Giorgio Levi's Festschrift Workshop

Abstract Interpretation of Resolution-Based Semantics

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Objective

Our objective

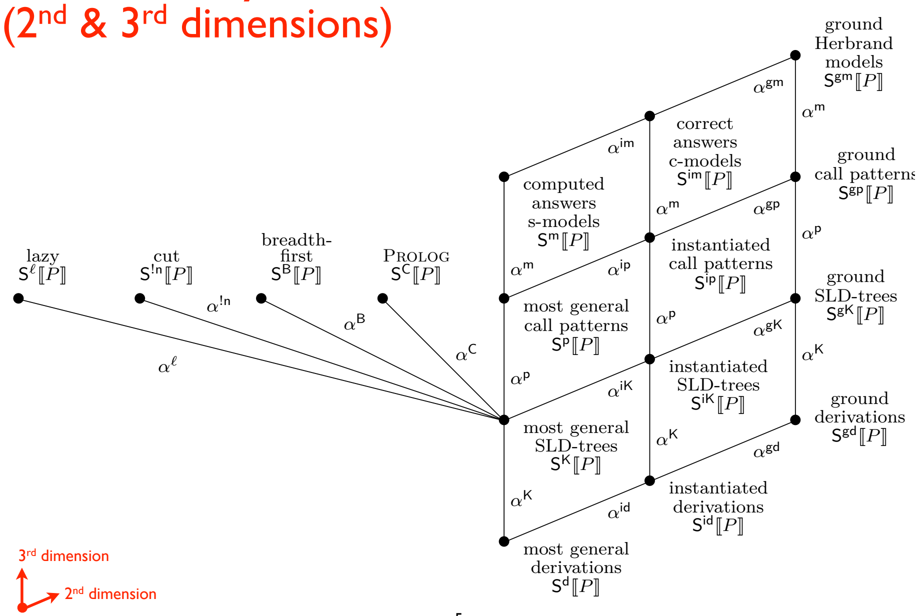
- To understand the work of Giorgio Levi on the semantics of logic programming languages for static analysis
- By reconstructing the semantics of Resolution-based/ Logic Programming…

  …by abstract interpretations of a concrete semantics

  …chosen to be a branching-time trace-based semantics (built from a state transition system)
- In passing, we get some novel semantics that tackle impure characteristics of real implementations.

Result

A Hierarchy of Abstractions and Semantics (2nd & 3rd dimensions)



Syntax

Syntax of logic programs

$f \in \mathbb{f}$	function symbols
$v \in \mathbb{v}$	variable symbols
$\vec{v} = v_1, \ldots, v_n$	sequences of variables ($\vec{\epsilon}$)
$T, U, \ldots \in \mathbb{t}$	terms built on $\mathbb{f}$ and $\vec{v} \in \vec{\mathbb{v}}$
$p \in \mathbb{p}$	predicate symbols
$A, B \in \mathbb{A}$	atoms built on $\mathbb{p}$ and $\mathbb{t}$
$\boldsymbol{B} = B_1 \ldots B_n \in \mathbb{B}$	sequences of atoms ($\boldsymbol{\varepsilon}$), body
$C = A \leftarrow \boldsymbol{B} \in \mathbb{C}$	definite clauses (unit clauses $\boldsymbol{B} = \boldsymbol{\varepsilon}$)
$P \in \mathbb{P}^n \triangleq [0, n[\mapsto \mathbb{C}$	Prolog programs

$$0: \quad \mathtt{n}(0) \leftarrow$$

$$1: \quad \mathtt{n}(\mathtt{s}(x)) \leftarrow \mathtt{n}(x)$$

$\alpha^{\mathbb{L}}(P) \triangleq \{P_1, \ldots, P_n\} \in \mathbb{L}$	abstraction to logic programs
$\mathbb{G} \triangleq \{p(v) \mid p \in \mathbb{p} \wedge v \in \mathbb{v}\}$	most general atomic goals

Substitutions

$\vartheta, \sigma \in \mathbb{S}$	substitutions (ε)
$\vartheta(T)$	application to a term T
$\vartheta\vert_e$	restriction to variables of expression e
$\vartheta \circ \sigma$	composition
$\vartheta \preceq \sigma$	pre-order
$\vartheta \simeq \vartheta'$	equivalence (renaming)
$\langle \mathbb{S}^{\circ}/_{\simeq}, \preceq \rangle$	complete lattice of idempotent substitutions up to renaming
$\mathscr{T}^{\emptyset}/_{\simeq}$	similarly for terms up to renaming

Unification

$mgu(\mathscr{T}) = \{\sigma\}$ most general unifier of a set $\mathscr{T}$ of terms

$\triangleq \varnothing$ not unifiable

$mgu(\mathcal{E})$ most general unifier of a set of equations $\mathcal{E} = \{T_i = U_i \mid i \in \Delta\}$

