Design of Syntactic Program Transformations by Abstract Interpretation of Semantic Transformations

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A Short Introduction to Abstract Interpretation

Abstract Interpretation

- Formalizes the idea of approximation of sets and set operations as considered in set (or category) theory;
- Mainly applied to the approximation of the semantics of programming languages/computer systems;

The Theory of Abstract Interpretation

- **Abstract interpretation** is a theory of conservative approximation of the semantics of computer systems.

  Approximation: observation of the behavior of a computer system at some level of abstraction, ignoring irrelevant details;

  Conservative: the approximation cannot lead to any erroneous conclusion.

Usefulness of Abstract Interpretation

- **Thinking tools**: the idea of abstraction is central to reasoning (in particular on computer systems);

- **Mechanical tools**: the idea of effective approximation leads to automatic semantics-based program manipulation tools.

Abstraction

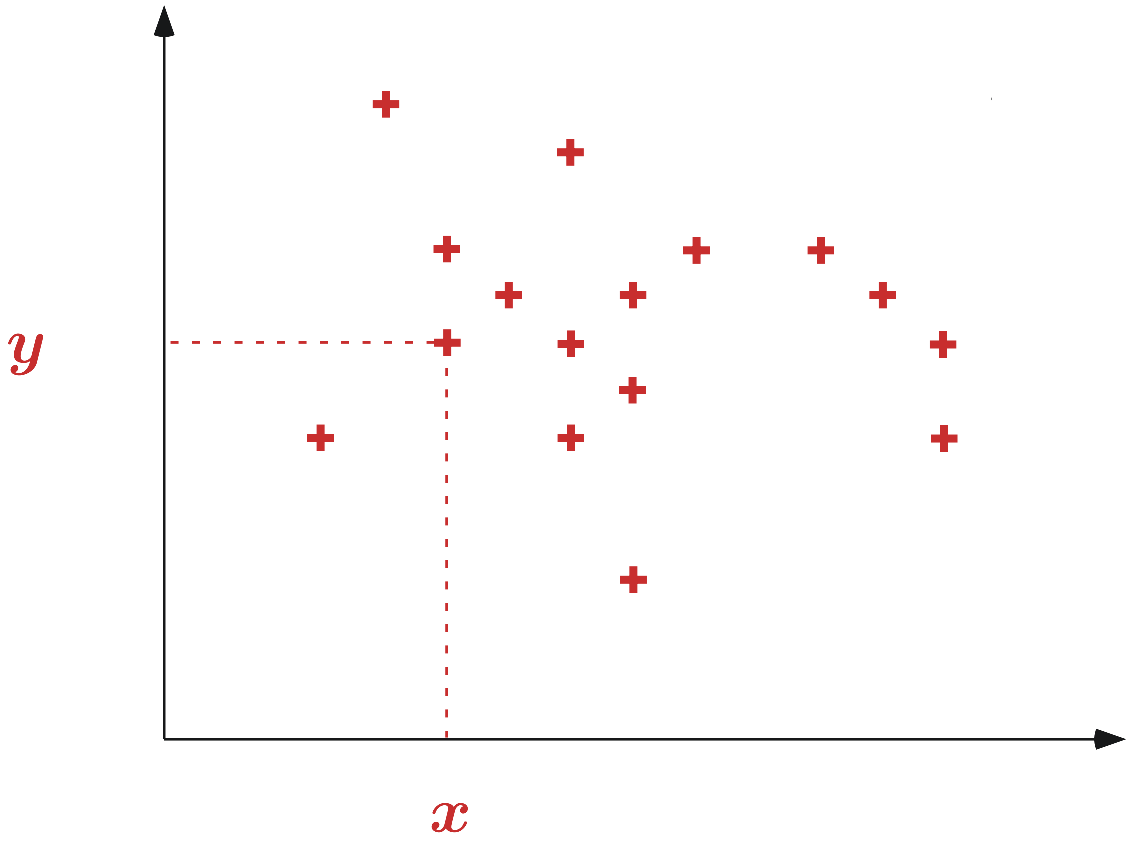
Abstraction: intuition

- Abstract interpretation formalizes the intuitive idea that a semantics is more or less precise according to the considered observation level of the program executions;

- Abstract interpretation theory formalizes this notion of approximation/abstraction in a mathematical setting which is independent of particular applications.

Intuition behind abstraction

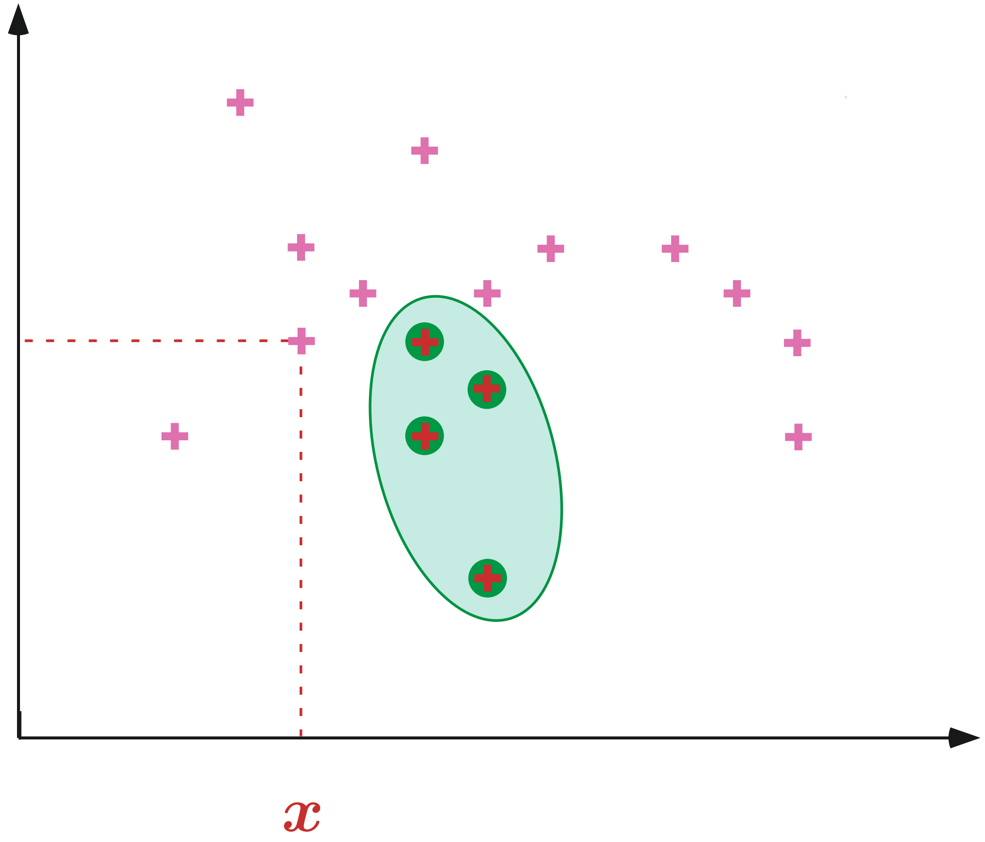
Approximations of an [in]finite set of points;



$$\{\ldots, \langle 19,\ 78\rangle, \ldots,\\ \langle 20,\ 01\rangle, \ldots\}$$

Approximations of an [in]finite set of points:

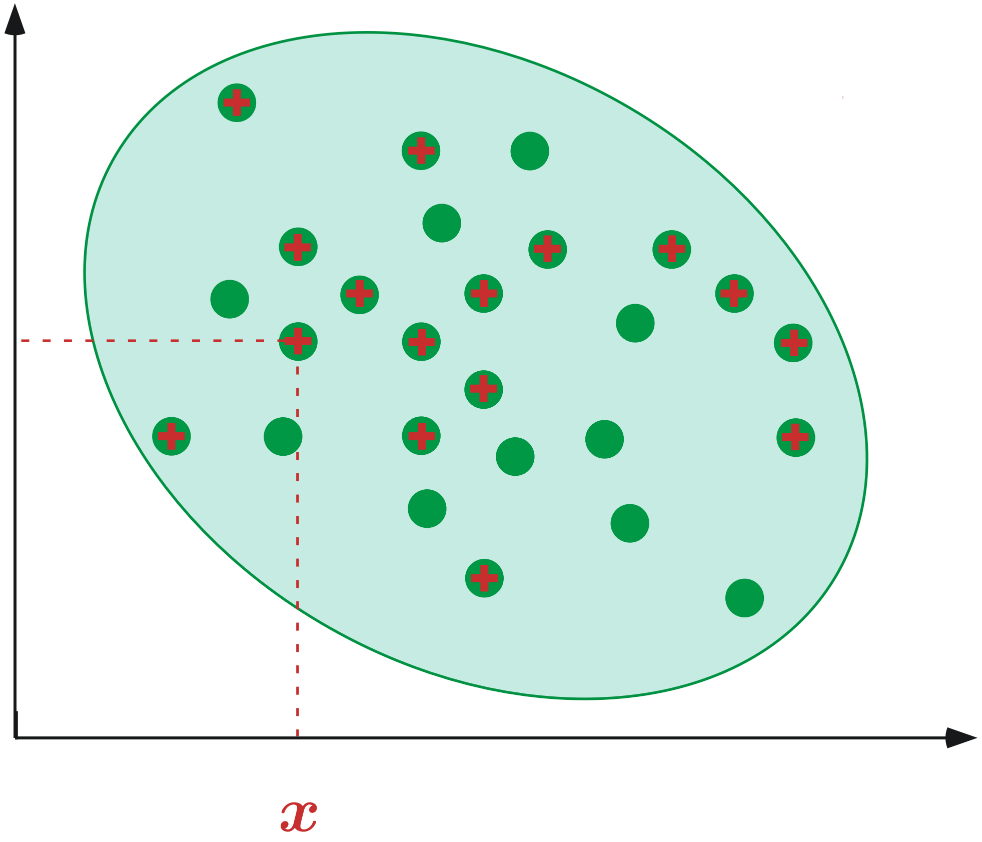
From Below



$$\{\ldots, \langle 19,\ 78\rangle, \ldots,\ \ldots\}$$

Approximations of an [in]finite set of points:

From Above

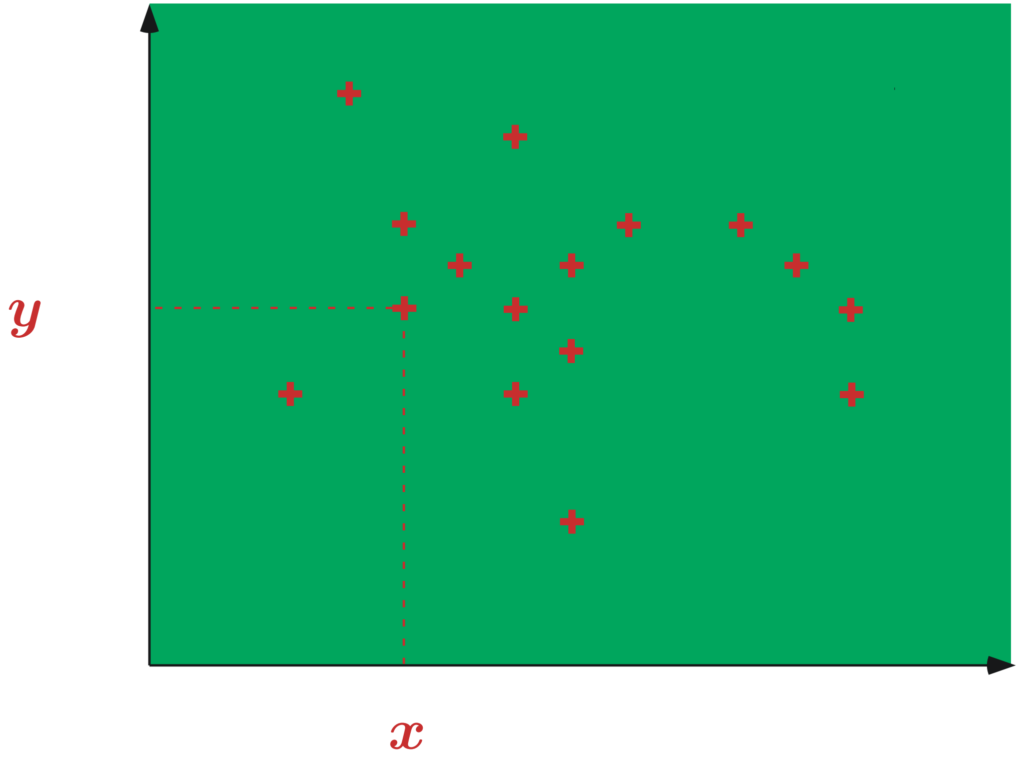


$$\{\ldots, \langle 19,\ 78\rangle, \ldots,$$

$$\langle 20,\ 01\rangle, \langle ?,\ ?\rangle, \ldots\}$$

Intuition Behind Effective Computable Abstraction

Effective computable approximations of an [in]finite set of points; Signs [1]

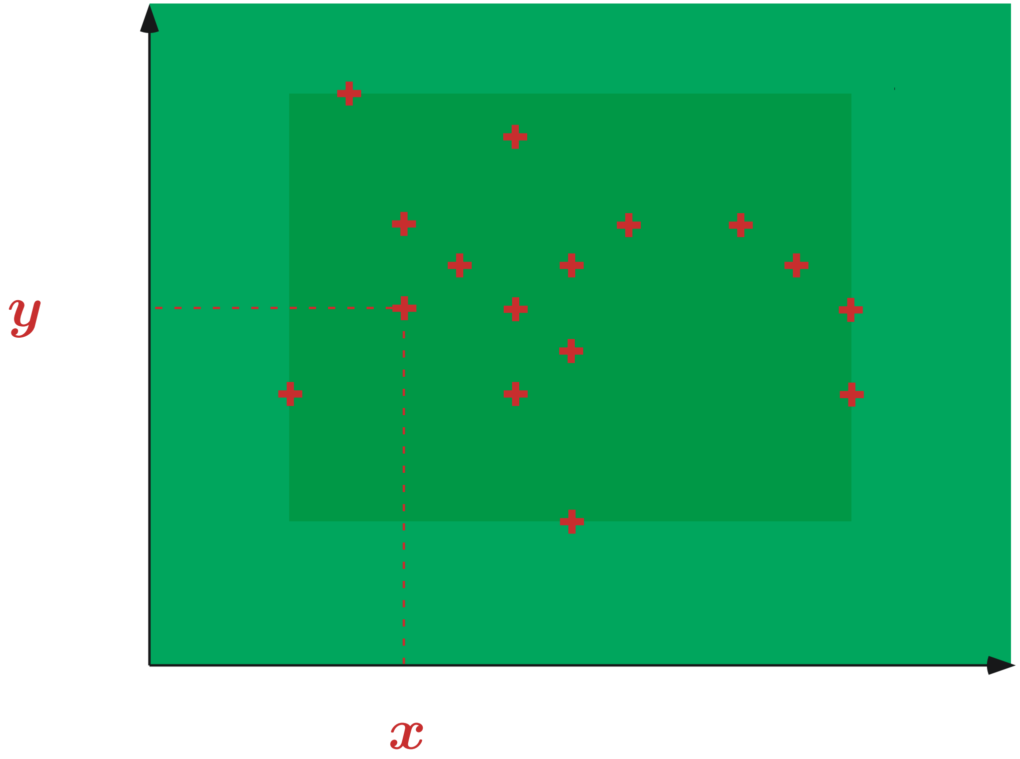


$$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$$

Reference

[1] P. Cousot and R. Cousot. Systematic design of program analysis frameworks. In *6th POPL*, pages 269–282, San Antonio, TX, 1979. ACM Press.

