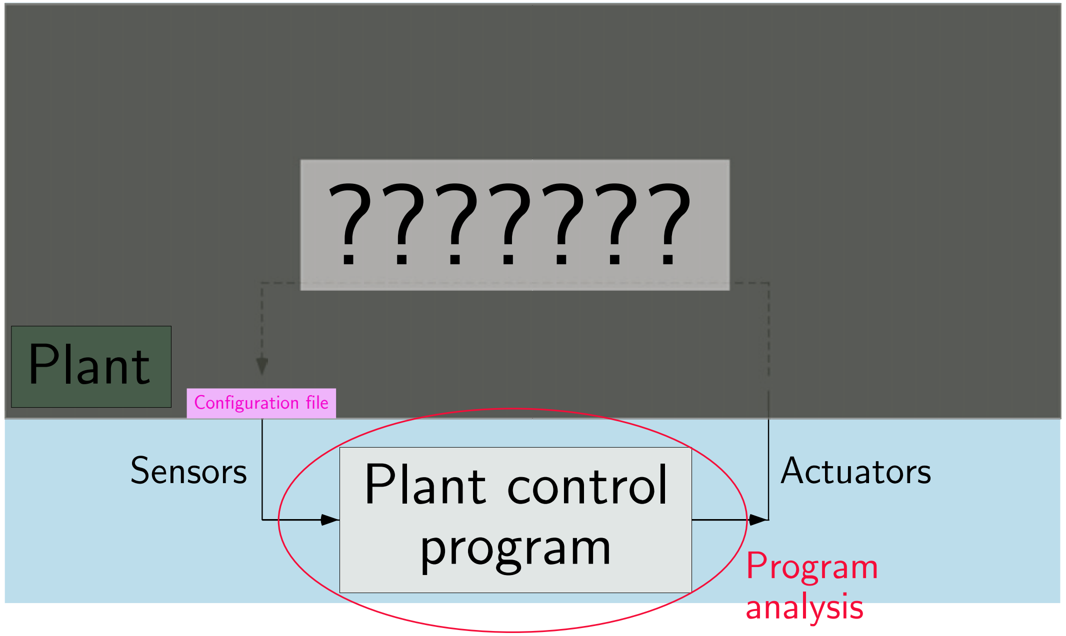
Computer controlled systems





Software analysis & verification with ASTRÉE



Abstractions: program → precise, system → coarse

Software analysis & verification with ASTRÉE

- Exhaustive: 100% coverage of RTE
- Can be made precise by specialization [8] to get no false alarm (so, the program does not go wrong whatever are the inputs [9]!)
- No specification of the controlled system (but for ranges of values of a few sensors [10])
- Impossible to prove essential properties of the controlled system (e.g. controlability, stability)

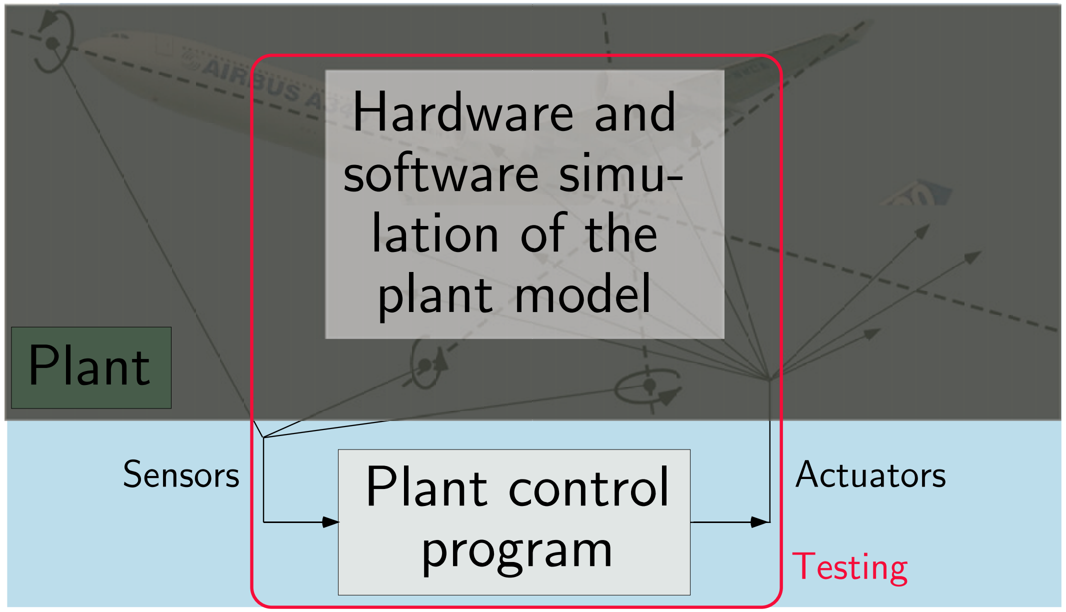
---

8 To specific families of properties and programs

9 but for a few inputs ...