$\uparrow \in \mathbb{S}^\circ/_\simeq \times \mathbb{S}^\circ/_\simeq \mapsto \mathbb{S}^\circ/_\simeq$ parallel composition of idempotent substitutions

Operational semantics defined by a labelled transition system

Labelled transition system

$$\mathsf{S}^{\mathsf{t}}[\![P]\!] \triangleq \langle \mathscr{E}, \mathscr{L}, \longrightarrow^{\mathsf{t}}, \mathscr{I} \rangle$$

Semantics

Program

States

Labels

Transition relation

Initial states

$\longrightarrow \in \wp(\mathscr{E} \times \mathscr{L} \times \mathscr{E})$

$\mathtt{0:}\quad \mathtt{n(0)} \leftarrow$

$\mathtt{1:}\quad \mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}$

States

$states\ \eta \in \mathscr{E} \triangleq \mathscr{S} \times \mathbb{S}$

$\eta = \langle \varpi, \vartheta \rangle$

Stack Substitution

Stacks

$\varpi \in \mathscr{S} \triangleq \mathscr{K}^+ \triangleq (\mathbb{C}^{\cdot} \cup \mathscr{M})^+$

- $[\vdash A]$ Initial stack for goal A
- $[\dashv\Box]$ Empty stack final marker

$\Big\}$ markers in $\mathscr{M}$

- $\mathbb{C}^{\cdot} \triangleq \{[\mathtt{i}:A \leftarrow \boldsymbol{B}.\boldsymbol{B}'] \mid \mathtt{i}:A \leftarrow \boldsymbol{B}\,\boldsymbol{B}' \in P\}$ specifying the control state of the derivation ($\boldsymbol{B}$ has been derived while $\boldsymbol{B}'$ is still to be derived) or a *marker* $\mathscr{M}$

Initial states

$$\mathscr{I} \triangleq \{\langle[\vdash A],\ \vartheta\rangle \mid A \in \mathbb{A} \land \vartheta \in \mathbb{S}\}$$

goal $\vartheta(A)$ (most often ϑ is chosen as the empty substitution ε)

Transition labels

- $(\!|\, \mathtt{i}:A' \leftarrow \boldsymbol{B}/\sigma$: apply renamed-apart clause $\mathtt{i}:A' \leftarrow \boldsymbol{B}$ to prove goal A, such that A and A' unify by $\sigma \in mgu(\vartheta(A), A')$
- $\mathtt{i}:A \leftarrow \boldsymbol{B}\,|\!)$: the proof of $\boldsymbol{B}$ is finished

Labelled transition relation $\xrightarrow{\ell}{}^{\mathsf{t}}$, $\ell \in \mathscr{L}$

- Start from goal $\vartheta(A)$, apply clause $\mathtt{i}:A \leftarrow \boldsymbol{B}$, prove new goal $\sigma \uparrow \vartheta(\boldsymbol{B})$:

$$\langle[\vdash A],\ \vartheta\rangle \xrightarrow{(\!|\, \mathtt{i}:A' \leftarrow \boldsymbol{B}/\sigma}{}^{\mathsf{t}} \langle[\dashv\Box][\mathtt{i}:A' \leftarrow \boldsymbol{.B}],\ \vartheta'\rangle$$
$$\text{if}\quad \mathtt{i}:A' \leftarrow \boldsymbol{B} \Subset P,\ \sigma \in mgu(\vartheta(A), A'),\ \vartheta' \in \sigma \uparrow \vartheta \qquad (2)$$

- Start from subgoal $\vartheta(B)$, apply clause $\mathtt{j}:B' \leftarrow \boldsymbol{B}''$, prove new goal $\sigma \uparrow \vartheta(\boldsymbol{B}'')$:

$$\langle\varpi[\mathtt{i}:A \leftarrow \boldsymbol{B.}B\boldsymbol{B}'],\ \vartheta\rangle \xrightarrow{(\!|\, \mathtt{j}:B' \leftarrow \boldsymbol{B}''/\sigma}{}^{\mathsf{t}} \langle\varpi[\mathtt{i}:A \leftarrow \boldsymbol{B}B\boldsymbol{.B}'][\mathtt{j}:B' \leftarrow \boldsymbol{.B}''],\ \vartheta'\rangle$$
$$\text{if}\quad \mathtt{i}:A \leftarrow \boldsymbol{B}B\boldsymbol{B}', \mathtt{j}:B' \leftarrow \boldsymbol{B}'' \Subset P,\ \sigma \in mgu(\vartheta(B), B'),\ \vartheta' \in \sigma \uparrow \vartheta \qquad (3)$$

Let $\mathtt{i}:A \leftarrow \boldsymbol{B} \Subset P$ means that $\mathtt{i}:A \leftarrow \boldsymbol{B}$ is a clause of the PROLOG program P renamed/standardized apart using fresh variables

