Abstract Interpretation

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This is an abstract interpretation

Scientific research

Scientific research

- In Mathematics/Physics:

  trend towards unification and synthesis through universal principles

- In Computer science:

  trend towards dispersion and parcellation through a ever-growing collection of local ad-hoc techniques for specific applications

  An exponential process, will stop!

Example: reasoning on computational structures

WCET

Axiomatic semantics

Confidentiality analysis

Program synthesis

Grammar analysis

Statistical model-checking

Invariance proof

Probabilistic verification

Parsing

Security protocole verification

Partial evaluation

Effect systems

Trace semantics

Symbolic execution

Type theory

Dataflow analysis

Model checking

Obfuscation

Denotational semantics

Theories combination

Code contracts

Quantum entanglement detection

Steganography

Systems biology analysis

Database query

Dependence analysis

CEGAR

Program transformation

Interpolants

Integrity analysis

Abstract model checking

Bisimulation

SMT solvers

Tautology testers

Operational semantics

Abstraction refinement

Type inference

Separation logic

Termination proof

Shape analysis

Malware detection

Code refactoring

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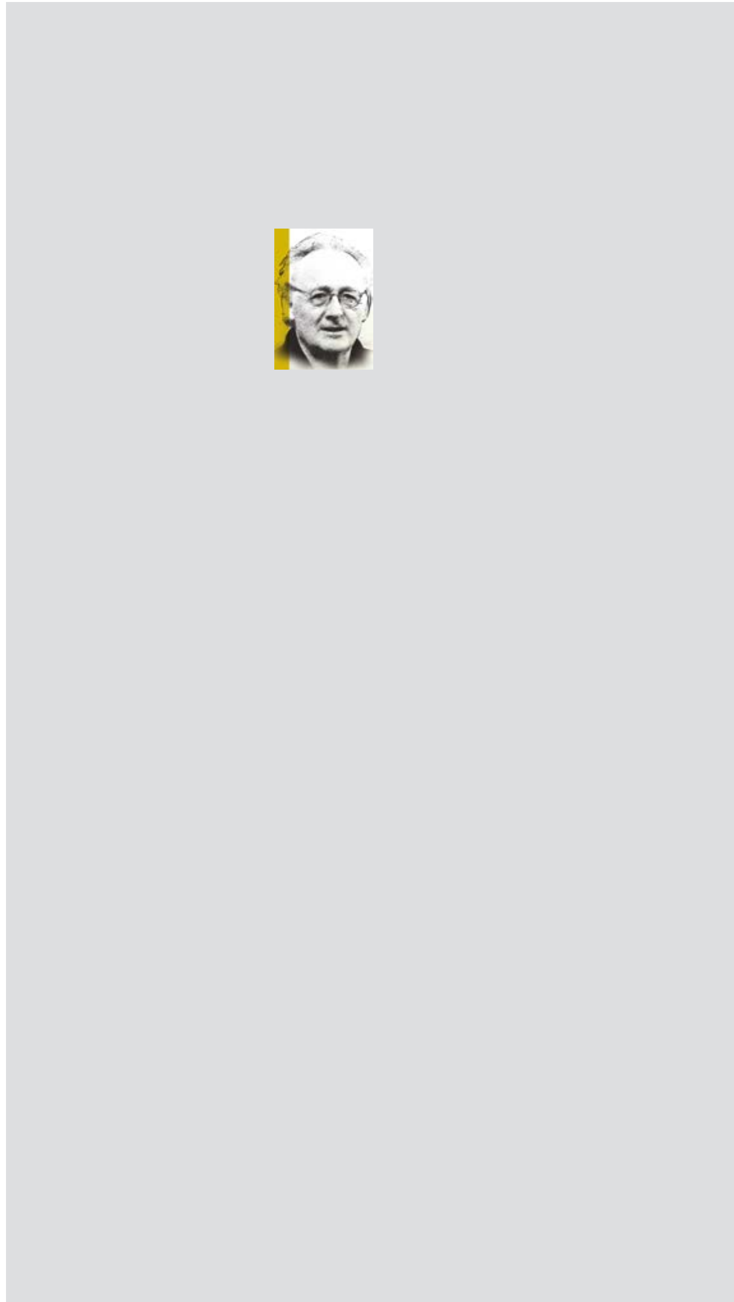
Intuition 1