10 ... specified in the *trusted configuration file*

State-of-the-art testing of the plant control program

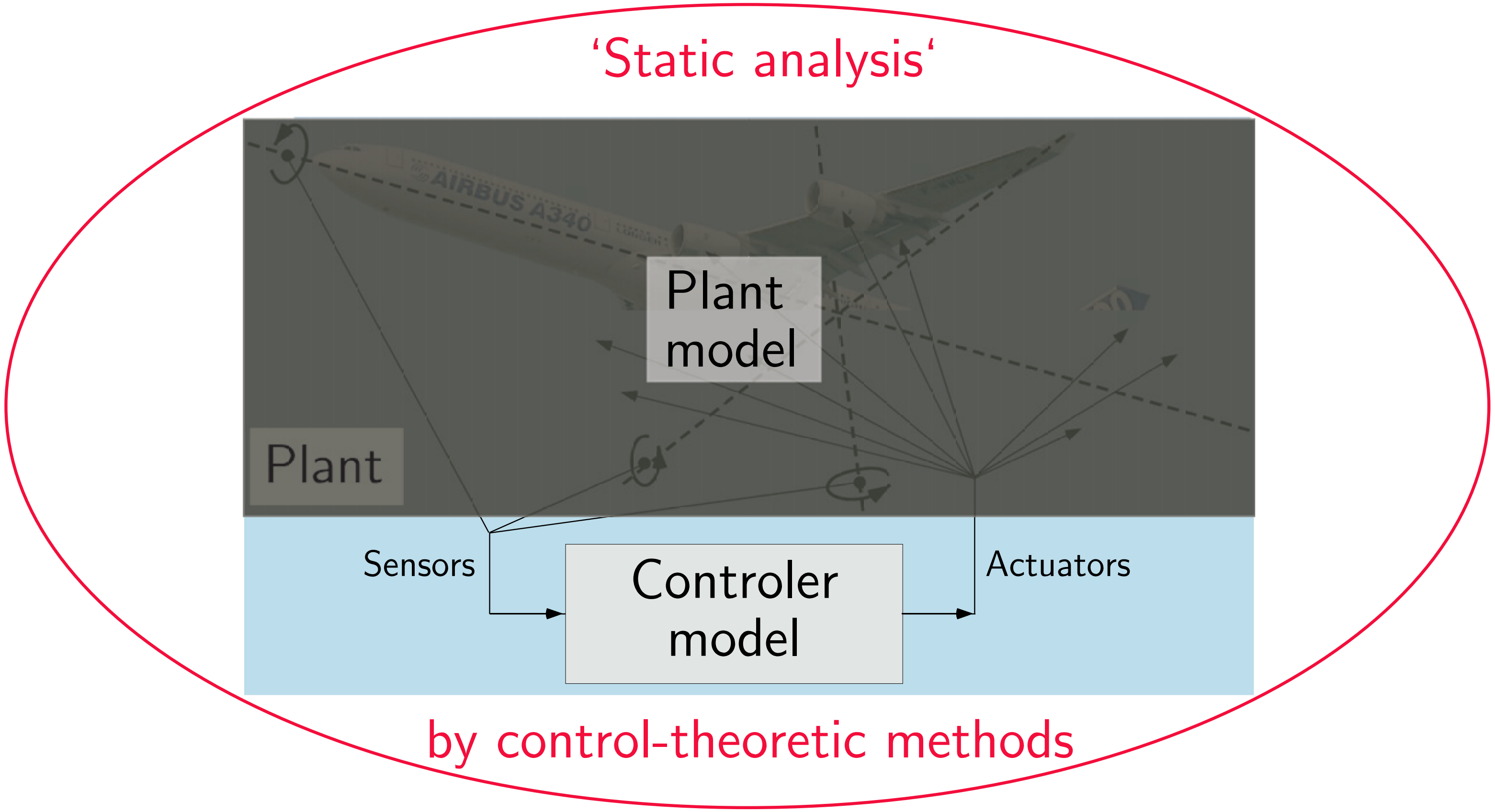


Abstractions: program → none, system → precise

State-of-the-art testing of the plant control program

- Extremely heavy and expensive (e.g. iron bird)
- Not exhaustive
- Extended during plant test period (e.g. certification flight tests)
- Late discovery of errors can delay the delivery by months (the whole software development process must be re-checked)