Labelled transition relation $\xrightarrow{\ell}{}^{\mathsf{t}}$, $\ell \in \mathscr{L}$

- Proof of $\boldsymbol{B}$ is finished, go back to previous goal on stack:

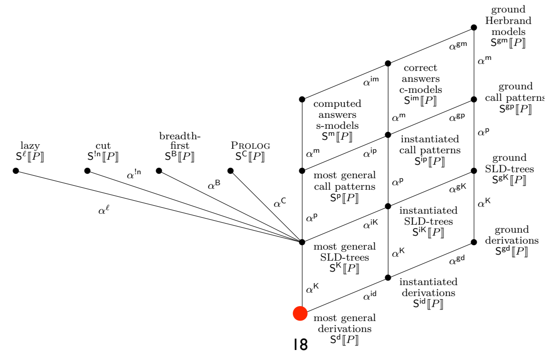
$$\langle\varpi[\mathtt{i}:A \leftarrow \boldsymbol{B.}],\ \vartheta\rangle \xrightarrow{\mathtt{i}:A \leftarrow \boldsymbol{B}\,|\!)}{}^{\mathsf{t}} \langle\varpi,\ \vartheta\rangle \qquad \text{if}\quad \mathtt{i}:A \leftarrow \boldsymbol{B} \Subset P\ . \qquad (4)$$

Example:

$$
\begin{array}{ll}
\mathtt{0:} & \mathtt{n(0)} \leftarrow \\
\mathtt{1:} & \mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}
\end{array}
$$

$$
\begin{array}{ll}
\langle [\vdash \mathtt{n(s(s(0)))}],\ \varepsilon\rangle & \wr\text{initial state}\wr \\
\xrightarrow{(\!| 1:\mathtt{n(s(}x\mathtt{))}\leftarrow\mathtt{n(}x\mathtt{)}/\{x\leftarrow\mathtt{s(0)}\}}{}_{\mathsf{t}} & \wr\text{by } (2)\wr \\
\langle [\dashv\square][1:\mathtt{n(s(}x\mathtt{))} \leftarrow \centerdot\mathtt{n(}x\mathtt{)}],\ \{x \leftarrow \mathtt{s(0)}\}\rangle & \\
\xrightarrow{(\!| 1:\mathtt{n(s(}x'\mathtt{))}\leftarrow\mathtt{n(}x'\mathtt{)}/\{x'\leftarrow\mathtt{0}\}}{}_{\mathsf{t}} & \wr\text{by } (3)\wr \\
\langle [\dashv\square][1:\mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}\centerdot][1:\mathtt{n(s(}x'\mathtt{))} \leftarrow \centerdot\mathtt{n(}x'\mathtt{)}],\ \{x \leftarrow \mathtt{s(0)}, x' \leftarrow \mathtt{0}\}\rangle & \\
\xrightarrow{(\!| 0:\mathtt{n(0)}\leftarrow/\varepsilon}{}_{\mathsf{t}} & \wr\text{by } (3)\wr \\
\langle [\dashv\square][1:\mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}\centerdot][1:\mathtt{n(s(}x'\mathtt{))} \leftarrow \mathtt{n(}x'\mathtt{)}\centerdot][0:\mathtt{n(0)} \leftarrow \centerdot], & \\
\quad \{x \leftarrow \mathtt{s(0)}, x' \leftarrow \mathtt{0}\}\rangle & \\
\xrightarrow{0:\mathtt{n(0)}\leftarrow |\!)}{}_{\mathsf{t}} & \wr\text{by } (4)\wr \\
\langle [\dashv\square][1:\mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}\centerdot][1:\mathtt{n(s(}x'\mathtt{))} \leftarrow \mathtt{n(}x'\mathtt{)}\centerdot],\ \{x \leftarrow \mathtt{s(0)}, x' \leftarrow \mathtt{0}\}\rangle & \\
\xrightarrow{1:\mathtt{n(s(}x'\mathtt{))}\leftarrow\mathtt{n(}x'\mathtt{)} |\!)}{}_{\mathsf{t}} & \wr\text{by } (4)\wr \\
\langle [\dashv\square][1:\mathtt{n(s(}x\mathtt{))} \leftarrow \mathtt{n(}x\mathtt{)}\centerdot],\ \{x \leftarrow \mathtt{s(0)}, x' \leftarrow \mathtt{0}\}\rangle & \\
\xrightarrow{1:\mathtt{n(s(}x\mathtt{))}\leftarrow\mathtt{n(}x\mathtt{)} |\!)}{}_{\mathsf{t}} & \wr\text{by } (4)\wr \\
\langle [\dashv\square],\ \{x \leftarrow \mathtt{s(0)}, x' \leftarrow \mathtt{0}\}\rangle & \square
\end{array}
$$