Effective computable approximations of an [in]finite set of points; Intervals [2]

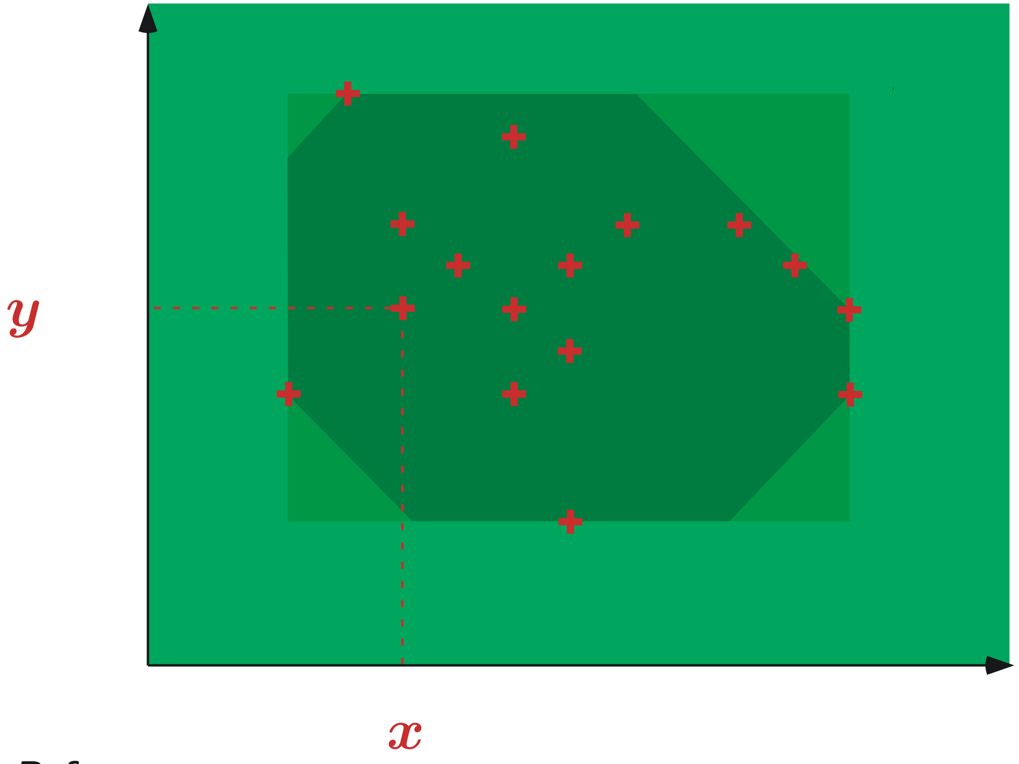


$$\begin{cases} x \in [19,\ 78] \\ y \in [20,\ 01] \end{cases}$$

Reference

[2] P. Cousot and R. Cousot. Static determination of dynamic properties of programs. In *2nd Int. Symp. on Programming*, pages 106–130. Dunod, 1976.

Effective computable approximations of an [in]finite set of points; Octagons [3]

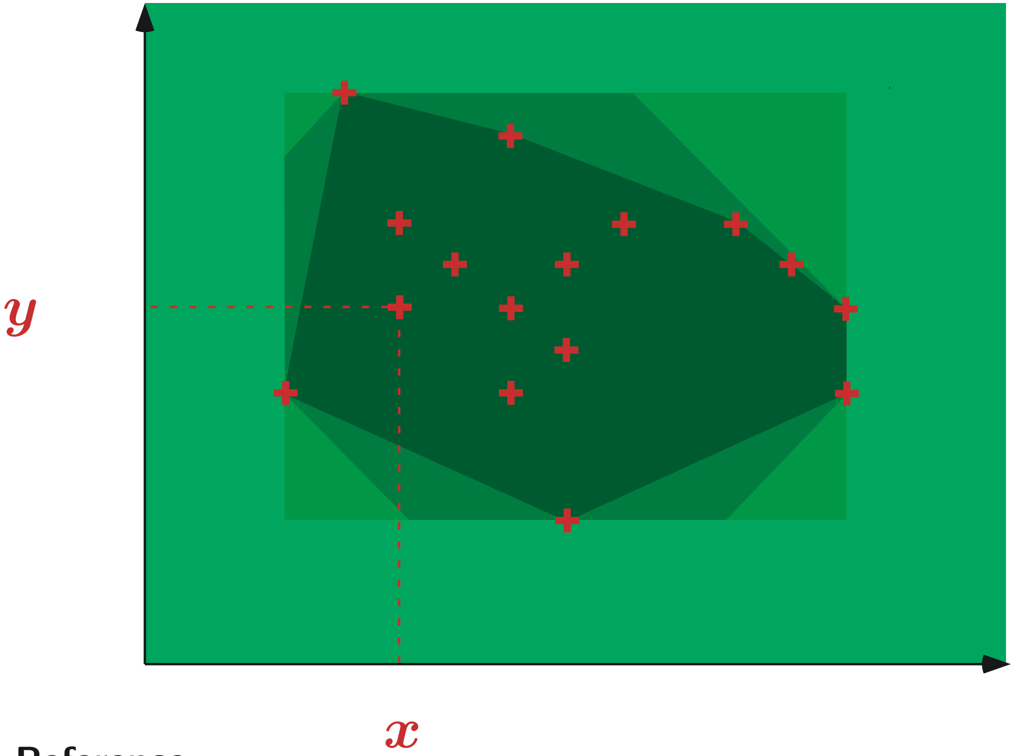


$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 78 \\ 1 \leq y \leq 9 \\ x - y \leq 99 \end{cases}$$

Reference

[3] A. Miné. A New Numerical Abstract Domain Based on Difference-Bound Matrices. In *PADO'2001*, LNCS 2053, Springer, 2001, pp. 155–172.

Effective computable approximations of an [in]finite set of points; Polyhedra [4]

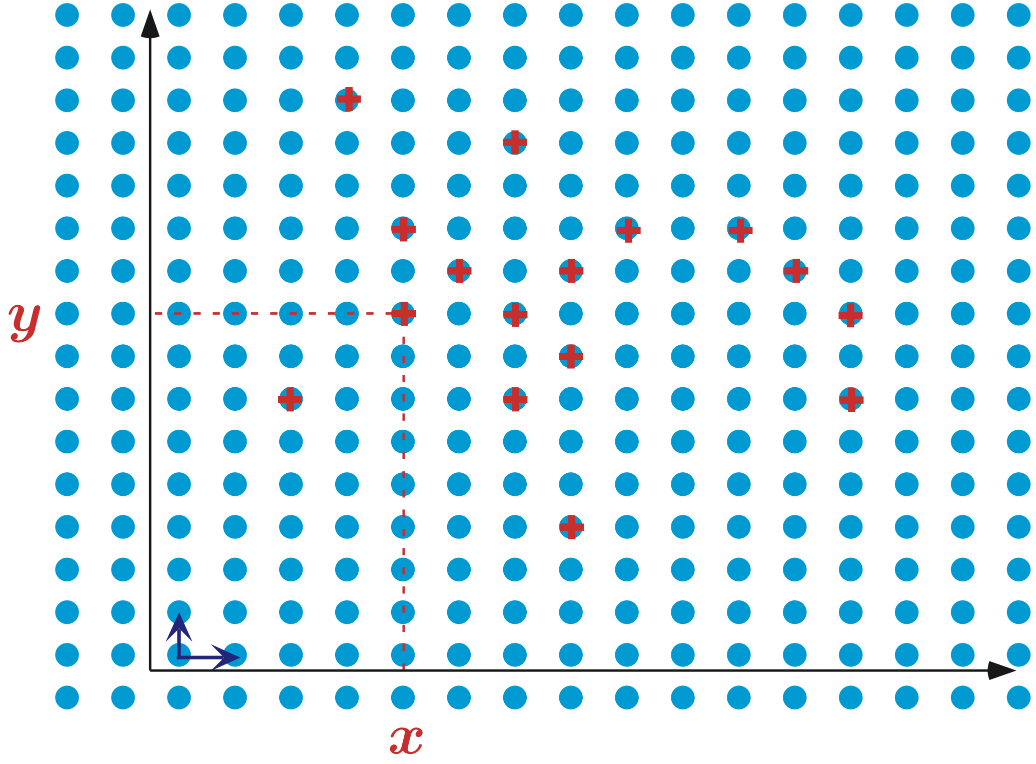


$$\begin{cases} 19x + 78y \leq 2000 \\ 20x + 01y \geq 0 \end{cases}$$

Reference

[4] P. Cousot and N. Halbwachs. Automatic discovery of linear restraints among variables of a program. In *5^th POPL*, pages 84–97, Tucson, AZ, 1978. ACM Press.

Effective computable approximations of an [in]finite set of points; Simple congruences [5]

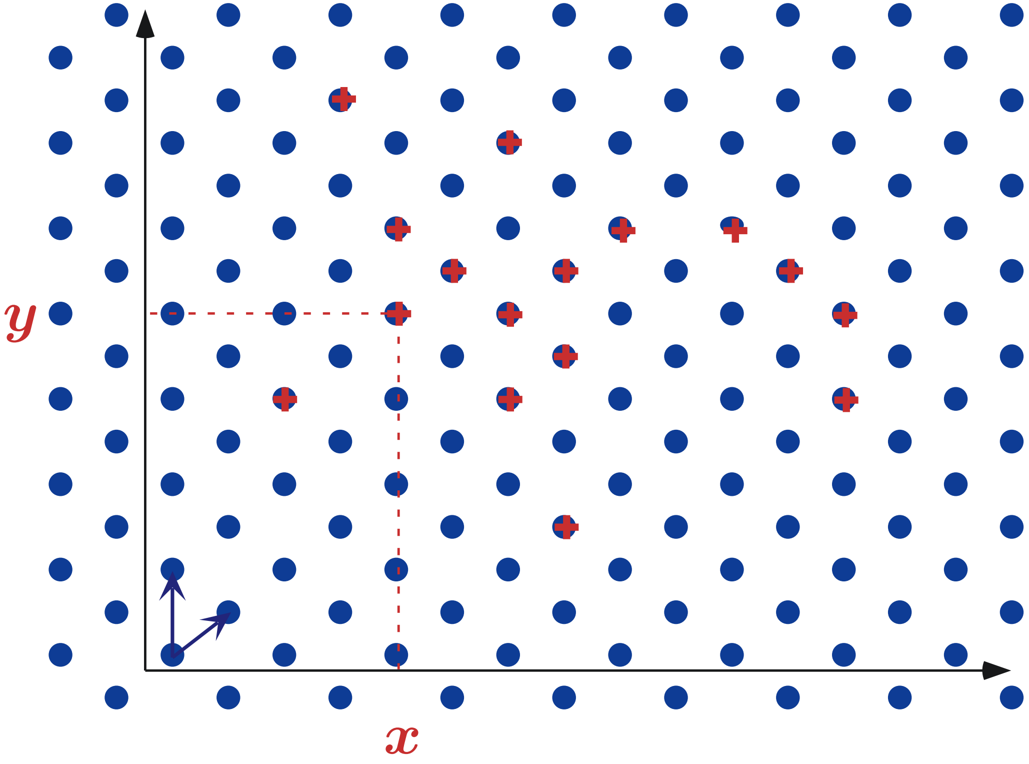


$$\begin{cases} x = 19 \bmod 78 \\ y = 20 \bmod 99 \end{cases}$$

Reference

[5] P. Granger. Static analysis of arithmetical congruences. *Int. J. Comput. Math.*, 30:165–190, 1989.