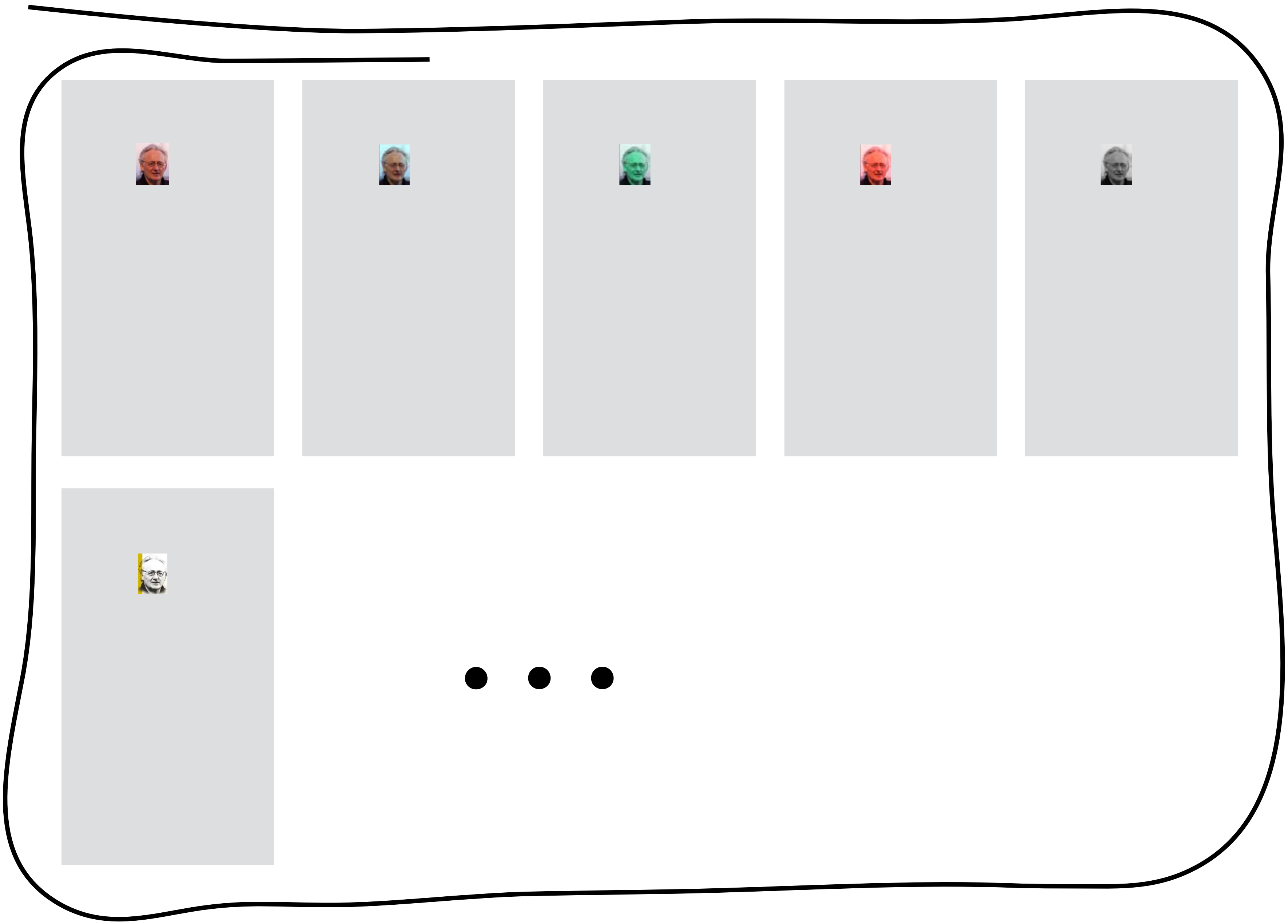
Concrete

Abstraction 1