Most general maximal terminal derivation semantics of logic programs



Transitional Most General Maximal Derivation Semantics

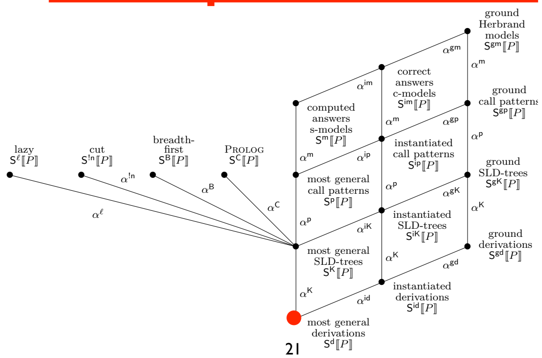
- Maximal traces generated by the transition system starting from most general goals:

$$
\begin{aligned}
\mathsf{S}^{\mathsf{d}}[\![P]\!] \triangleq \{\eta_0 \xrightarrow{\ell_0} \eta_1 \ldots \eta_{n-1} \xrightarrow{\ell_{n-1}} \eta_n \in \boldsymbol{\Theta}[n+1] \mid n \geqslant 0 \wedge {} \\
\eta_0 = \langle [\vdash p(v)],\ \varepsilon\rangle \wedge p \in \mathbb{p} \wedge v \in \mathbb{v} \wedge \forall i \in [0, n-1] : \eta_i \xrightarrow{\ell_i}{}^{\mathsf{t}} \eta_{i+1} \wedge {} \\
\forall \eta \in \mathscr{S} : \forall \ell \in \mathscr{L} : \neg(\eta_n \xrightarrow{\ell}{}^{\mathsf{t}} \eta)\}\ .
\end{aligned}
$$

Final states

- *answer substitution* states in $\mathscr{E}^{\mathrm{AS}} \triangleq \{\langle [\dashv\square],\ \vartheta\rangle \mid \vartheta \in \mathbb{S}\}$ for *successful traces*, or
- *finite failure* states in $\mathscr{E}^{\mathrm{FF}} \triangleq \{\langle \varpi[\mathtt{i}:A \leftarrow \boldsymbol{B}\centerdot B\boldsymbol{B}'],\ \vartheta\rangle \mid \forall\ \mathtt{j}:B' \leftarrow \boldsymbol{B}'' \Cap P : mgu(\vartheta(B), B') = \varnothing\}$ for *failing traces*.

Most general maximal terminal derivation semantics of logic programs in fixpoint form



Transitional Most General Maximal Derivation Semantics in Fixpoint Form

Theorem 20 $\mathsf{S}^{\mathsf{d}}[\![P]\!] = \mathit{lfp}^{\subseteq}\, \hat{\mathsf{F}}^{\mathsf{d}}[\![\overline{P}]\!]$.

$\hat{\mathsf{F}}^{\mathsf{d}}[\![P]\!] \in \wp(\boldsymbol{\Theta}) \mapsto \wp(\boldsymbol{\Theta})$

$$\hat{\mathsf{F}}^{\mathsf{d}}[\![P]\!] \triangleq \boldsymbol{\lambda}\Theta\boldsymbol{\cdot} \bigcup_{\mathtt{i}:A\leftarrow B\in P,\, p\in\mathbb{p},\, v\in\mathbb{V},\, \vartheta\in mgu(p(v),A)} \langle[\vdash p(v)],\, \varepsilon\rangle \xrightarrow{(\!|\mathtt{i}:A\leftarrow B/\vartheta|\!)} \hat{\mathsf{F}}^{\mathsf{d}}_{\cdot}[\mathtt{i}:A \leftarrow \boldsymbol{.B}]\,\vartheta\,\Theta \qquad (9)$$

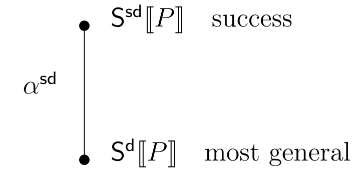
$\hat{\mathsf{F}}^{\mathsf{d}}_{\cdot}[\mathtt{i}:A \leftarrow \boldsymbol{B.B'}] \in \mathbb{S} \mapsto \wp(\boldsymbol{\Theta}) \mapsto \wp(\boldsymbol{\Theta})$

$$\hat{\mathsf{F}}^{\mathsf{d}}_{\cdot}[\mathtt{i}:A \leftarrow \boldsymbol{B.BB'}] \triangleq \boldsymbol{\lambda}\vartheta\boldsymbol{\cdot}\boldsymbol{\lambda}\Theta\boldsymbol{\cdot} \qquad (10)$$
$$\{(\langle[\dashv\Box][\mathtt{i}:A\leftarrow \boldsymbol{B.BB'}],\ [\dashv\Box][\mathtt{i}:A\leftarrow \boldsymbol{BB.B'}],\ \vartheta\rangle \Uparrow^{\mathsf{d}} \eta \xrightarrow{\ell} \langle\varpi,\, \vartheta'\rangle)\,\mathring{\varsigma}\,\theta \mid$$
$$\eta \xrightarrow{\ell} \langle\varpi,\, \vartheta'\rangle \in \Theta.B' \wedge \sigma \in mgu(B,B') \wedge \theta \in \hat{\mathsf{F}}^{\mathsf{d}}_{\cdot}[\mathtt{i}:A\leftarrow \boldsymbol{BB.B'}]\,(\vartheta\uparrow\sigma\uparrow\vartheta'^{\,3})\,\Theta\}$$