Effective computable approximations of an [in]finite set of points; Linear congruences [6]

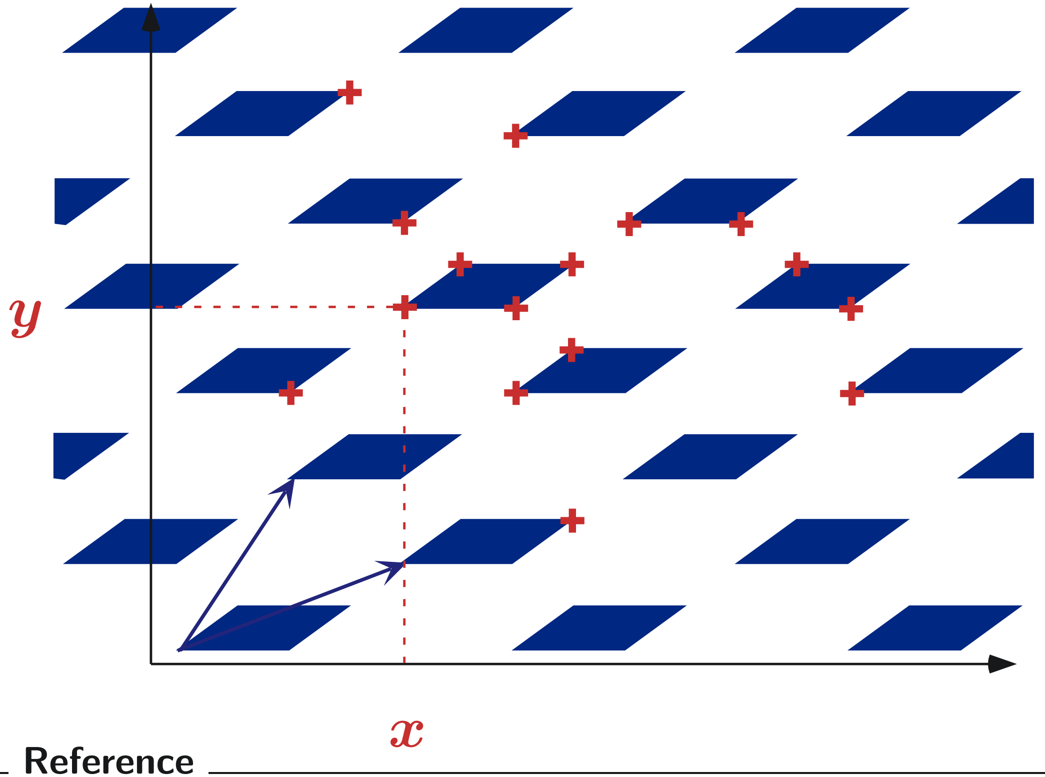


$$\begin{cases} 1x + 9y = 7 \bmod 8 \\ 2x - 1y = 9 \bmod 9 \end{cases}$$

Reference

[6] P. Granger. Static analysis of linear congruence equalities among variables of a program. *CAAP '91*, LNCS 493, pp. 169–192. Springer, 1991.

Effective computable approximations of an [in]finite set of points; Trapezoidal linear congruences [7]



$$\begin{cases} 1x + 9y \in [0, 78] \bmod 10 \\ 2x - 1y \in [0, 99] \bmod 11 \end{cases}$$

Reference

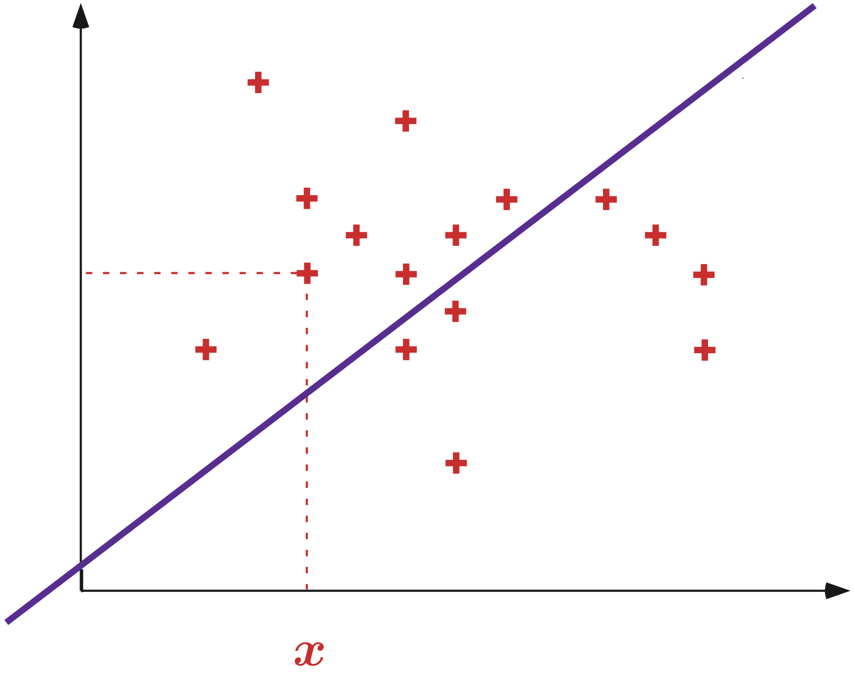
[7] F. Masdupuy. Array operations abstraction using semantic analysis of trapezoid congruences. In *ACM Int. Conf. on Supercomputing, ICS '92*, pages 226–235, 1992.

Conservative Approximation and Information Loss

Intuition Behind Sound/Conservative Approximation

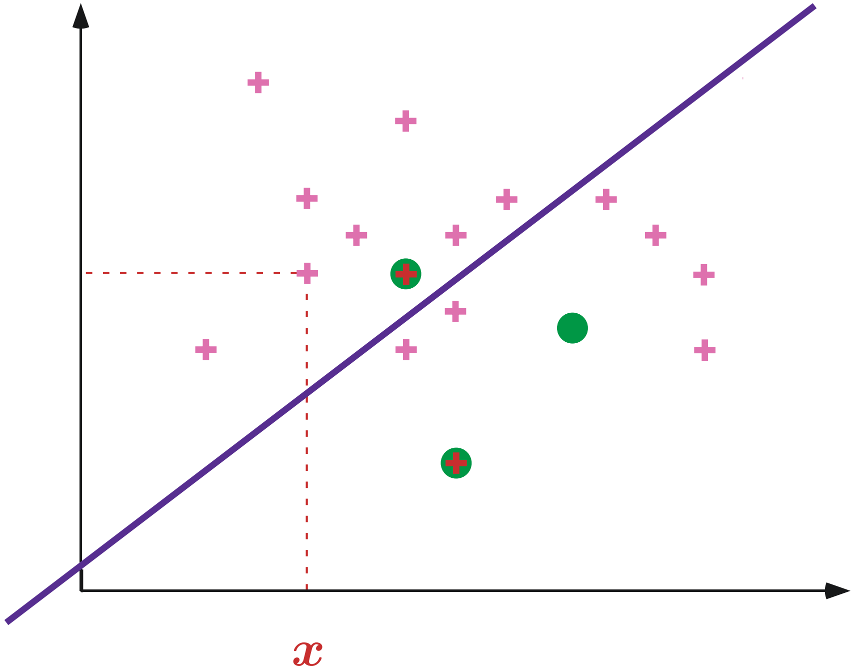
Conservative Approximation

- Is the operation `1/(x+1-y)` well defined at run-time?
- Concrete semantics: **yes**



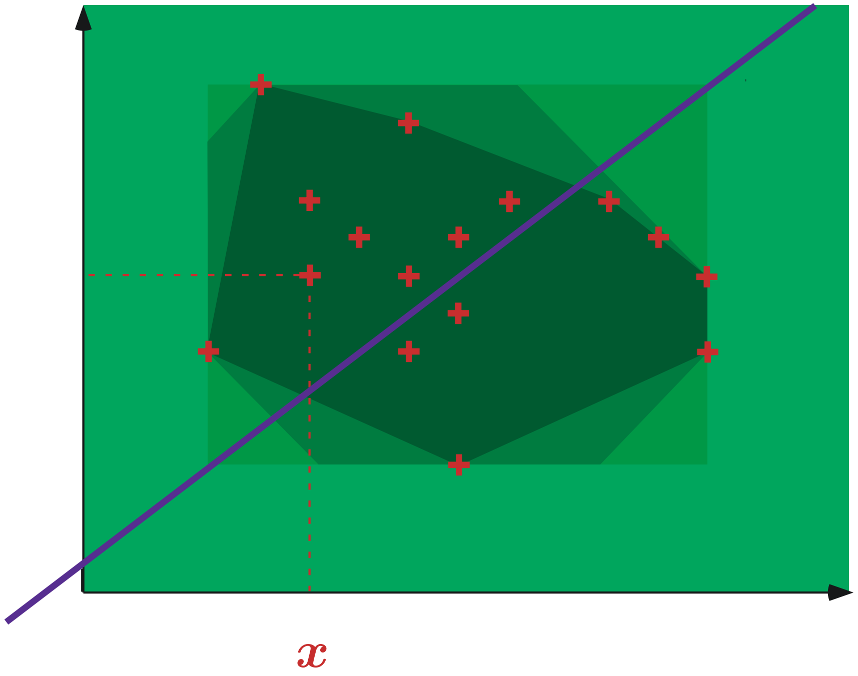
Conservative Approximation

- Is the operation `1/(x+1-y)` well defined at run-time?
- Testing : **You <u>never</u> know!**



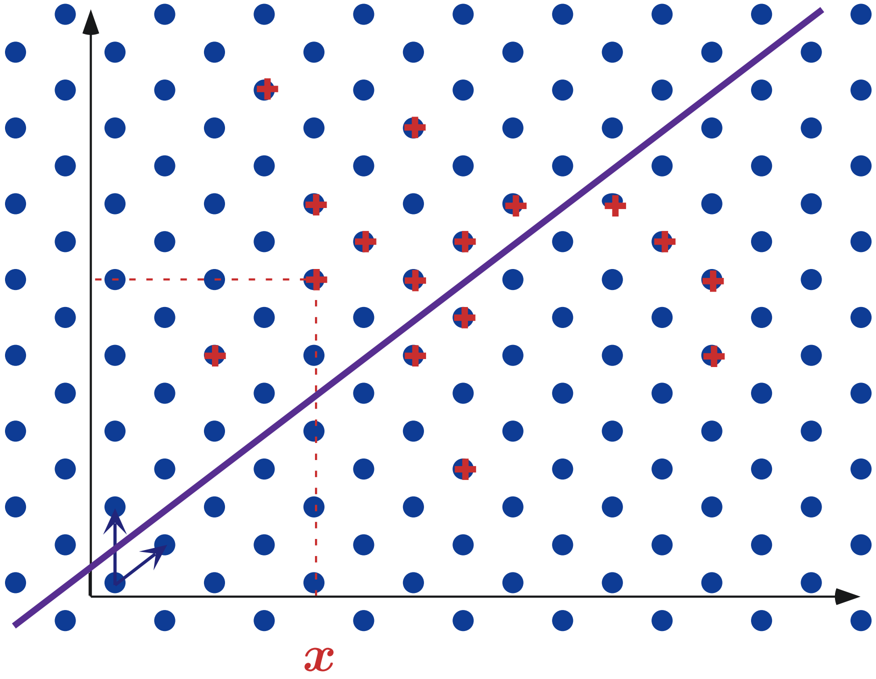
Conservative Approximation

- Is the operation `1/(x+1-y)` well defined at run-time?
- Abstract semantics 1: **I don't know**



Conservative Approximation

- Is the operation `1/(x+1-y)` well defined at run-time?
- Abstract semantics 2: **yes**



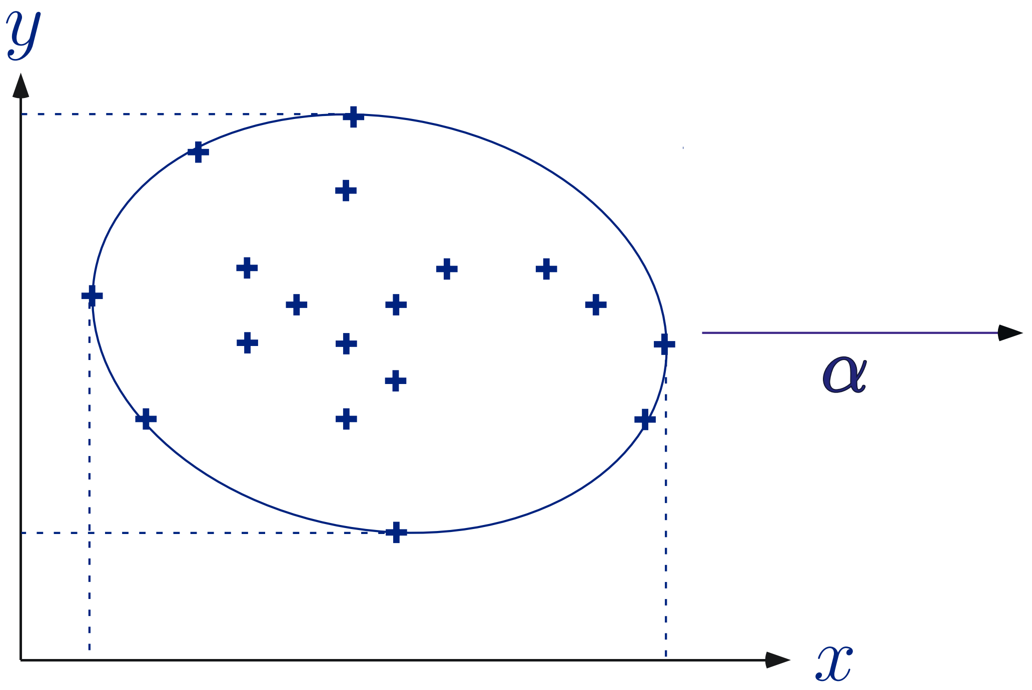
Intuition Behind Information Loss

Information Loss

- All answers given by the abstract semantics are always correct with respect to the concrete semantics;
- Because of the information loss, not all questions can be definitely answered with the abstract semantics;
- The more concrete semantics can answer more questions;
- The more abstract semantics are more simple.