System analysis & verification by control engineers



Abstractions: program → imprecise, system → precise (for control laws only)

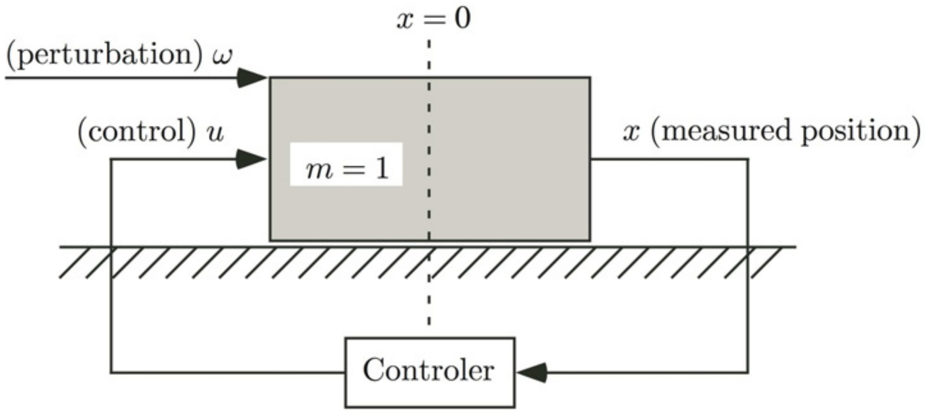
System analysis & verification by control engineers

- The controler model is a rough abstraction of the control program:
  - Continuous, not discrete
  - Limited to control laws
  - Does not take into account fault-tolerance to failures and computer-related system dependability.
- In theory, SDP-based search of system invariants (Lyapunov-like functions) can be used to prove reachability and inevitability properties
- Does not scale up (e.g. over long periods of time)

– In practice, the system/controler model is explored by discrete simulations (testing)

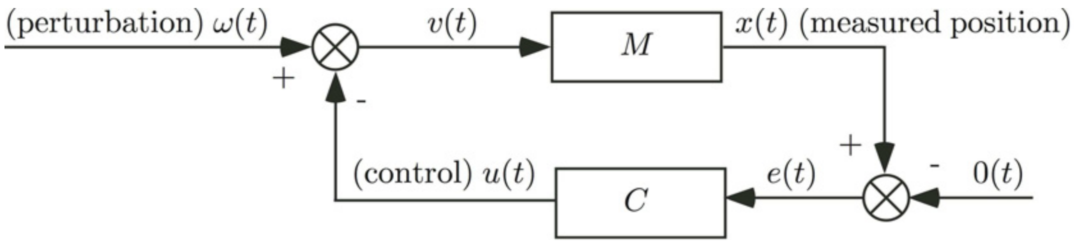
A simple example

– Physical system:



A simple example (cont'd)

– Block diagram representation:



– Evolution of this system with time t[11]:

$$\begin{cases} x(t) = M(\omega|_{[0,t]} - u|_{[0,t]})(t) & t \in [0, +\infty[\\ u(t) = C(x|_{[0,t]})(t) \end{cases}$$

– The transfer function M is known through the differential equation of motion given by Newton's law:

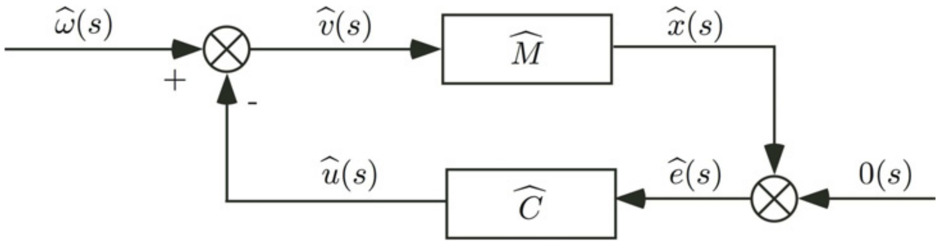
$$m\frac{d^2}{dt^2}x(t) = \omega(t) - u(t) \ .$$

where $m = 1kg$.