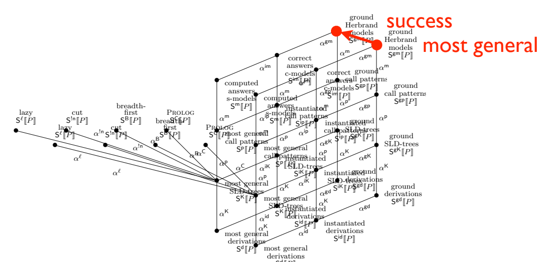
$$\hat{\mathsf{F}}^{\mathsf{d}}_{\cdot}[\mathtt{i}:A \leftarrow \boldsymbol{B.}] \triangleq \boldsymbol{\lambda}\vartheta\boldsymbol{\cdot}\boldsymbol{\lambda}\Theta\boldsymbol{\cdot}\{\langle[\dashv\Box][\mathtt{i}:A\leftarrow \boldsymbol{B.}],\ \vartheta\rangle \xrightarrow{\mathtt{i}:A\leftarrow B|\!)} \langle[\dashv\Box],\ \vartheta\rangle\}\ . \quad (11)$$

Abstractions of the trace semantics

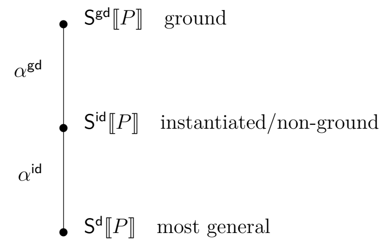
1st dimension: Partial correctness Abstractions



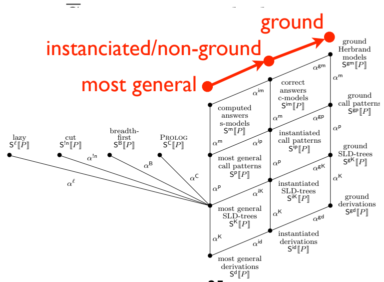
The *success abstraction* eliminates finite failures



2nd dimension: Instantiation Abstractions

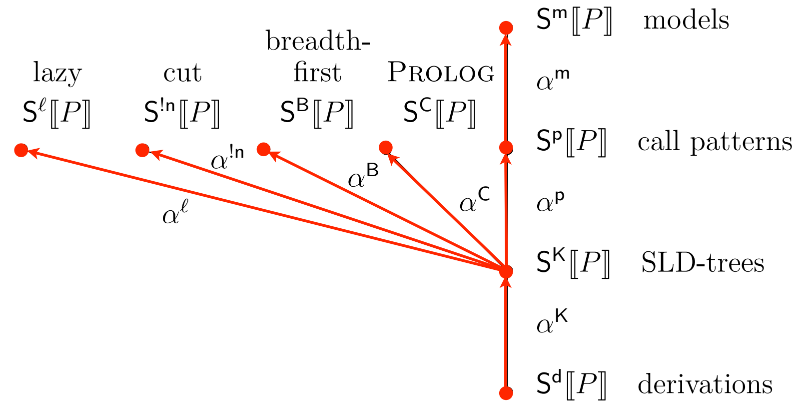


The *derivation ground instantiation abstraction* maps derivations for non-ground goals to derivations for ground instantiations of these goals.



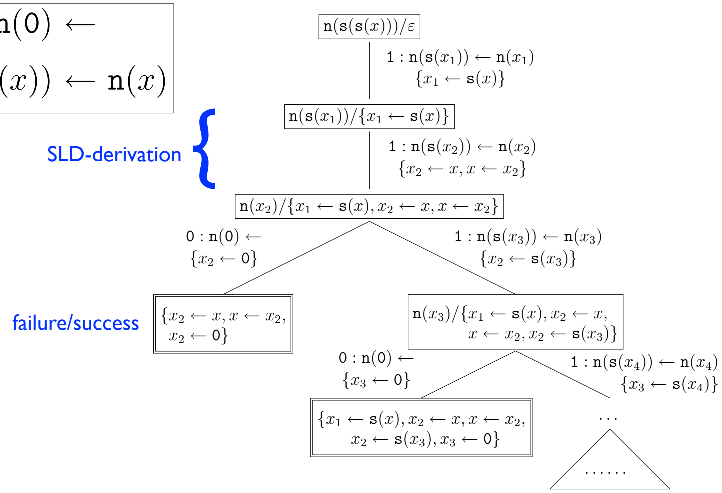
3rd dimension: Computational Information Abstractions