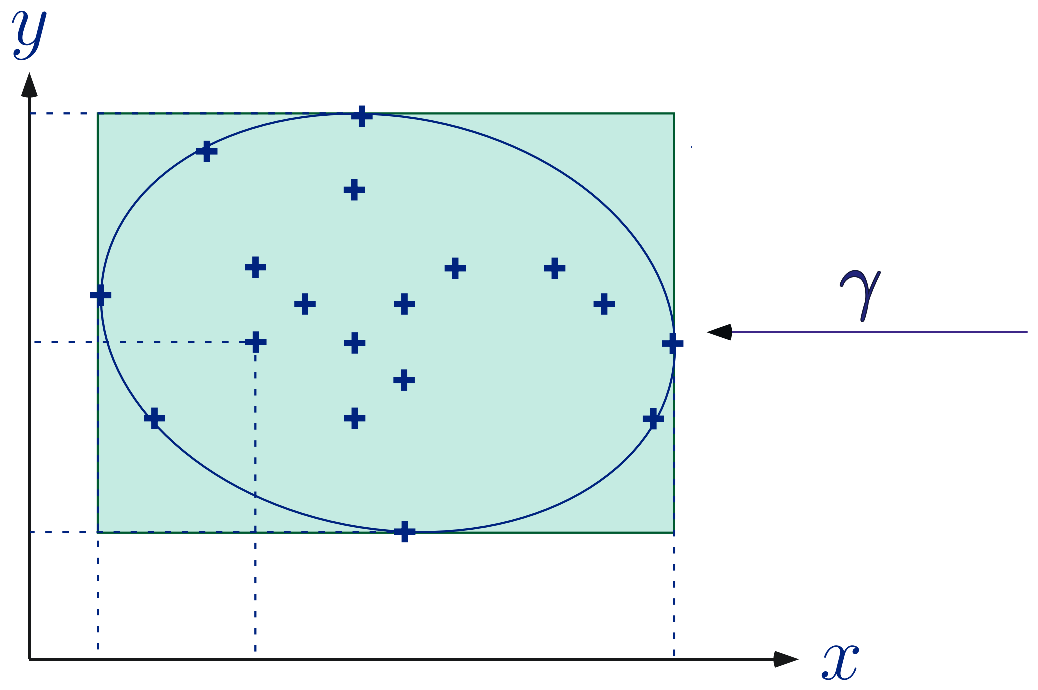
Very Basic Elements of Abstract Interpretation Theory

Abstraction α



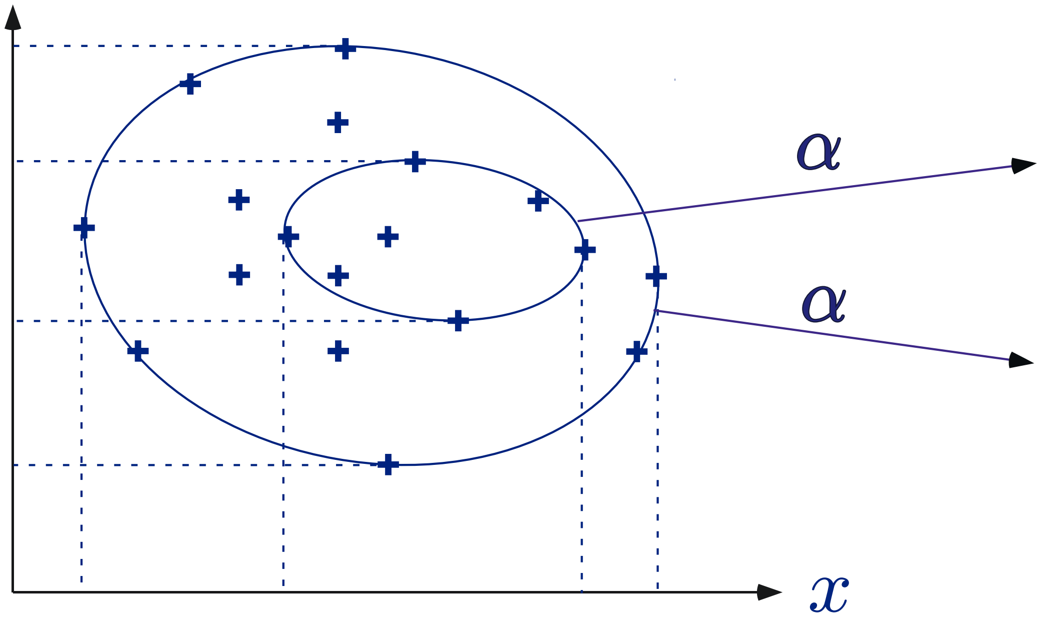
$$\{x : [1,99], y : [2,77]\}$$

Concretization γ



$\{x : [1, 99], y : [2, 77]\}$

The Abstraction α is Monotone



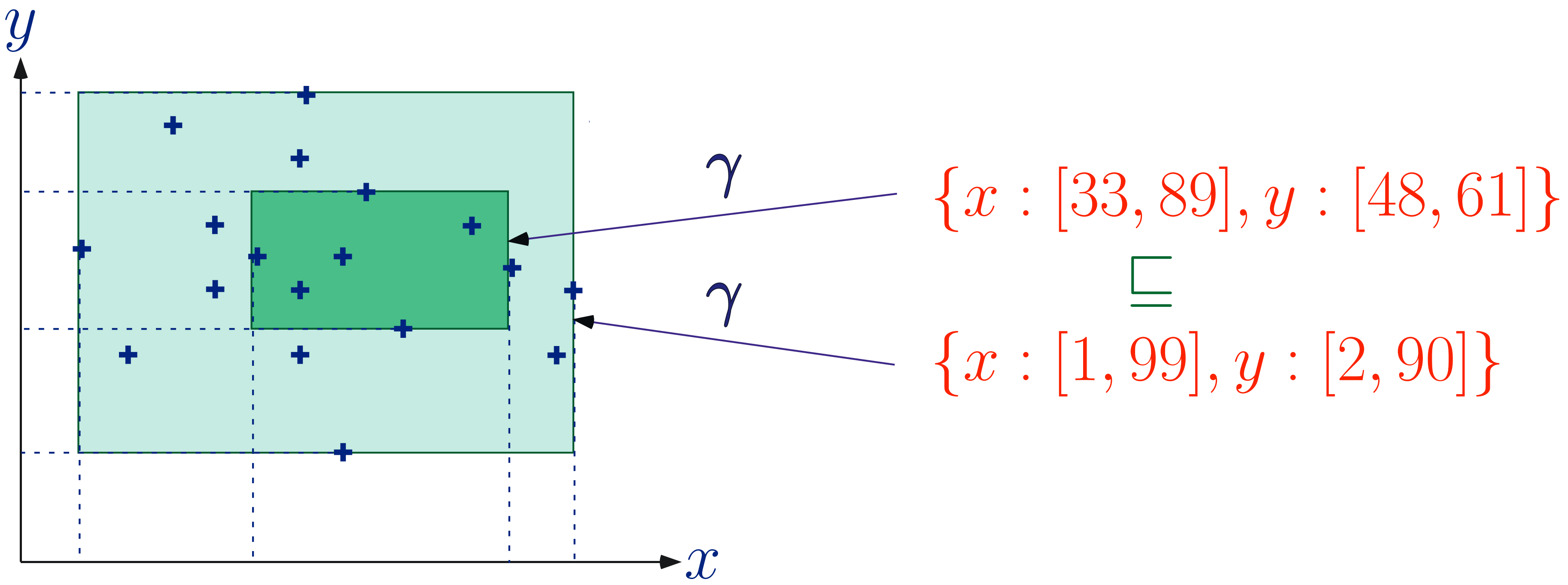
$$\{x : [33, 89], y : [48, 61]\}$$

$$\sqsubseteq$$

$$\{x : [1, 99], y : [2, 90]\}$$

$$X \subseteq Y \Rightarrow \alpha(X) \sqsubseteq \alpha(Y)$$

The Concretization γ is Monotone



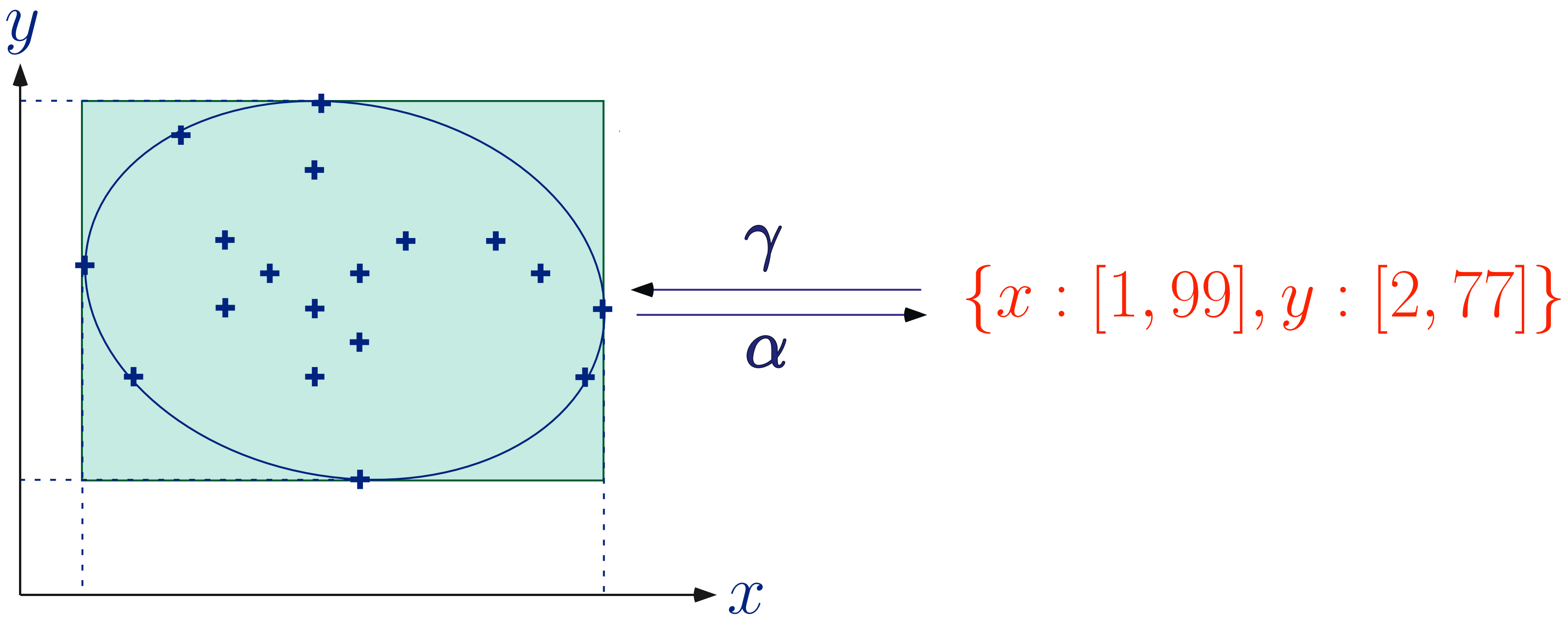
$\{x : [33, 89], y : [48, 61]\}$

$\sqsubseteq$

$\{x : [1, 99], y : [2, 90]\}$

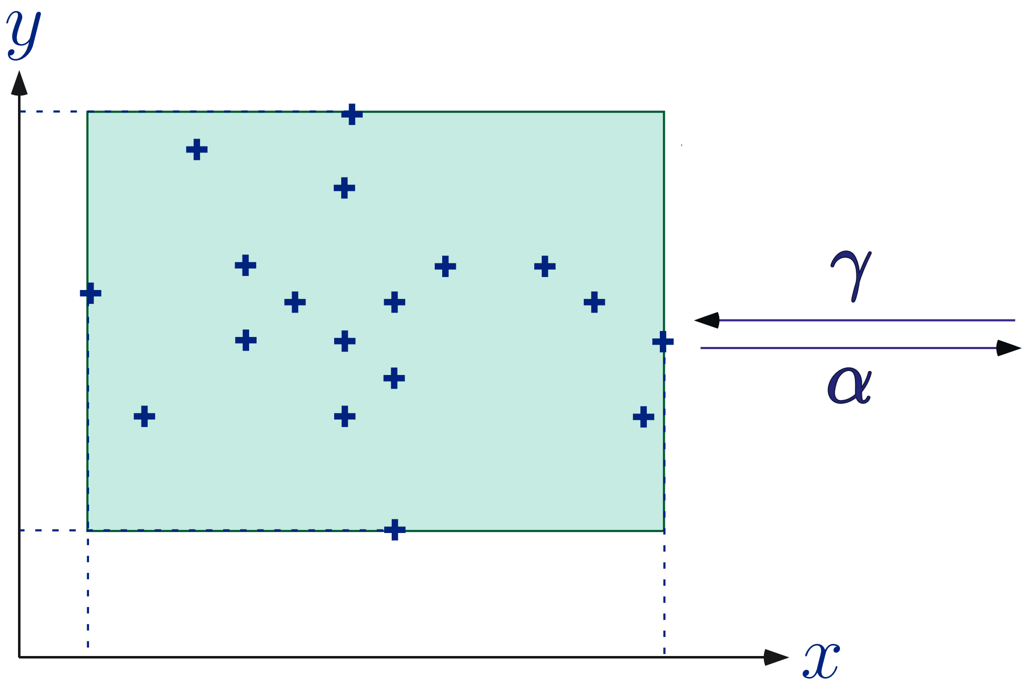
$$X \sqsubseteq Y \Rightarrow \gamma(X) \subseteq \gamma(Y)$$

The $\gamma \circ \alpha$ Composition



$$X \subseteq \gamma \circ \alpha(X)$$

The $\alpha \circ \gamma$ Composition



$$\{x : [1, 99], y : [2, 77]\}$$

$$=$$

$$\{x : [1, 99], y : [2, 77]\}$$

$$\alpha \circ \gamma(Y) = Y$$

Galois Connection [1]

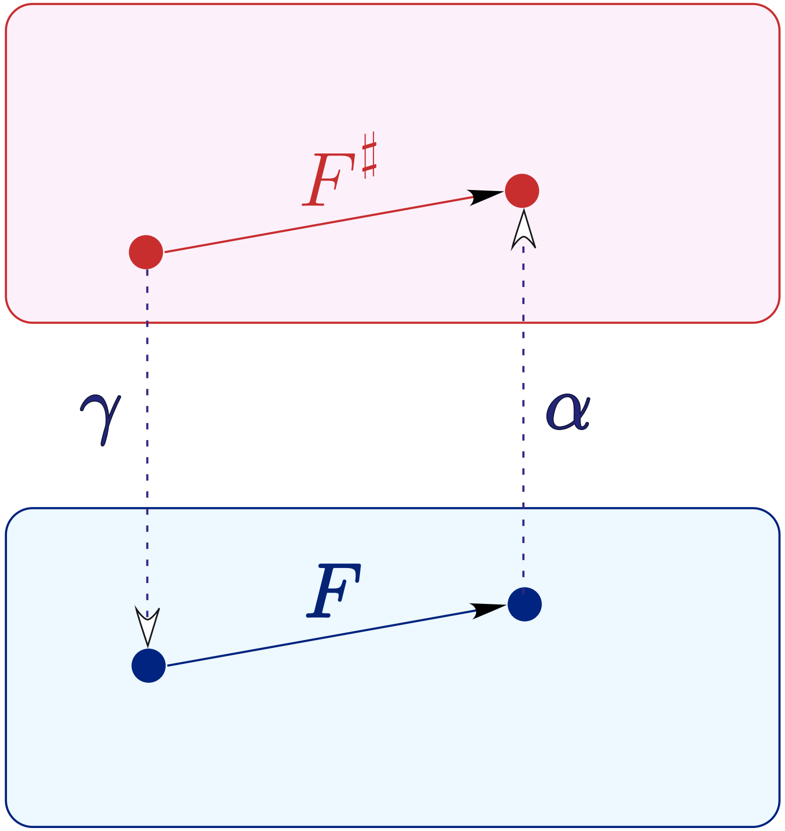
$$\langle P,\ \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle Q,\ \sqsubseteq\rangle$$

iff

- α is monotone
- γ is monotone
- $X \subseteq \gamma \circ \alpha(X)$
- $\alpha \circ \gamma(Y) \sqsubseteq Y$

[1] formalizations using closure operators, ideals, etc. are equivalent.