[11] $f|_{[a,b]}$ is the restriction of function f to the interval $[a, b]$. So the system has the ability to record the past evolution from time 0.

A simple example (cont'd)

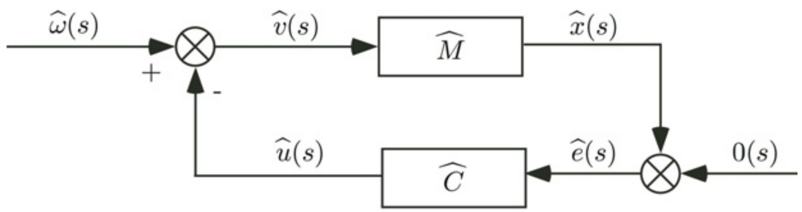
– Laplace transform (to transform differential equations into algebraic equations [12]):



[12] The Laplace transform of $\hat{f}(s)$ of $f(t)$ (also denoted $\mathcal{L}[f(t)]$) is the partial function $\hat{f} = \lambda s \in \mathbb{C} \cdot \int_0^\infty f(t)e^{-st}dt$ of the complex variable s. The Laplace transform is linear in that $a\widehat{f(t) + b}g(t) = \lambda s \cdot a\hat{f}(s) + b\hat{g}(s)$. For differentiation, $\widehat{\frac{d}{dt}f(t)} = \lambda s \cdot s\hat{f}(s) - f(0)$ and so $\widehat{\frac{d^2}{dt^2}f(t)} = \lambda s \cdot s^2\hat{f}(s) - s\frac{d}{dt}f(0) - f(0)$ if $f(t)$ is continuously differentiable in $[0, \infty[$.

A simple example (cont'd)

– Phase lead controler [13]:



$$\hat{C}(s) = \frac{\hat{u}(s)}{\hat{e}(s)} = k\frac{s+K_z}{s+K_p}$$

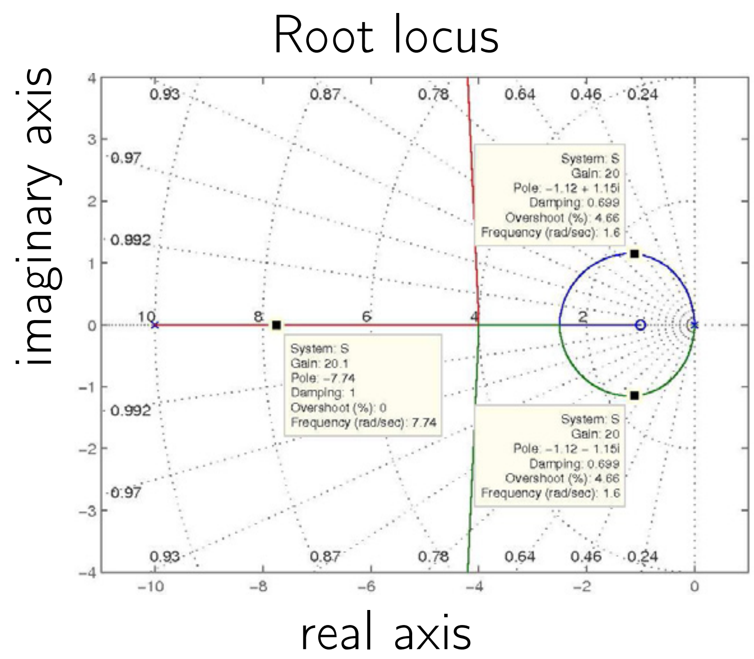
with $K_p > K_z$, for example:

$$\hat{C}(s) = \frac{\hat{u}(s)}{\hat{e}(s)} = k\frac{s+1}{s+10}\ .$$

[13] Well-chosen among many possibilities such as proportional, derivative, integral, lead compensation, lead compensation with proportional integral correction, ... controllers

A simple example (cont'd)