- Abstract away the information provided by a computation



SLD trees

0:	$\mathtt{n}(0) \leftarrow$
1:	$\mathtt{n}(\mathtt{s}(x)) \leftarrow \mathtt{n}(x)$



SLD abstraction

- The SLD-abstraction collects the nodes of the SLD-tree from the states of traces.
- The SLD-trees are built from traces by grouping their common prefixes in the order of the PROLOG program clauses.

$$\alpha^{\mathsf{K}}(\langle[\vdash A],\ \vartheta\rangle) \triangleq \boxed{\leftarrow \vartheta(A)/\vartheta}$$
$$\alpha^{\mathsf{K}}(\langle\varpi,\ \vartheta\rangle) \triangleq \boxed{\leftarrow \langle\alpha'^{\mathsf{K}}(\langle\varpi,\ \vartheta\rangle),\ \vartheta\rangle}$$
$$\alpha'^{\mathsf{K}}(\langle\varpi[\mathtt{i}:A \leftarrow \boldsymbol{B}\mathbf{.}B\boldsymbol{B}'],\ \vartheta\rangle) \triangleq \vartheta(B\boldsymbol{B}')\alpha'^{\mathsf{K}}(\langle\varpi,\ \vartheta\rangle)$$
$$\alpha'^{\mathsf{K}}(\langle[\dashv\Box],\ \vartheta\rangle) \triangleq \boldsymbol{\varepsilon}$$
$$\alpha^{\mathsf{K}}(\Theta) \triangleq \{\alpha^{\mathsf{K}}(\eta)[\![\mathtt{i}_1:\ell_1\alpha^{\mathsf{K}}(\Theta_1);\ldots;\mathtt{i}_n:\ell_n\alpha^{\mathsf{K}}(\Theta_n)]\!] \mid \eta \in \mathscr{E} \wedge \mathtt{i}_1 < \ldots < \mathtt{i}_n \wedge$$
$$\Theta.\eta = \bigcup_{k=1}^{n} \Theta_k \wedge \forall k \in [1,n] : \Theta_k = \{\theta \mid \eta \xrightarrow{(\!|\mathtt{i}_k:\ell_k}{}^{\mathsf{t}}\ \theta \in \Theta.\eta\} \neq \varnothing\} \cup$$
$$\alpha^{\mathsf{K}}(\{\theta \mid \eta \xrightarrow{\mathtt{i}:C|\!)}{}^{\mathsf{t}}\ \theta \in \Theta\}) \cup \{\boxed{\boxed{\vartheta}}\,[\![]\!] \mid \exists\vartheta : \langle[\dashv\Box],\ \vartheta\rangle \in \Theta\} \quad .$$

Call-patterns abstractions

- The *call-patterns abstraction* collects the goal, call-patterns and the answer substitution for each derivation, including those leading to finite failures

$$\alpha^{\mathsf{P}}(\langle \xi_i,\ i \in \Delta\rangle) \triangleq \bigcup\{\alpha^{\mathsf{P}}(\xi_i) \mid i \in \Delta\} \qquad \text{SLD derivation forest}$$

$$\alpha^{\mathsf{P}}(\boxed{\leftarrow A/\sigma}\ [\![\mathtt{i}_1 : A_1 \leftarrow \boldsymbol{B}_1/\vartheta_1\, \xi_1; \ldots ; \mathtt{i}_n : A_n \leftarrow \boldsymbol{B}_n/\vartheta_n\, \xi_n]\!]) \triangleq \qquad \text{SLD tree}$$

$$\alpha'^{\mathsf{P}}(\boxed{\leftarrow A/\sigma}\ [\![\mathtt{i}_1 : A_1 \leftarrow \boldsymbol{B}_1/\vartheta_1\, \xi_1; \ldots ; \mathtt{i}_n : A_n \leftarrow \boldsymbol{B}_n/\vartheta_n\, \xi_n]\!])(\sigma(A))$$

$$\alpha'^{\mathsf{P}}(\boxed{\leftarrow B\boldsymbol{B}/\sigma}\ [\![\mathtt{i}_1 : A_1 \leftarrow \boldsymbol{B}_1/\vartheta_1\, \xi_1; \ldots ; \mathtt{i}_n : A_n \leftarrow \boldsymbol{B}_n/\vartheta_n\, \xi_n]\!])A' \triangleq$$

$$\{\langle \sigma(A'),\ \sigma(B)\rangle\} \cup \bigcup_{i=1}^{n} \alpha'^{\mathsf{P}}(\xi_i)(A')$$

$$\alpha'^{\mathsf{P}}(\boxed{\leftarrow \boldsymbol{B}/\sigma}\ [\![]\!])A' \triangleq \varnothing \qquad \text{failure}$$

$$\alpha'^{\mathsf{P}}(\boxed{\boxed{\sigma}}\ [\![]\!])A' \triangleq \{\langle \sigma(A'),\ [\dashv\Box]\rangle\} \qquad \text{success.}$$