Function Abstraction

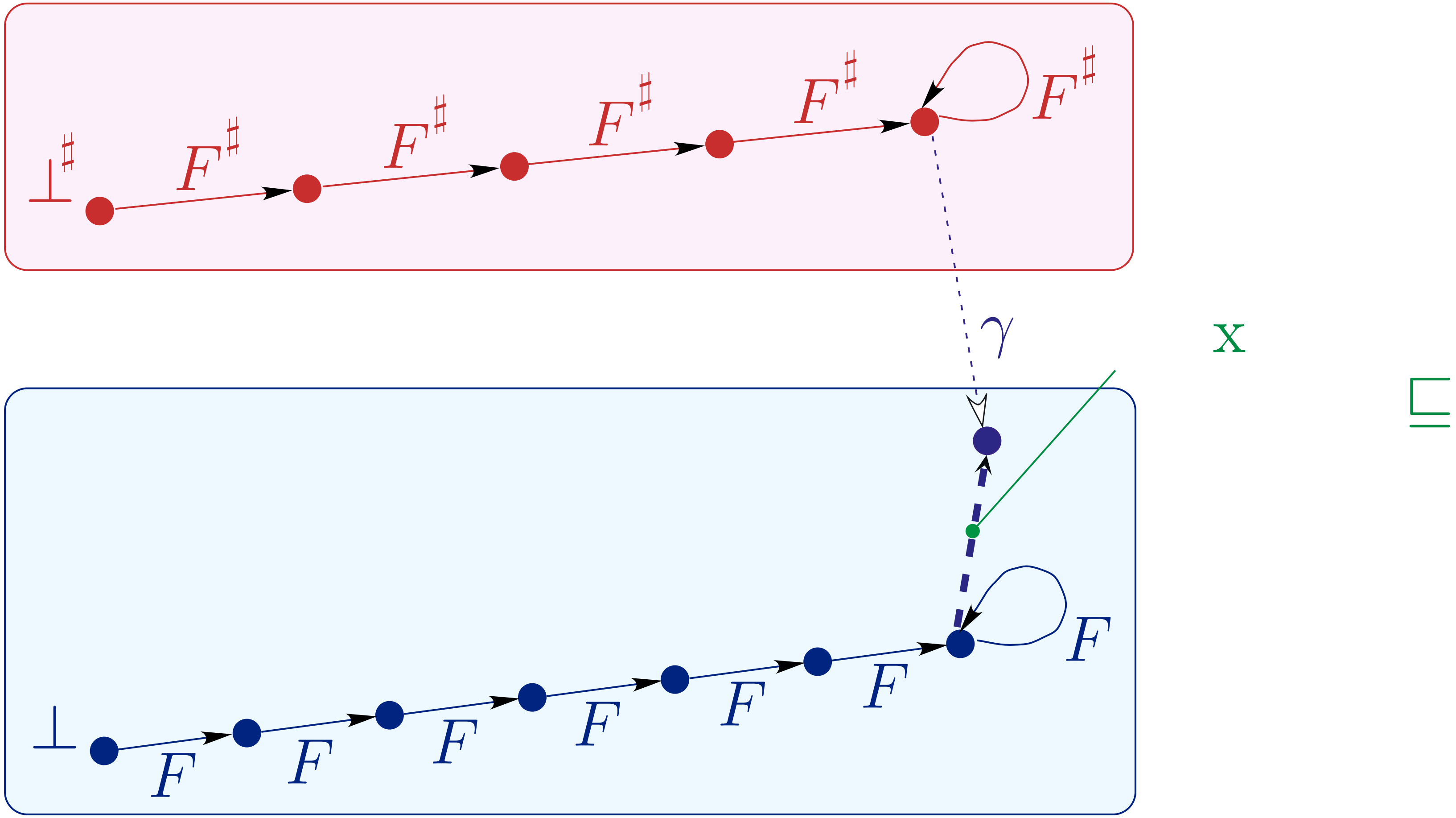


$$F^\sharp = \alpha \circ F \circ \gamma$$

$$\langle P, \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle Q, \sqsubseteq\rangle \Rightarrow$$

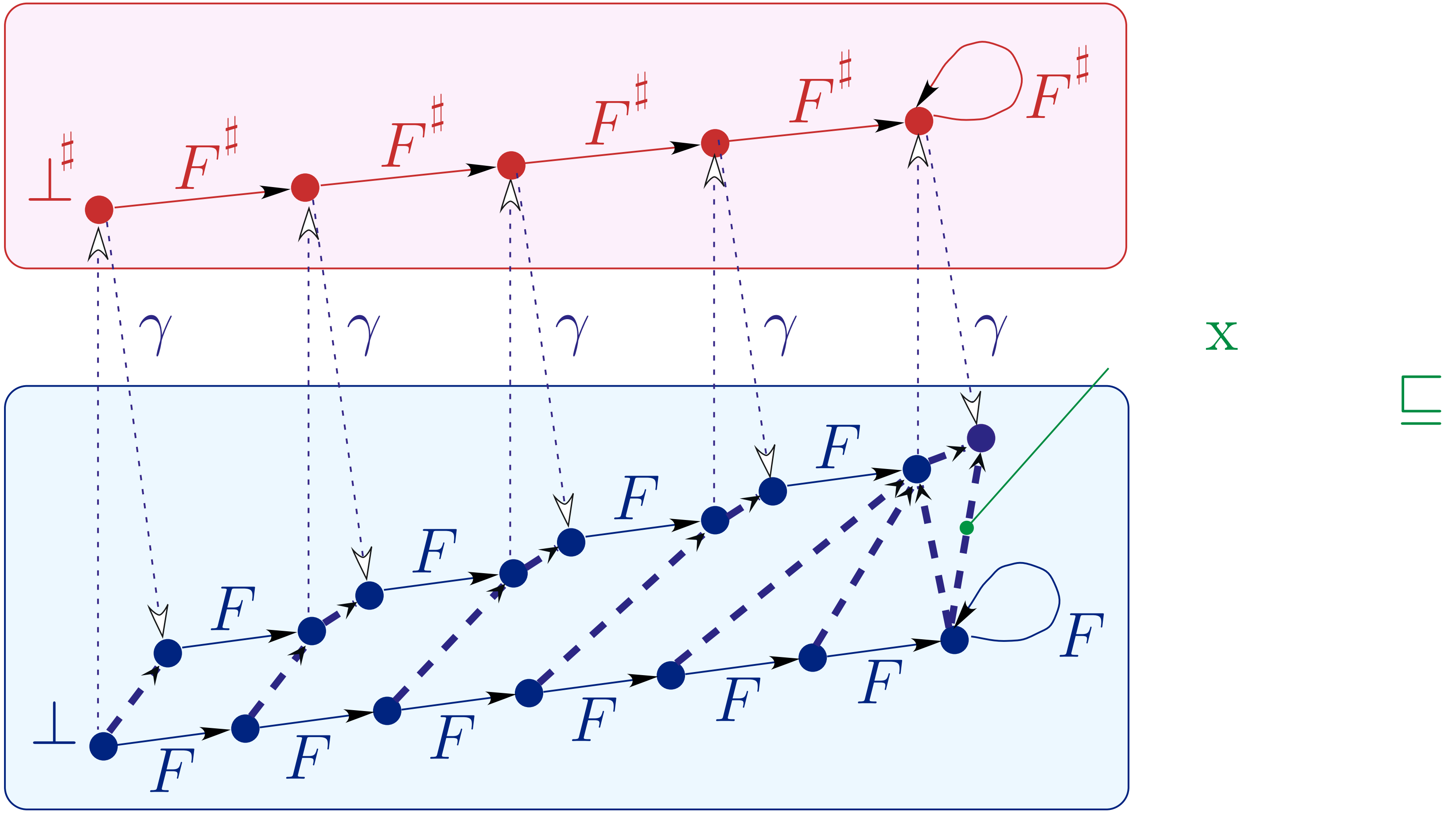
$$\langle P \xmapsto{\text{mon}} P, \dot{\subseteq}\rangle \underset{\lambda F \cdot \alpha\circ F\circ\gamma}{\overset{\lambda F^\sharp \cdot \gamma\circ F^\sharp\circ\alpha}{\leftrightarrows}} \langle Q \xmapsto{\text{mon}} Q, \dot{\sqsubseteq}\rangle$$

Fixpoint Abstraction



$$\mathbf{lfp}\ F \sqsubseteq \gamma(\mathbf{lfp}\ F^\sharp)$$

Fixpoint Abstraction



$$F^\sharp = \alpha \circ F \circ \gamma \Rightarrow \textsf{lfp}\, F \sqsubseteq \gamma(\textsf{lfp}\, F^\sharp)$$

Exact/Approximate Fixpoint Abstraction

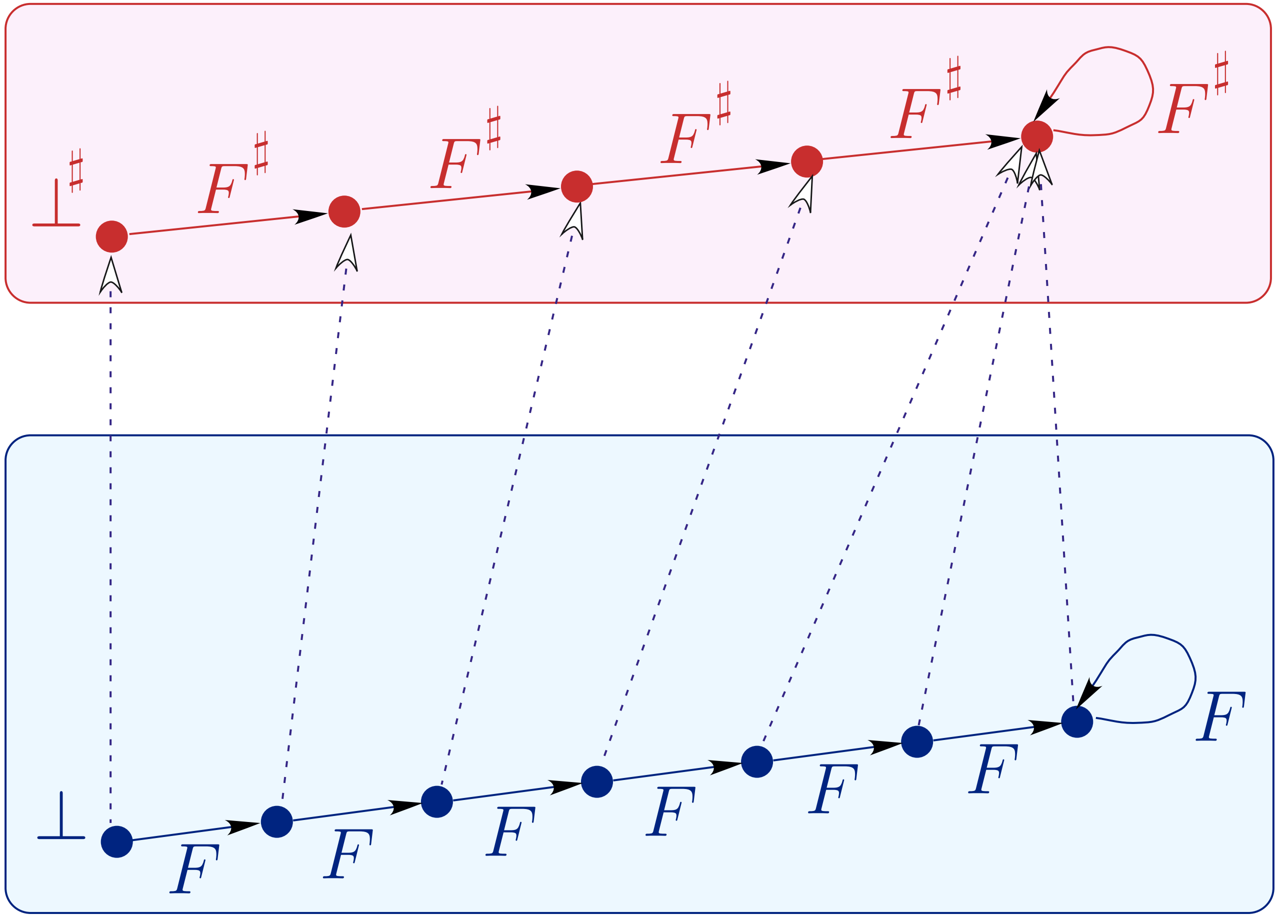
Exact Abstraction:

$$\alpha(\mathbf{lfp}\, F) = \mathbf{lfp}\, F^\sharp$$

Approximate Abstraction:

$$\alpha(\mathbf{lfp}\, F) \sqsubseteq^\sharp \mathbf{lfp}\, F^\sharp$$

Exact Fixpoint Abstraction



$$\alpha \circ F = F^\sharp \circ \alpha \;\Rightarrow\; \alpha(\mathbf{lfp}\, F) = \mathbf{lfp}\, F^\sharp$$

A Few References on Foundations

- P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4th POPL*, pages 238–252, Los Angeles, CA, 1977. ACM Press.