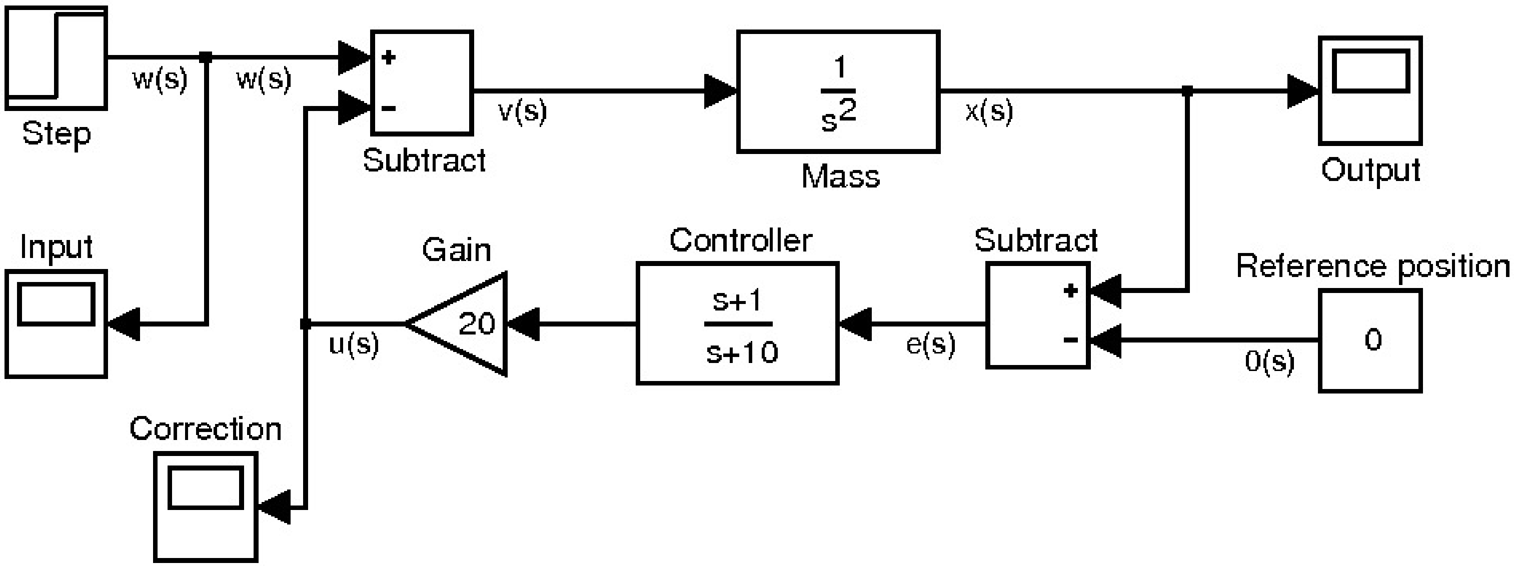
– Design of the controler parameter k (e.g. by Evans' root locus method with Matlab™ [14]):



[14] The choice of k is a compromise between larger negative real parts of the complex roots/eigenvalues to improve stability and large gains to improve speed of reaction but may lead to instability

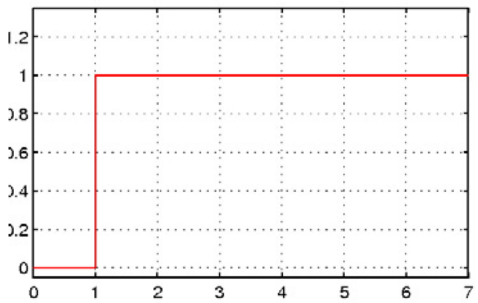
A simple example (cont'd)

– Simulation (Simulink™ continuous model of the system):

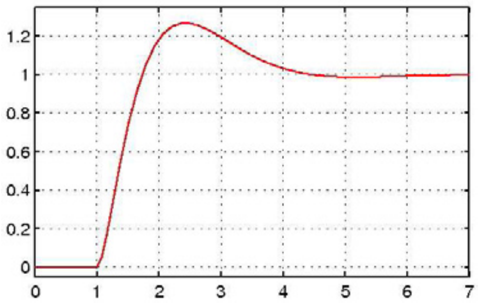


A simple example (cont'd)