The model abstraction

- The *model abstraction* collects answers in the call patterns

$$\alpha^{\mathsf{m}}(K) \triangleq \{A \in \mathbb{A} \mid \langle A,\ [\dashv\Box]\rangle \in K\}$$

The PROLOG abstraction

- The PROLOG *abstraction* abstracts a forest $\langle \xi_i,\ i \in \Delta\rangle$ of SLD-trees $\xi_i,\ i \in \Delta$ into the set of execution traces corresponding to a depth-first traversal of these SLD-trees ξ_i (as in the PROLOG interpreter).
- SLD-trees may have infinite branches so the execution sequence, defined by transfinite recursion, may be transfinite (and is truncated to ω by PROLOG interpreters, which is a further abstraction).

$$\alpha^{\mathsf{C}}(\langle \xi_i,\ i \in \Delta\rangle) \triangleq \langle \alpha^{\mathsf{C}}(\xi_i),\ i \in \Delta\rangle$$

$$\alpha^{\mathsf{C}}(\boxed{\leftarrow \boldsymbol{B}/\sigma}\ [\![i_1 : A_1 \leftarrow \boldsymbol{B}_1/\vartheta_1 \xi_1; \ldots ; i_n : A_n \leftarrow \boldsymbol{B}_n/\vartheta_n\, \xi_n]\!]) \triangleq$$

$$\boxed{\leftarrow \boldsymbol{B}/\sigma}\ i_1 : A_1 \leftarrow \boldsymbol{B}_1/\vartheta_1 \alpha^{\mathsf{C}}(\xi_1) \ldots i_n : A_n \leftarrow \boldsymbol{B}_n/\vartheta_n \alpha^{\mathsf{C}}(\xi_n)$$

$$\alpha^{\mathsf{C}}(\boxed{\leftarrow \boldsymbol{B}/\sigma}\ [\![]\!]) \triangleq \epsilon$$

$$\alpha^{\mathsf{C}}(\boxed{\boxed{\sigma}}\ [\![]\!]) \triangleq \sigma \quad .$$

Fixpoint abstract semantics

Abstract semantics

1. Define an abstraction of the trace semantics
2. Constructively derive the abstract semantics in fixpoint form (by proving commutation and applying the exact fixpoint transfer theorem)

Computational design of the abstract fixpoint semantics

- The trace semantics is in fixpoint form $\mathsf{S}^{\mathsf{d}}[\![P]\!] = \mathit{lfp}^{\subseteq}\, \hat{\mathsf{F}}^{\mathsf{d}}[\![\overline{P}]\!]$
- So, by abstraction, the abstract fixpoint semantics also have a fixpoint definition
- Example: *Fixpoint s-semantics*

Theorem 24 (G. Levi et al.) $\mathsf{S}^{\mathsf{s}}[\![P]\!] = \mathit{lfp}^{\subseteq}\, \hat{\mathsf{F}}^{\mathsf{s}}[\![P]\!]$.

Let us define the *bottom-up call-patterns transformer* $\hat{\mathsf{F}}^{\mathsf{s}}[\![P]\!] \in \wp(\mathbb{A}) \mapsto \wp(\mathbb{A})$ for a PROLOG program $P \in \mathbb{P}$ as

$$\hat{\mathsf{F}}^{\mathsf{s}}[\![P]\!] \triangleq \boldsymbol{\lambda} \mathscr{A} \cdot \bigcup_{\mathtt{i}:A\leftarrow B\in P} \{\vartheta(A) \mid \vartheta \in \hat{\mathsf{F}}^{\mathsf{s}}_{\bullet}[\mathtt{i}:A \leftarrow .\boldsymbol{B}]\,\mathscr{A}\,\{\varepsilon\}\} \tag{12}$$

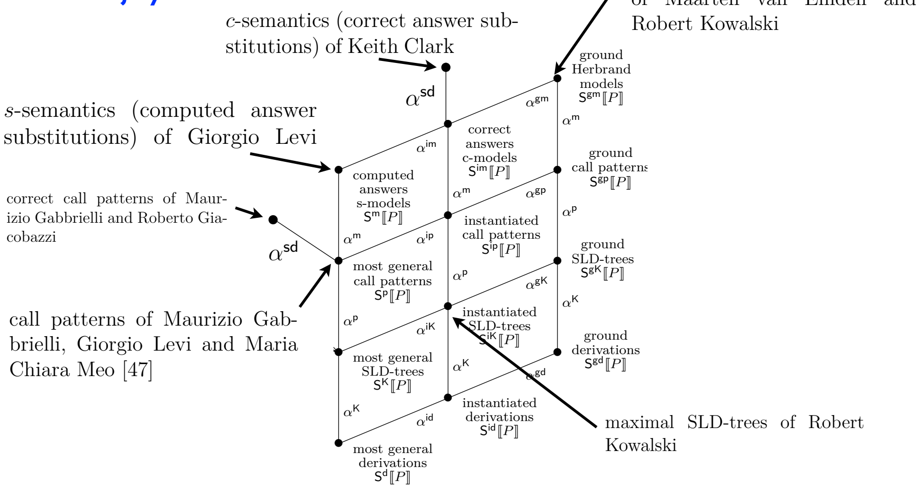
where the *clause transformer* $\hat{\mathsf{F}}^{\mathsf{s}}_{\bullet}[\mathtt{i}:A \leftarrow \boldsymbol{B}.\boldsymbol{B}'] \in \wp(\boldsymbol{\Theta}) \mapsto \wp(\mathbb{S}) \mapsto \wp(\mathbb{S})$ is defined as