- P. Cousot and R. Cousot. Systematic design of program analysis frameworks. In *6th POPL*, pages 269–282, San Antonio, TX, 1979. ACM Press.

- P. Cousot and R. Cousot. Abstract interpretation frameworks. *J. Logic and Comp.*, 2(4):511–547, 1992.

Applications of Abstract Interpretation

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL77,78,79,...]
- **Program Proofs** [HTCS 90]
- **Hierarchies of Semantics** [POPL 92]
- **Typing** [POPL 97]
- **Model Checking** [POPL 00]
- **Program Transformation** [ICLP'01,POPL 02]

All these techniques involves approximations that can be formalized by abstract interpretation.

Applications of Abstract Interpretation to Logic/Constraint Programming

- **Numerous contributors**, a.o.:

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and many more!

A New Application of Abstract Interpretation: Program Transformation

Objectives of this (Ongoing) Work

Program Transformation & Abstract Interpretation

In semantics-based program transformation , such as:

- constant propagation ,
- partial evaluation ,
- slicing ,

abstract interpretation is used:

- in a preliminary program static analysis phase
- to collect the information about the program runtime behaviors, which is necessary
- to validate the applicable transformations.

Present Objective

Our present objective is **quite different**:

- Formalize the program transformation itself as an abstract interpretation;
- Two subgoals:
  - Understand correctness proofs of program transformations as abstract interpretations;
  - Imagine and apply a program transformation design method by abstract interpretation.

Example Program Transformation: Constant Propagation

The Programming Language

```
a : X := ? → b;          random assignment/input
b : Y := 1 → c;          assignment
c : (X ≤ 0) → f;         nondeterministic guard
c : (X > 0) → d;
d : X := X − Y → e;
e : skip → c;            branching
f : stop;                stop
```

Program Transformation: The Syntactic Point of View

Subject program:

$\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};$

$\mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c};$

$\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};$

$\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};$

$\mathtt{d} : \mathtt{X} := \mathtt{X} - \mathtt{Y} \longrightarrow \mathtt{e};$

$\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};$

$\mathtt{f} : \mathtt{stop};$

Transformed program:

$\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};$

$\mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c};$

$\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};$

$\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};$

$\mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{1}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e};$

$\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};$

$\mathtt{f} : \mathtt{stop};$

Syntactic program transformations

- Program transformations are performed at the syntactic level;

$$\begin{array}{c}\text{Subject}\\ \text{program } P \in \mathbb{P}\end{array} \xmapsto[\text{transformation } \mathbb{t}]{\text{Syntactic}} \begin{array}{c}\text{Transformed}\\ \text{program } \mathbb{t}[\![P]\!] \in \mathbb{P}\end{array}$$

Program Transformation: The Semantic Point of View

- The subject and transformed programs should have "similar"/ "equivalent" semantics;

  (cannot be the "same" semantics).

The Prefix Trace Semantics

$$
\begin{array}{l}
\texttt{a} : \texttt{X} := \texttt{?} \longrightarrow \texttt{b}; \\
\texttt{b} : \texttt{Y} := \texttt{1} \longrightarrow \texttt{c}; \\
\texttt{c} : (\texttt{X} \leq 0) \longrightarrow \texttt{f}; \\
\texttt{c} : (\texttt{X} > 0) \longrightarrow \texttt{d}; \\
\texttt{d} : \texttt{X} := \texttt{X} - \texttt{Y} \longrightarrow \texttt{e}; \\
\texttt{e} : \texttt{skip} \longrightarrow \texttt{c}; \\
\texttt{f} : \texttt{stop};
\end{array}
$$

The semantics is the set of prefixes of all traces similar to that one (with different inputs) ↓

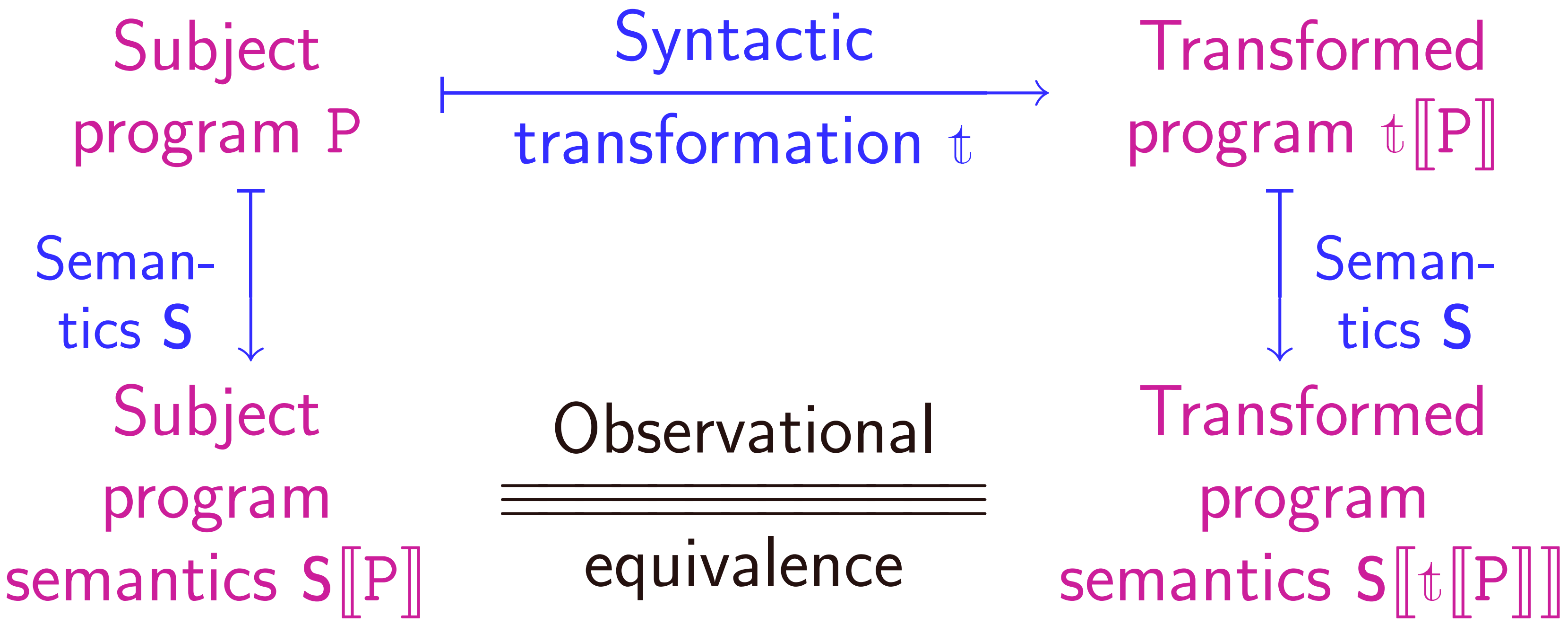
$$
\begin{array}{l}
\langle \texttt{a} : \texttt{X} := \texttt{?} \longrightarrow \texttt{b};, [\texttt{X} : \mho, \texttt{Y} : \mho] \rangle \\
\langle \texttt{b} : \texttt{Y} := \texttt{1} \longrightarrow \texttt{c};, [\texttt{X} : 1, \texttt{Y} : \mho] \rangle \\
\langle \texttt{c} : (\texttt{X} > 0) \longrightarrow \texttt{d};, [\texttt{X} : 1, \texttt{Y} : 1] \rangle \\
\langle \texttt{d} : \texttt{X} := \texttt{X} - \texttt{Y} \longrightarrow \texttt{e};, [\texttt{X} : 1, \texttt{Y} : 1] \rangle \\
\langle \texttt{e} : \texttt{skip} \longrightarrow \texttt{c};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle \\
\langle \texttt{c} : (\texttt{X} \leq 0) \longrightarrow \texttt{f};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle \\
\langle \texttt{f} : \texttt{stop};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle
\end{array}
$$

Semantics of the Transformed program

$\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};$	$\langle \mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};, [\mathtt{X} : \mho, \mathtt{Y} : \mho]\rangle$
$\mathtt{b} : \mathtt{Y} := \mathtt{1} \longrightarrow \mathtt{c};$	$\langle \mathtt{b} : \mathtt{Y} := \mathtt{1} \longrightarrow \mathtt{c};, [\mathtt{X} : 1, \mathtt{Y} : \mho]\rangle$
$\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};$	
$\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};$	$\langle \mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$
$\mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{\mathtt{1}}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e};$	$\langle \mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{\mathtt{1}}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$
$\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};$	$\langle \mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$
	$\langle \mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$
$\mathtt{f} : \mathtt{stop};$	$\langle \mathtt{f} : \mathtt{stop};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$

Validation of program transformations

- Program transformations are validated at a semantic level.



Subject program P

Syntactic transformation $\mathbb{t}$

Transformed program $\mathbb{t}[\![\mathrm{P}]\!]$

Semantics $\mathbf{S}$

Semantics $\mathbf{S}$

Subject program semantics $\mathbf{S}[\![\mathrm{P}]\!]$

Observational equivalence

Transformed program semantics $\mathbf{S}[\![\mathbb{t}[\![\mathrm{P}]\!]]\!]$

Observational semantics

$$\begin{array}{ll}
\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b}; & \langle \mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};,\ [\mathtt{X} : \mho, \mathtt{Y} : \mho]\rangle \\
\mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c}; & \langle \mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c};,\ [\mathtt{X} : 1, \mathtt{Y} : \mho]\rangle \\
\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f}; & \\
\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d}; & \langle \mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};,\ [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle \\
\mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{1}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e}; & \langle \mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{1}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e};,\ [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle \\
\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c}; & \langle \mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};,\ [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle \\
 & \langle \mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};,\ [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle \\
\mathtt{f} : \mathtt{stop}; & \langle \mathtt{f} : \mathtt{stop};,\ [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle
\end{array}$$

Online Versus Offline Program Transformation

Online Transformation : use the actual values of variables (i.e. the program concrete semantics);

Offline Transformation : use a preliminary static analysis of the program (i.e. the program abstract semantics).

(Constant propagation is an offline program transformation, otherwise the transformation might loop for non-constants.)

Basic Elements of Program Transformation Design by Abstract Interpretation

Element 1: Observational Equivalence

- The semantics of the subject and transformed programs should be equivalent at some level of observation:

$$\begin{tikzcd}
\text{Subject program } P \arrow[r, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, mapsto, "\text{Semantics } \mathsf{S}"'] & \text{Transformed program } \mathbb{t}[\![P]\!] \arrow[d, mapsto, "\text{Semantics } \mathsf{S}"] \\
\text{Subject program semantics } \mathsf{S}[\![P]\!] \arrow[r, equal, "\text{Observational}", "\text{equivalence } \equiv_{\mathcal{O}}"'] & \text{Transformed program semantics } \mathsf{S}[\![\mathbb{t}[\![P]\!]]\!]
\end{tikzcd}$$

Element 1: Observational Equivalence, Cont'd

- Observational equivalence can be formalized as an abstract interpretation:

$$\begin{tikzcd}
\begin{array}{c}\text{Subject}\\ \text{program}\\ \text{semantics}\\ \mathsf{S}[\![\mathrm{P}]\!]\end{array} \arrow[rr, equal, "\text{Observational}", "\text{equivalence } \equiv_{\mathcal{O}}"'] \arrow[dr, mapsto, "\alpha_{\mathcal{O}}"'] & & \begin{array}{c}\text{Transformed}\\ \text{program}\\ \text{semantics}\\ \mathsf{S}[\![\mathfrak{t}[\![\mathrm{P}]\!]]\!]\end{array} \arrow[dl, mapsto, "\alpha_{\mathcal{O}}"] \\
& \text{Observational semantics} &
\end{tikzcd}$$

Example: Constant Propagation

Subject/Transformed Semantics	Observational Semantics
$\langle \mathtt{a} : \mathtt{X} := \mathtt{?} \longrightarrow \mathtt{b};, [\mathtt{X} : \mho, \mathtt{Y} : \mho]\rangle$	$\langle \mathtt{a} : \quad \longrightarrow \mathtt{b};, [\mathtt{X} : \mho, \mathtt{Y} : \mho]\rangle$
$\langle \mathtt{b} : \mathtt{Y} := \mathtt{1} \longrightarrow \mathtt{c};, [\mathtt{X} : 1, \mathtt{Y} : \mho]\rangle$	$\langle \mathtt{b} : \quad \longrightarrow \mathtt{c};, [\mathtt{X} : 1, \mathtt{Y} : \mho]\rangle$
$\langle \mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$	$\langle \mathtt{c} : \quad \longrightarrow \mathtt{d};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$
$\langle \mathtt{d} : \mathtt{X} := \mathtt{X} - \overset{\mathtt{1}}{\cancel{\mathtt{Y}}} \longrightarrow \mathtt{e};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$	$\langle \mathtt{d} : \quad \longrightarrow \mathtt{e};, [\mathtt{X} : 1, \mathtt{Y} : 1]\rangle$
$\langle \mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$	$\langle \mathtt{e} : \quad \longrightarrow \mathtt{c};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$
$\langle \mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$	$\langle \mathtt{c} : \quad \longrightarrow \mathtt{f};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$
$\langle \mathtt{f} : \mathtt{stop};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$	$\langle \mathtt{f} : \mathtt{stop};, [\mathtt{X} : 0, \mathtt{Y} : 1]\rangle$

Element 2: Induced Semantic Transformation

- A syntactic program transformation induces a semantic program transformation:

$$\begin{tikzcd}
\text{Subject program } P \arrow[r, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, mapsto, "\text{Semantics } \mathbf{S}"'] & \text{Transformed program } \mathbb{t}\llbracket P \rrbracket \arrow[d, mapsto, "\text{Semantics } \mathbf{S}"'] \\
\text{Subject semantics } \mathbf{S}\llbracket P \rrbracket \arrow[r, mapsto, "\text{Semantic}", "\text{transformation } t"'] & \begin{array}{c}\text{Transformed semantics}\\ t[\mathbf{S}\llbracket P \rrbracket] \equiv_{\mathcal{O}} \mathbf{S}\llbracket \mathbb{t}\llbracket P \rrbracket \rrbracket\end{array}
\end{tikzcd}$$

- We study this semantic transformation from an abstract interpretation point of view.