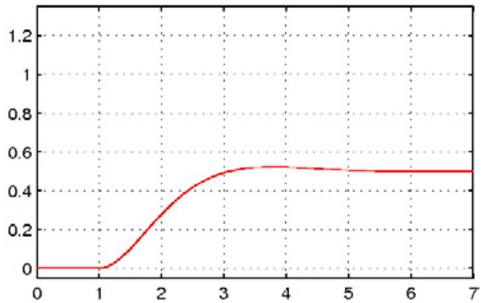
– Response *testing* by simulation (e.g. response to a step input of 1 N):



Input $\omega(s)$



Correction $u(s)$



Output $x(s)$

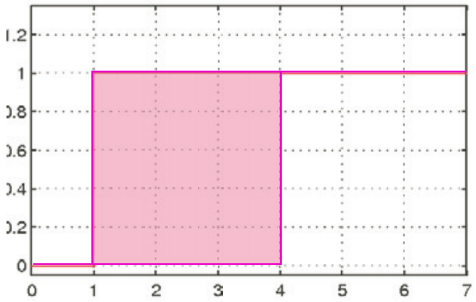
A simple example (cont'd)

– System (plant+control) discrete simulation program (e.g. $\Delta t = \frac{1}{100}$ s):

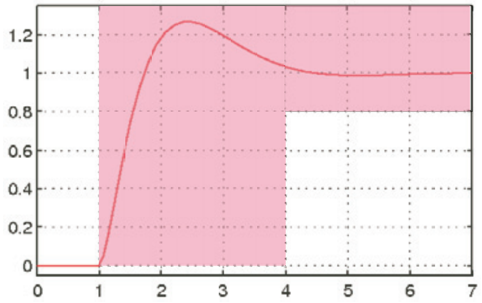
```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT;
static float U, X, Z, E;
volatile float W;
const float Dt = 0.01;
const float K = 20.0; /* controller gain */

void control ()
{  float X_1;
   if (INIT) {
      U = 0.0;
      X = 0.0;
      Z = -X;
      E = 0.0;
   } else {
      X_1 = X;
      U = K*(E-Z);
      X = X_1 - K*Dt*E + K*Dt*Z + Dt*W;
      Z = (1.0 - 10.0*Dt)*Z + 9.0*Dt*E;
      E = E + Dt*X_1;
   }
}

void main()
{
   INIT = TRUE;
   while (TRUE) {
      control();
      INIT = FALSE;
      wait_for_clock();
   }
}
```

A simple example (cont'd)

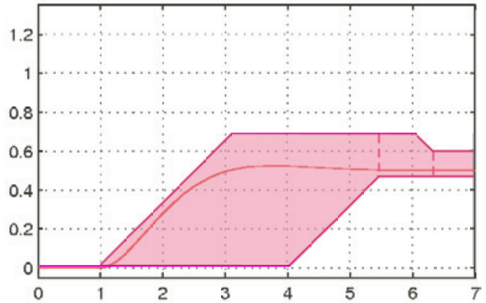
– *Abstract response analysis* by abstract interpretation:



Abstract input $\omega(s)$



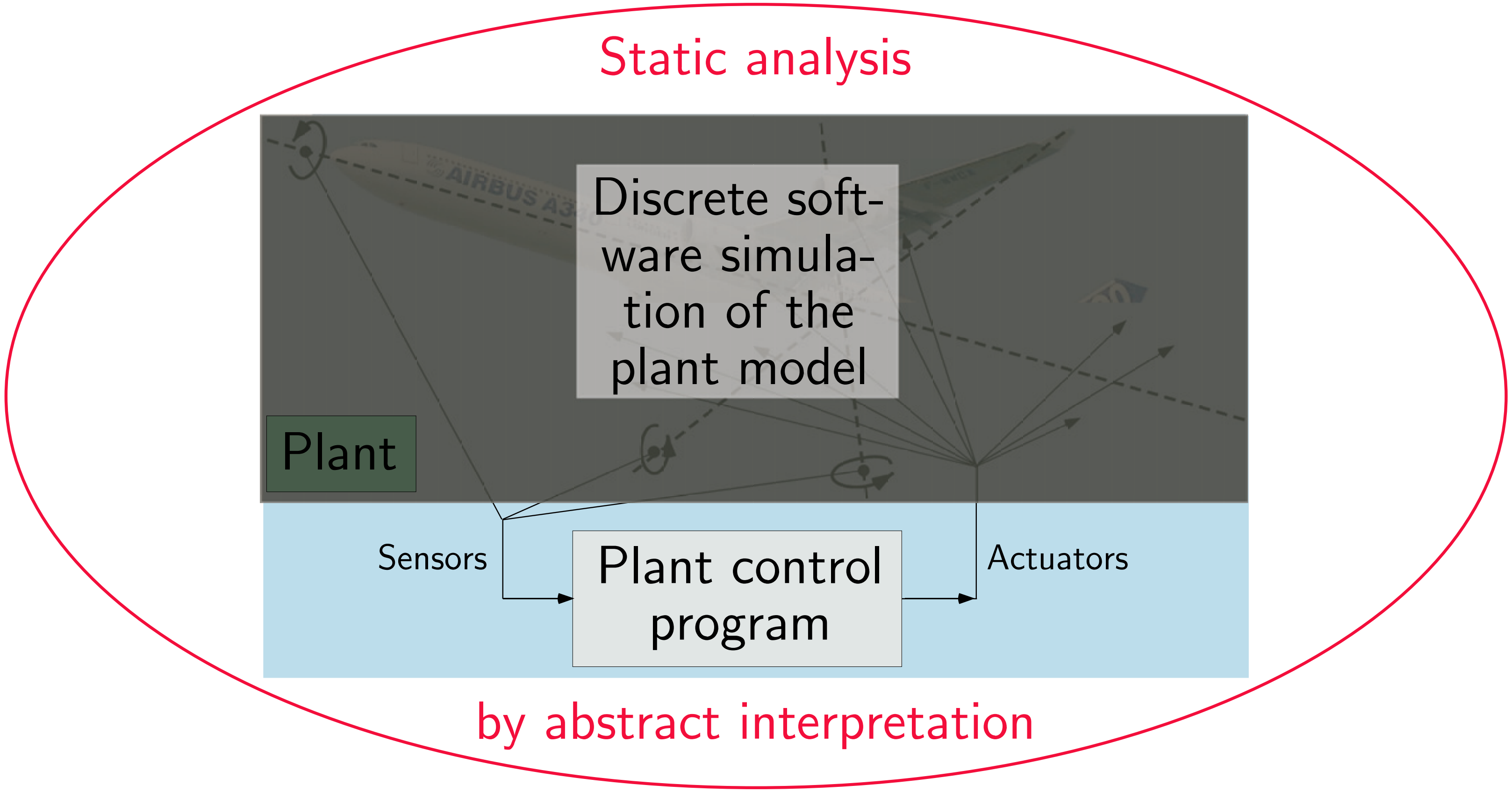
Abstract correction $u(s)$



Abstract output $x(s)$

Exploring new avenues in static analysis

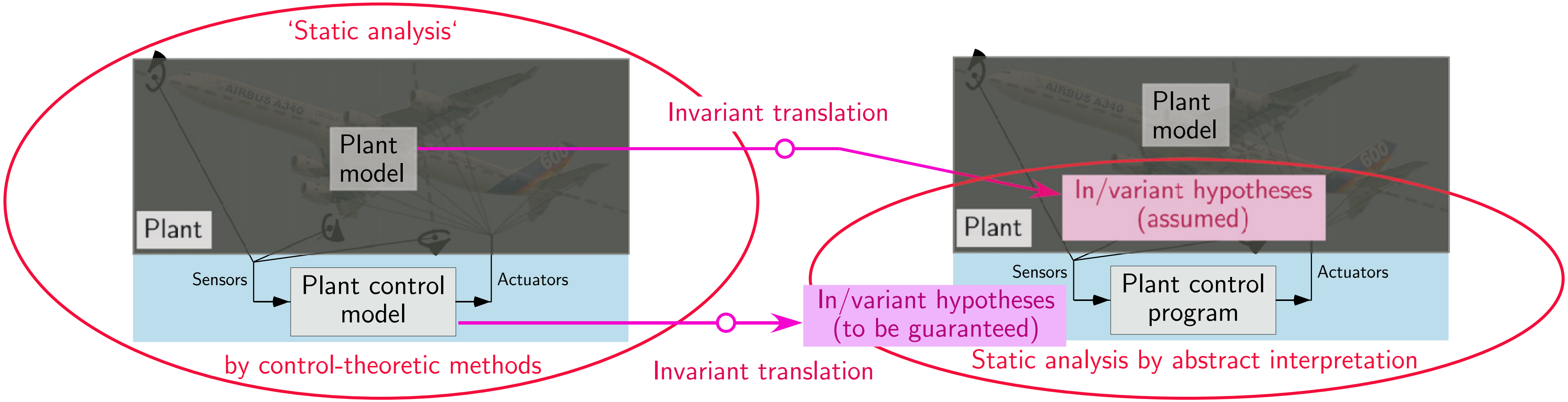
System analysis & verification, Avenue 1