$$\hat{\mathsf{F}}^{\mathsf{s}}_{\bullet}[\mathtt{i}:A \leftarrow \boldsymbol{B}.B\boldsymbol{B}'] \triangleq \boldsymbol{\lambda} \mathscr{A} \cdot \boldsymbol{\lambda} \mathscr{S} \cdot \{\vartheta' \mid B' \in \mathscr{A} \wedge \sigma \in mgu(B, B') \wedge \vartheta \in \mathscr{S} \wedge \vartheta' \in \hat{\mathsf{F}}^{\mathsf{s}}_{\bullet}[\mathtt{i}:A \leftarrow \boldsymbol{B}B.\boldsymbol{B}']\,\mathscr{A}\,(\vartheta \uparrow \sigma)\} \tag{13}$$

$$\hat{\mathsf{F}}^{\mathsf{s}}_{\bullet}[\mathtt{i}:A \leftarrow \boldsymbol{B}.] \triangleq \boldsymbol{\lambda} \mathscr{A} \cdot \boldsymbol{\lambda} \mathscr{S} \cdot \mathscr{S}\ . \tag{14}$$

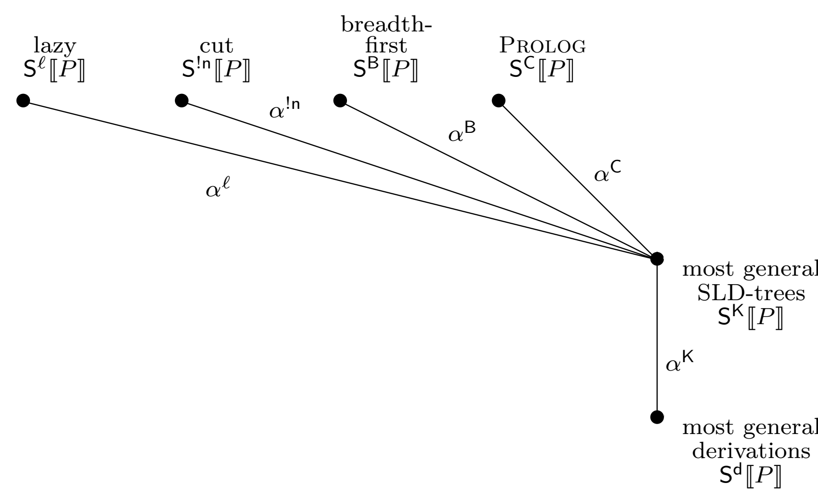
Conclusion

1. The joy of research



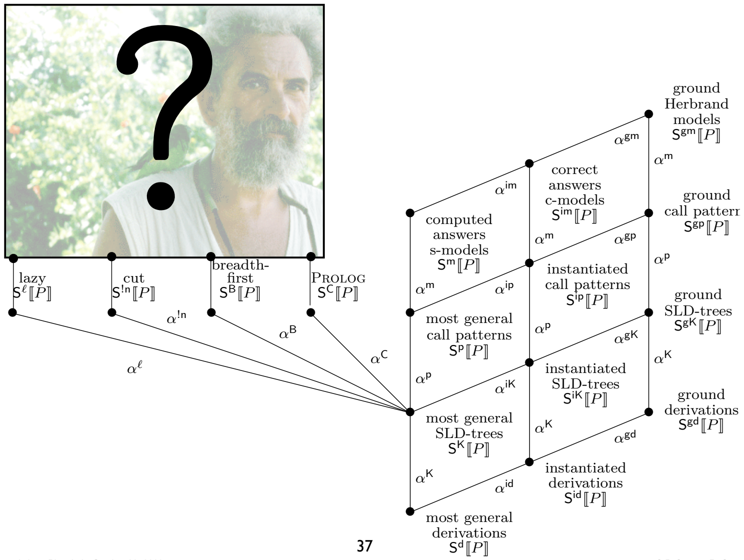
Conclusion (cont'd)

2. Life is hard!



Conclusion (cont'd)

3. Future work for Giorgio



Thank you