Example: Semantic Constant Propagation

$a : X := ? \longrightarrow b;$	$\langle a : X := ? \longrightarrow b;, [X : \mho, Y : \mho]\rangle$
$b : Y := 1 \longrightarrow c;$	$\langle b : Y := 1 \longrightarrow c;, [X : 1, Y : \mho]\rangle$
$c : (X \leq 0) \longrightarrow f;$	
$c : (X > 0) \longrightarrow d;$	$\langle c : (X > 0) \longrightarrow d;, [X : 1, Y : 1]\rangle$
$d : X := X - Y \longrightarrow e;$	$\langle d : X := X - \overset{1}{\cancel{Y}} \longrightarrow e;, [X : 1, Y : 1]\rangle$
$e : skip \longrightarrow c;$	$\langle e : skip \longrightarrow c;, [X : 0, Y : 1]\rangle$
	$\langle c : (X \leq 0) \longrightarrow f;, [X : 0, Y : 1]\rangle$
$f : stop;$	$\langle f : stop;, [X : 0, Y : 1]\rangle$

Element 3: Semantic Correctness

- From a validation point of view, the correctness of a syntactic transformation can be proved by reasoning on the induced semantic transformation:

$$\begin{tikzcd}
\text{Subject program } \mathrm{P} \arrow[r, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, mapsto, "\text{Seman-tics } \mathbf{S}"'] & \text{Transformed program } \mathbb{t}[\![\mathrm{P}]\!] \arrow[d, mapsto, "\text{Seman-tics } \mathbf{S}"'] \\
\text{Subject semantics } \mathbf{S}[\![\mathrm{P}]\!] \arrow[r, mapsto, "\text{Semantic}", "\text{transformation } \mathrm{t}"'] & \text{Transformed semantics } \mathrm{t}[\mathbf{S}[\![\mathrm{P}]\!]]
\end{tikzcd}$$

$$\mathbf{S}[\![\mathrm{P}]\!] \equiv_{\mathcal{O}} \mathrm{t}[\mathbf{S}[\![\mathrm{P}]\!]] \equiv_{\mathcal{O}} \mathbf{S}[\![\mathbb{t}[\![\mathrm{P}]\!]]\!]$$

Element 4: Correspondence Between Syntax and Semantics

- The program syntax forgets details about the program execution semantics:
  - The sequence of values of variables during execution is forgotten, but:
    - their existence and maybe their type are recorded;
    - the sequence (partial order, ...) of (denotations of) the performed actions is recorded;
  - Program execution times are completely abstracted (but might be included in the operational semantics);

Element 4: Correspondence Between Syntax and Semantics, Cont'd

- The correspondence between syntax and semantics is an abstract interpretation:

$$\mathsf{po}\langle\mathfrak{D};\ \sqsubseteq\rangle \;\underset{\mathbb{p}}{\overset{\mathsf{S}}{\mathrel{\substack{\longleftarrow\\ \twoheadrightarrow}}}}\; \mathsf{po}\langle\mathbb{P}/_{\equiv};\ \sqsubseteq\rangle$$

Example: Syntax to Prefix Trace Semantics

- Fixpoint semantics:

$$\mathbf{S}^*[\![\mathrm{P}]\!] = \mathbf{lfp}^{\subseteq}\ \mathbf{F}^*[\![\mathrm{P}]\!]$$

$$\mathbf{F}^*[\![\mathrm{P}]\!]\mathcal{T} = \mathcal{I}[\![\mathrm{P}]\!] \cup \{\sigma s s' \mid \sigma s \in \mathcal{T} \wedge s' \in \mathbf{S}[\![\mathrm{P}]\!]s\},$$

Example: Prefix Trace Semantics to Syntax

- Collect commands along traces.

Element 5: Semantics-Based Design

- From a constructive design point of view, the syntactic transformation can be formally derived from a correct semantic transformation:

$$\begin{tikzcd}[row sep=huge, column sep=huge]
\text{Subject program } P \arrow[r, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, shift right, "\mathsf{S}"'] & \text{Transformed program } \mathbb{t}[\![P]\!] = \mathbb{p}[\mathsf{t}[\mathsf{S}[\![P]\!]]] \arrow[d, shift right, "\mathsf{S}"'] \\
\text{Subject program semantics } \mathsf{S}[\![P]\!] \arrow[u, twoheadrightarrow, shift right, "\mathbb{p}"'] \arrow[r, mapsto, "\text{Semantic}", "\text{transformation } \mathsf{t}"'] & \text{Transformed program semantics } \mathsf{t}[\mathsf{S}[\![P]\!]] \arrow[u, twoheadrightarrow, shift right, "\mathbb{p}"']
\end{tikzcd}$$

Semantic to Syntactic Constant Propagation

$\texttt{a} : \texttt{X} := ? \longrightarrow \texttt{b};$	$\langle \texttt{a} : \texttt{X} := ? \longrightarrow \texttt{b};, [\texttt{X} : \mho, \texttt{Y} : \mho] \rangle$
$\texttt{b} : \texttt{Y} := 1 \longrightarrow \texttt{c};$	$\langle \texttt{b} : \texttt{Y} := 1 \longrightarrow \texttt{c};, [\texttt{X} : 1, \texttt{Y} : \mho] \rangle$
$\texttt{c} : (\texttt{X} \leq 0) \longrightarrow \texttt{f};$	
$\texttt{c} : (\texttt{X} > 0) \longrightarrow \texttt{d};$	$\langle \texttt{c} : (\texttt{X} > 0) \longrightarrow \texttt{d};, [\texttt{X} : 1, \texttt{Y} : 1] \rangle$
$\texttt{d} : \texttt{X} := \texttt{X} - \overset{1}{\cancel{\texttt{Y}}} \longrightarrow \texttt{e};$	$\langle \texttt{d} : \texttt{X} := \texttt{X} - \overset{1}{\cancel{\texttt{Y}}} \longrightarrow \texttt{e};, [\texttt{X} : 1, \texttt{Y} : 1] \rangle$
$\texttt{e} : \texttt{skip} \longrightarrow \texttt{c};$	$\langle \texttt{e} : \texttt{skip} \longrightarrow \texttt{c};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle$
	$\langle \texttt{c} : (\texttt{X} \leq 0) \longrightarrow \texttt{f};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle$
$\texttt{f} : \texttt{stop};$	$\langle \texttt{f} : \texttt{stop};, [\texttt{X} : 0, \texttt{Y} : 1] \rangle$

Element 6: Transformations as Approximations

- A semantic program transformation is a loss of information on the semantics of the subject program;

  $\longrightarrow$ This can be formalized by abstract interpretation;

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Subject program } P \arrow[r, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, shift right, "\mathsf{S}"'] & \text{Transformed program } \mathbb{t}[\![P]\!] \sqsupseteq \mathbb{p}[t[\mathsf{S}[\![P]\!]]] \arrow[d, shift right, "\mathsf{S}"'] \\
\text{Subject program semantics } \mathsf{S}[\![P]\!] \arrow[u, shift right, "\mathbb{p}"'] \arrow[r, shift right, "\text{Semantic transformation } t"'] & \text{Transformed program semantics } t[\mathsf{S}[\![P]\!]] \sqsubseteq \mathsf{S}[\![\mathbb{t}[\![P]\!]]\!] \arrow[u, shift right, "\mathbb{p}"'] \arrow[l, shift right, "\gamma_t"']
\end{tikzcd}$$

Element 6: Transformations as Approximations (Cont'd)

- By composition, the syntactic program transformation is also a loss of information on subject program;

$\longrightarrow$ This can be formalized by abstract interpretation;

$$\begin{tikzcd}
\text{Subject program } P \arrow[r, shift right, "\text{Syntactic transformation } \mathbb{t}"'] \arrow[d, shift right, "\mathsf{S}"'] & \text{Transformed program } \mathbb{t}[\![P]\!] \sqsupseteq \mathbb{p}[\mathrm{t}[\mathsf{S}[\![P]\!]]] \arrow[l, shift right, "\gamma_{\mathbb{t}}"'] \arrow[d, shift right, "\mathsf{S}"'] \\
\text{Subject program semantics } \mathsf{S}[\![P]\!] \arrow[r, shift right, "\text{Semantic transformation } \mathrm{t}"'] \arrow[u, shift right, "\mathbb{p}"'] & \text{Transformed program semantics } \mathrm{t}[\mathsf{S}[\![P]\!]] \sqsubseteq \mathsf{S}[\![\mathbb{t}[\![P]\!]]] \arrow[l, shift right, "\gamma_{\mathrm{t}}"'] \arrow[u, shift right, "\mathbb{p}"']
\end{tikzcd}$$

Intuition for Transformations as Abstractions

$\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};$

$\mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c};$

$\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};$

$\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};$

$\ldots$

$\mathtt{X} - (2 * \mathtt{Y} - 1)$

$\mathtt{X} - \mathtt{Y}$

$\mathtt{d} : \mathtt{X} := \mathtt{X} - 1 \longrightarrow \mathtt{e};$

$\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};$

$\mathtt{f} : \mathtt{stop};$

$\mathtt{a} : \mathtt{X} := ? \longrightarrow \mathtt{b};$

$\mathtt{b} : \mathtt{Y} := 1 \longrightarrow \mathtt{c};$

$\mathtt{c} : (\mathtt{X} \leq 0) \longrightarrow \mathtt{f};$

$\mathtt{c} : (\mathtt{X} > 0) \longrightarrow \mathtt{d};$

$\mathtt{d} : \mathtt{X} := \mathtt{X} - 1 \longrightarrow \mathtt{e};$

$\mathtt{e} : \mathtt{skip} \longrightarrow \mathtt{c};$

$\mathtt{f} : \mathtt{stop};$

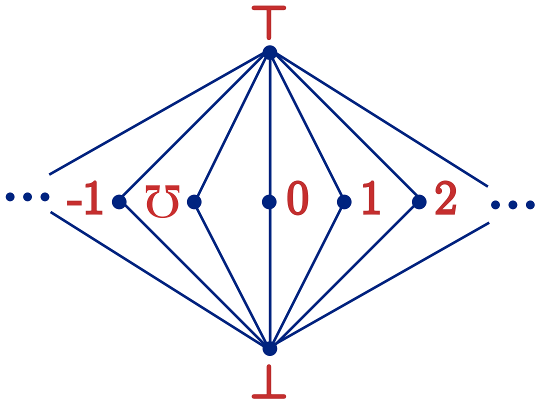
Element 7: offline Transformations

- A semantic program transformation can be restricted to use the only semantic information which can be discovered by a static program analysis;

  ⟶ This can be formalized by abstract interpretation.

Example: Kildall's Constant Propagation

- Kildall's lattice (POPL'73):



$$\begin{aligned}\gamma^c(\top) &= \mathbb{Z} \cup \{\mho\}\\ \gamma^c(x) &= \{x\}, \quad x \in \mathbb{Z} \cup \{\mho\}\\ \gamma^c(\bot) &= \emptyset\end{aligned}$$

- Pointwise extension to variable environments and program labels;