Abstractions: program $\rightarrow$ precise, system $\rightarrow$ precise

- Exhaustive (contrary to current simulations)
- The plant model discretization errors are similar to those of simulation methods (but for the use of the *actual* control program instead of a model!)
- In general, polyhedral abstractions are unstable or of very high complexity
- New abstractions have to be studied (e.g. ellipsoidal abstractions)!

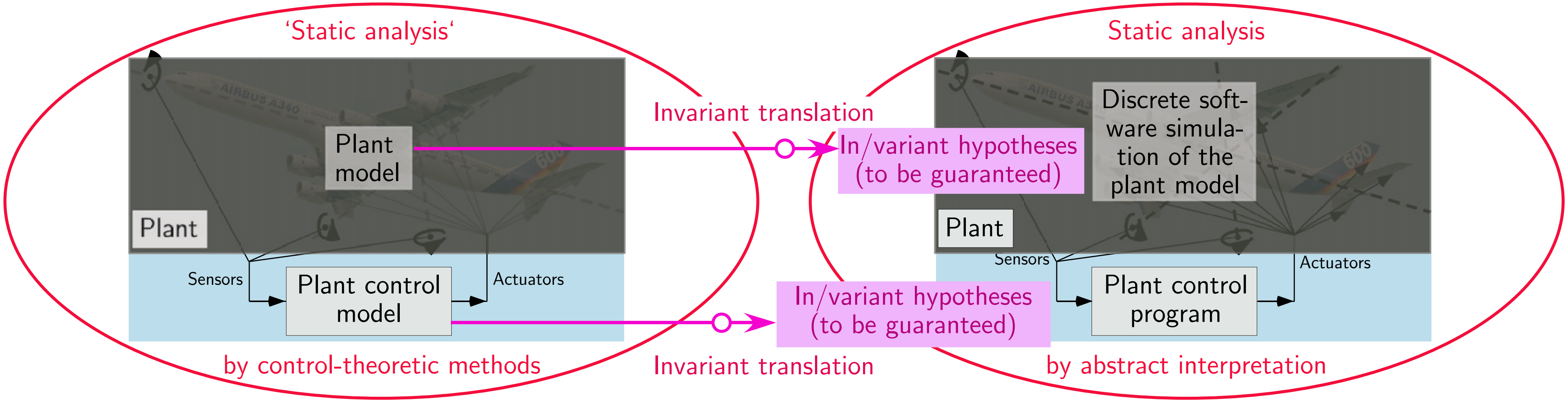
System analysis & verification, Avenue 2



Abstractions: program → precise, system → precise

- The control-theoretic ‘static analysis‘ is easier on the plant/controller model using continuous optimization methods
- The in/variant hypotheses on the controlled plant are assumed to be true in the analysis of the plant control program
- It is now sufficient to perform the analysis analysis control program under these in/variant hypotheses
- The results can then be checked on the whole system (plant simulation + control program)

System analysis & verification, Avenue 3



Abstractions: program → precise, system → precise

- The translated in/variants can be checked for the plant simulator/control program (easier than in/variant discovery)
- Should scale up (since these complex in/variants are relevant to a small part of the control program only [15])

[15] e.g. the plant model assumes perfect sensors/actuators/computers whereas the control program must be made dependable by using redundant failing sensors/actuators/computers

Conclusion

Conclusions

1. On soundness and completeness:
   – Software checking (e.g. [abstract] testing): unsound
   – Software static analysis (for a language): sound but unprecise
   – Software verification (for a well-defined family of programs): theoretically possible [SARA '00], practically feasible [PLDI '03]

Reference

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Conclusions (cont'd)

2. On specifications for static verification:
   - Implicit: e.g. from a language semantics (e.g. RTE) → extremely easy for engineers
   - Explicit:
     - By a logic → very hard for engineers
     - By a model → easy for engineers / hard for static analysis
     - By a program automatically generated from a model → easy for engineers / easy for static analysis

THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.

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