Example: Kildall's Constant Propagation, Cont'd

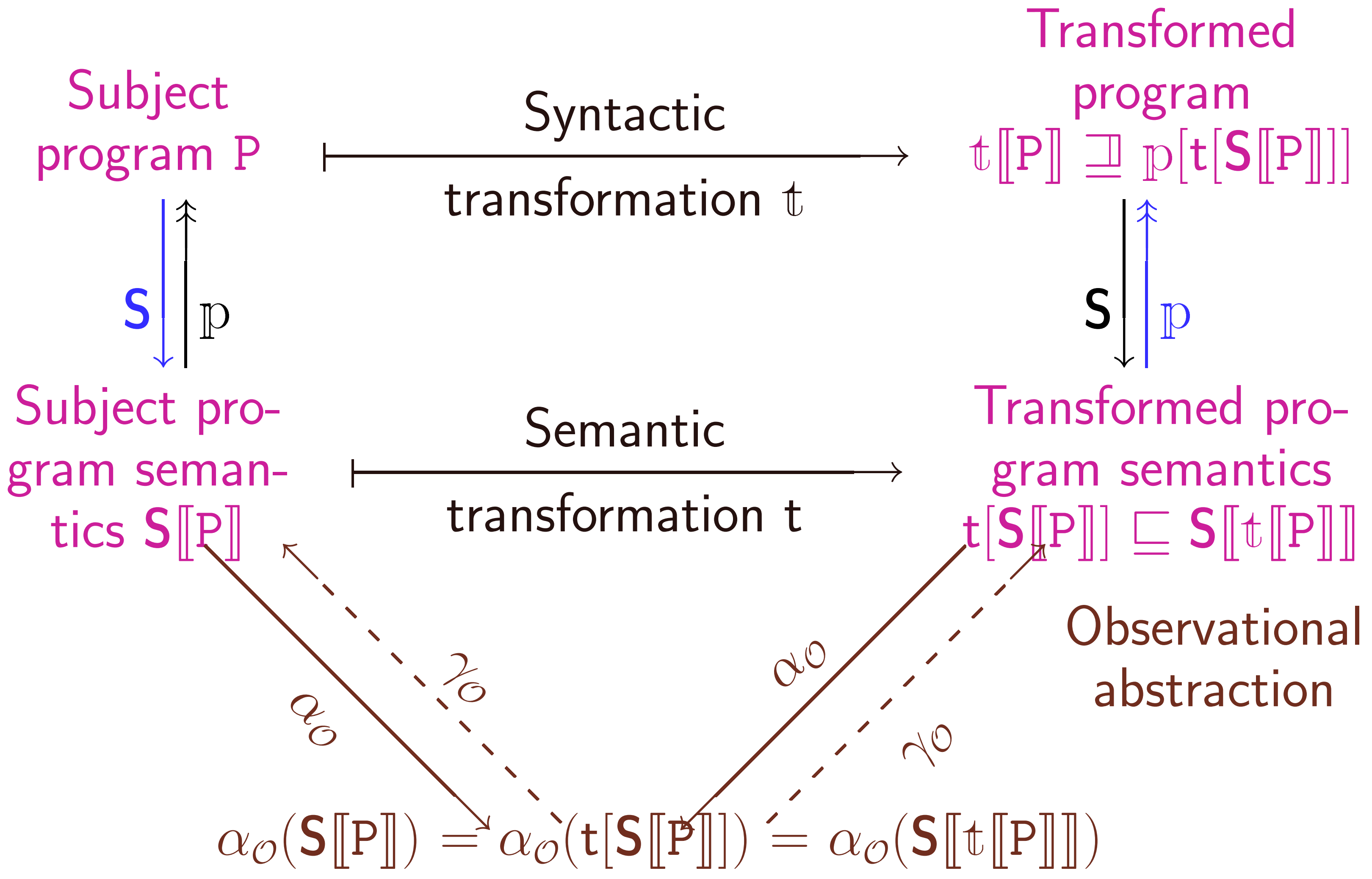
- Elementwise abstraction of a set $\mathcal{T}$ of traces:

$$\begin{aligned}\alpha^c(\mathcal{T}) = \lambda \mathtt{L} \cdot \lambda \mathtt{X} \cdot \ddot{\bigsqcup}\{\rho(\mathtt{X}) \mid \exists \sigma \in \mathcal{T} : \exists \mathtt{C} \in \mathbb{C} : \exists i :&\\ \sigma_i = \langle \rho,\ \mathtt{C} \rangle \wedge \mathsf{lab}[\![\mathtt{C}]\!] = \mathtt{L}\}&\end{aligned}$$

where $\ddot{\bigsqcup}$ is the pointwise extension of the lub in Kildall's lattice

Principle of the Formalization of Program Transformation by Abstract Interpretation

Principle of Online Program Transformation



Subject program P

Syntactic transformation $\mathbb{t}$

Transformed program $\mathbb{t}[\![\mathrm{P}]\!] \sqsupseteq \mathbb{p}[\mathsf{t}[\mathbf{S}[\![\mathrm{P}]\!]]]$

Subject program semantics $\mathbf{S}[\![\mathrm{P}]\!]$

Semantic transformation t

Transformed program semantics $\mathsf{t}[\mathbf{S}[\![\mathrm{P}]\!]] \sqsubseteq \mathbf{S}[\![\mathbb{t}[\![\mathrm{P}]\!]]\!]$

Observational abstraction

$$\alpha_{\mathcal{O}}(\mathbf{S}[\![\mathrm{P}]\!]) = \alpha_{\mathcal{O}}(\mathsf{t}[\mathbf{S}[\![\mathrm{P}]\!]]) = \alpha_{\mathcal{O}}(\mathbf{S}[\![\mathbb{t}[\![\mathrm{P}]\!]]\!])$$

Principle of Offline Program Transformation (1)

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{array}{c}\text{Subject}\\ \text{program } \mathrm{P}\end{array} \arrow[r, mapsto, "\text{Syntactic annotation}", "\mathbb{a}"'] \arrow[d, shift right, "\mathsf{S}"'] & \begin{array}{c}\text{Annotated}\\ \text{subject program}\\ \underline{\mathrm{P}} = \mathbb{a}[\![\mathrm{P}]\!]\end{array} \arrow[r, mapsto, "\text{Syntactic transformation}", "\mathbb{t}"'] \arrow[d, shift left, "\mathsf{S}"] & \begin{array}{c}\text{Transformed}\\ \text{program}\\ \mathbb{t}[\![\underline{\mathrm{P}}]\!] \sqsupseteq \mathbb{p}[\mathrm{t}[\mathsf{S}[\![\underline{\mathrm{P}}]\!]]]\end{array} \arrow[d, shift right, "\mathsf{S}"'] \\
\begin{array}{c}\text{Subject}\\ \text{program}\\ \text{semantics } \mathsf{S}[\![\mathrm{P}]\!]\end{array} \arrow[u, shift right, two heads, "\mathbb{p}"'] \arrow[r, mapsto, "\text{Semantic annotation}", "\mathrm{a}"'] \arrow[dr, "\alpha_{\mathcal{O}}"'] & \begin{array}{c}\text{Annotated}\\ \text{subject program}\\ \text{semantics } \mathsf{S}[\![\underline{\mathrm{P}}]\!]\end{array} \arrow[u, shift left, two heads, "\mathbb{p}"] \arrow[r, mapsto, "\text{Semantic transformation}", "\mathrm{t}"'] \arrow[d, shift right, "\alpha_{\mathcal{O}}"'] & \begin{array}{c}\text{Transformed}\\ \text{program semantics}\\ \mathrm{t}[\mathsf{S}[\![\underline{\mathrm{P}}]\!]] \sqsubseteq \mathsf{S}[\![\mathbb{t}[\![\underline{\mathrm{P}}]\!]]\!]\end{array} \arrow[u, shift right, two heads, "\mathbb{p}"'] \arrow[dl, "\alpha_{\mathcal{O}}"] \\
 & \alpha_{\mathcal{O}}(\mathsf{S}[\![\mathrm{P}]\!]) = \alpha_{\mathcal{O}}(\mathsf{S}[\![\underline{\mathrm{P}}]\!]) = \alpha_{\mathcal{O}}(\mathrm{t}[\mathsf{S}[\![\underline{\mathrm{P}}]\!]]) = \alpha_{\mathcal{O}}(\mathsf{S}[\![\mathbb{t}[\![\underline{\mathrm{P}}]\!]]\!]) \arrow[ul, dashed, "\gamma_{\mathcal{O}}"'] \arrow[u, shift right, dashed, "\gamma_{\mathcal{O}}"'] \arrow[ur, dashed, "\gamma_{\mathcal{O}}"] &
\end{tikzcd}$$

Observational abstraction

Principle of Offline Program Transformation (2)

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{array}{c}\text{Subject}\\ \text{program}\\ P\end{array} \arrow[r, mapsto, "\text{Static program}", "\text{analysis } \overline{\mathsf{S}}"'] \arrow[d, shift right, "\mathsf{S}"'] & \langle P,\ \overline{\mathsf{S}}[\![P]\!]\rangle \arrow[r, mapsto, "\text{Syntactic trans-}", "\text{formation } \mathbb{t}"'] \arrow[d, shift left, "\mathsf{S}"] & \begin{array}{c}\text{Transformed program}\\ \mathbb{t}[\![P]\!] \sqsupseteq \mathbb{p}[\mathrm{t}[\mathsf{S}[\![P]\!], \alpha(\mathsf{S}[\![P]\!])]]\end{array} \arrow[d, shift right, "\mathsf{S}"'] \\
\begin{array}{c}\text{Subject}\\ \text{program}\\ \text{semantics}\\ \mathsf{S}[\![P]\!]\end{array} \arrow[u, shift right, "\mathbb{p}"'] \arrow[r, mapsto, "\text{Semantic}", "\text{abstraction } \alpha"'] \arrow[dr, "\alpha_{\mathcal{O}}"', shift right] & \langle \mathsf{S}[\![P]\!],\ \alpha(\mathsf{S}[\![P]\!])\rangle \arrow[u, shift left, "\mathbb{p}"] \arrow[u, dotted, shift right=6, "\sqsupseteq"] \arrow[r, mapsto, "\text{Semantic trans-}", "\text{formation t}"'] \arrow[d, shift right, "\alpha_{\mathcal{O}}"'] & \begin{array}{c}\text{Transformed}\\ \text{program semantics}\\ \mathrm{t}[\mathsf{S}[\![P]\!], \alpha(\mathsf{S}[\![P]\!])] \sqsubseteq \mathsf{S}[\![\mathbb{t}[\![P]\!]]\!]\end{array} \arrow[u, shift right, "\mathbb{p}"'] \arrow[dl, shift left, "\alpha_{\mathcal{O}}"] \\
& \alpha_{\mathcal{O}}(\mathsf{S}[\![P]\!]) = \alpha_{\mathcal{O}}(\mathrm{t}[\mathsf{S}[\![P]\!], \alpha(\mathsf{S}[\![P]\!])) = \alpha_{\mathcal{O}}(\mathsf{S}[\![\mathbb{t}[\![P]\!]]\!]) \arrow[ul, dashed, shift right, "\gamma_{\mathcal{O}}"'] \arrow[u, dashed, shift right, "\gamma_{\mathcal{O}}"'] \arrow[ur, dashed, shift left, "\gamma_{\mathcal{O}}"] & \begin{array}{c}\text{Observational}\\ \text{abstraction}\end{array}
\end{tikzcd}$$

Design of Program Transformations by Abstract Interpretation

Back to Principles ...

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Subject program } \mathrm{P} \arrow[rr, mapsto, "\text{Syntactic}", "\text{transformation } \mathbb{t}"'] \arrow[d, shift right, "\mathsf{S}"'] & & \text{Transformed program } \boxed{\mathbb{t}\llbracket \mathrm{P} \rrbracket \sqsupseteq \mathbb{p}[\mathsf{t}[\mathsf{S}\llbracket \mathrm{P} \rrbracket]]} \arrow[d, shift right, "\mathsf{S}"'] \\
\text{Subject program semantics } \mathsf{S}\llbracket \mathrm{P} \rrbracket \arrow[u, shift right, two heads, "\mathbb{p}"'] \arrow[rr, mapsto, "\text{Semantic}", "\text{transformation } \mathsf{t}"'] \arrow[dr, "\alpha_{\mathcal{O}}"'] & & \text{Transformed program semantics } \mathsf{t}[\mathsf{S}\llbracket \mathrm{P} \rrbracket] \sqsubseteq \mathsf{S}\llbracket \mathbb{t}\llbracket \mathrm{P} \rrbracket \rrbracket \arrow[u, shift right, two heads, "\mathbb{p}"'] \arrow[dl, "\alpha_{\mathcal{O}}"] \arrow[d, phantom, "\text{Observational abstraction}"] \\
& \alpha_{\mathcal{O}}(\mathsf{S}\llbracket \mathrm{P} \rrbracket) = \alpha_{\mathcal{O}}(\mathsf{t}[\mathsf{S}\llbracket \mathrm{P} \rrbracket]) = \alpha_{\mathcal{O}}(\mathsf{S}\llbracket \mathbb{t}\llbracket \mathrm{P} \rrbracket \rrbracket) \arrow[ul, dashed, bend left, "\gamma_{\mathcal{O}}"'] \arrow[ur, dashed, bend right, "\gamma_{\mathcal{O}}"] & \;
\end{tikzcd}$$

Design of Program Transformation Algorithms

$$\begin{aligned}
\mathbb{t}[\![\mathrm{P}]\!] &\sqsupseteq \mathbb{p}[\mathsf{t}[\mathsf{S}[\![\mathrm{P}]\!]]] \\
&= \mathbb{p}[\mathsf{t}[\mathbf{lfp}^{\subseteq}\ \mathsf{F}^{*}[\![\mathrm{P}]\!]]] \\
&\sqsupseteq \ldots \\
&= \mathbf{lfp}^{\subseteq^{\sharp}}\ \mathbb{F}^{\sharp}[\![\mathrm{P}]\!]
\end{aligned}$$

← apply fixpoint transfer /approximation theorems

The Iterative Constant Propagation Algorithm

```
**ConstantPropagation**$(\mathtt{P}\,,\ \rho^\sharp) =$
    $\mathtt{Q} := \emptyset$;
    **forall**  label $\mathtt{L}$ of $\mathtt{P}$ such that $\rho^\sharp(\mathtt{L}) \neq \dot{\bot}$  **do**
        **forall**  $\mathtt{L} : \mathtt{A} \longrightarrow \mathtt{L}_1 ;\ \in \mathtt{P}$  **do**
            $\mathtt{A}_c :=$ **Simplify**$[\![\mathtt{A}]\!](\rho^\sharp(\mathtt{L}))$;
            $\mathtt{Q} := \mathtt{Q} \cup \{\mathtt{L} : \mathtt{A}_c \longrightarrow \mathtt{L}_1 ;\}$
        **end**;
        **if**  $\mathtt{L} : \mathtt{stop};\ \in \mathtt{P}$  **then**
            $\mathtt{Q} := \mathtt{Q} \cup \{\mathtt{L} : \mathtt{stop};\}$
        **end**
    **end**;
    **return** $\mathtt{Q}$.
```

Other Program Transformations Formally Handled in the Same Way

- In this talk, the approach was illustrated on the trivial constant propagation example;
- The same approach has been successfully applied to:
  - Blocking command elimination (ENTCS v. 45);
  - Program monitoring (POPL'02);
  - Program reduction (e.g. transition compression);
  - Slicing;

Conclusion

Conclusion

- Program transformation is understood as an abstraction of a semantic transformation of run-time execution;
- Leads to a unified framework for semantics-based program analysis and transformation;
- The benefit is presently purely foundational and conceptual;
- Practical application: reanalysis of assembler code from source requires the formalization of the compilation process.