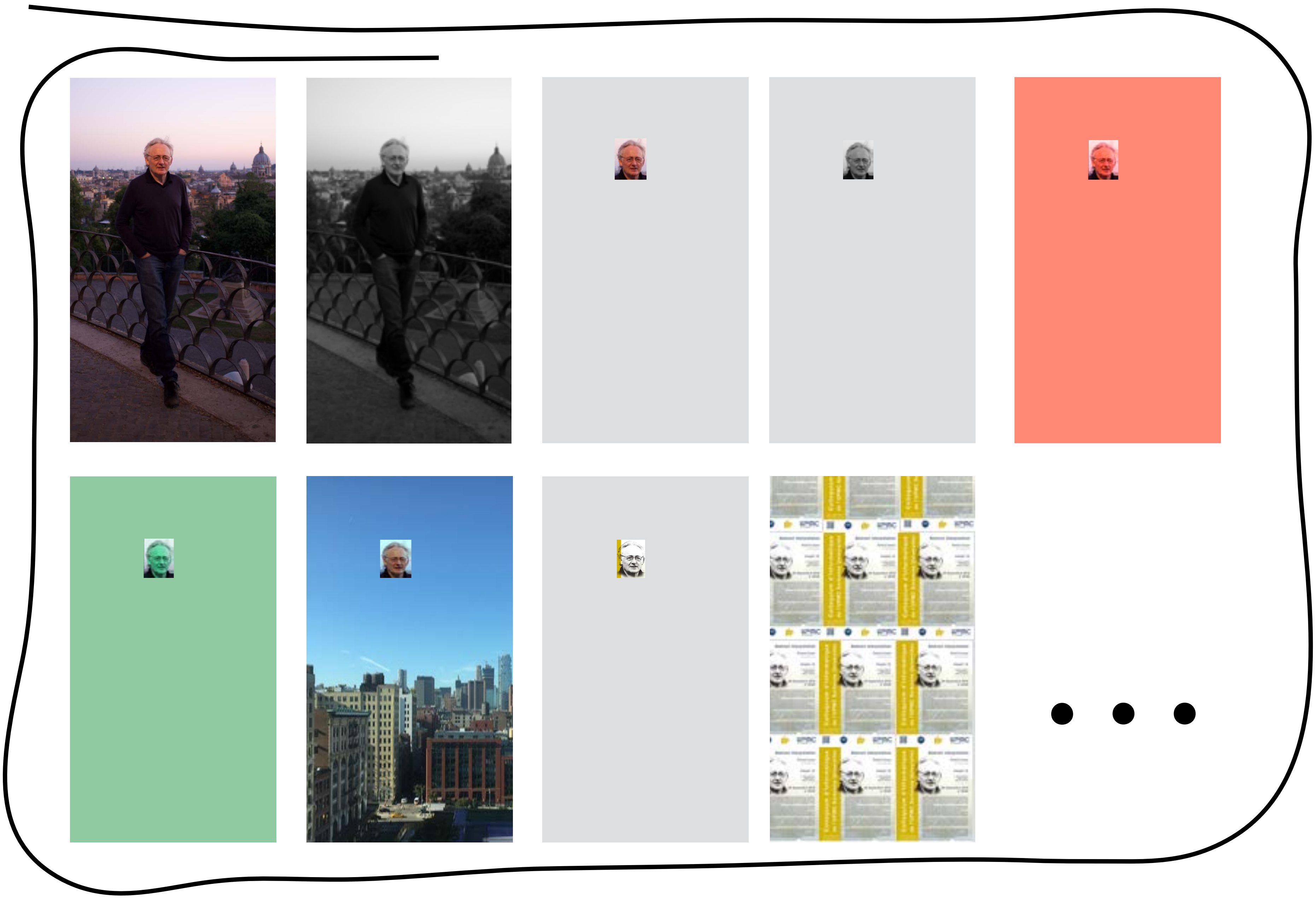
Abstraction 2



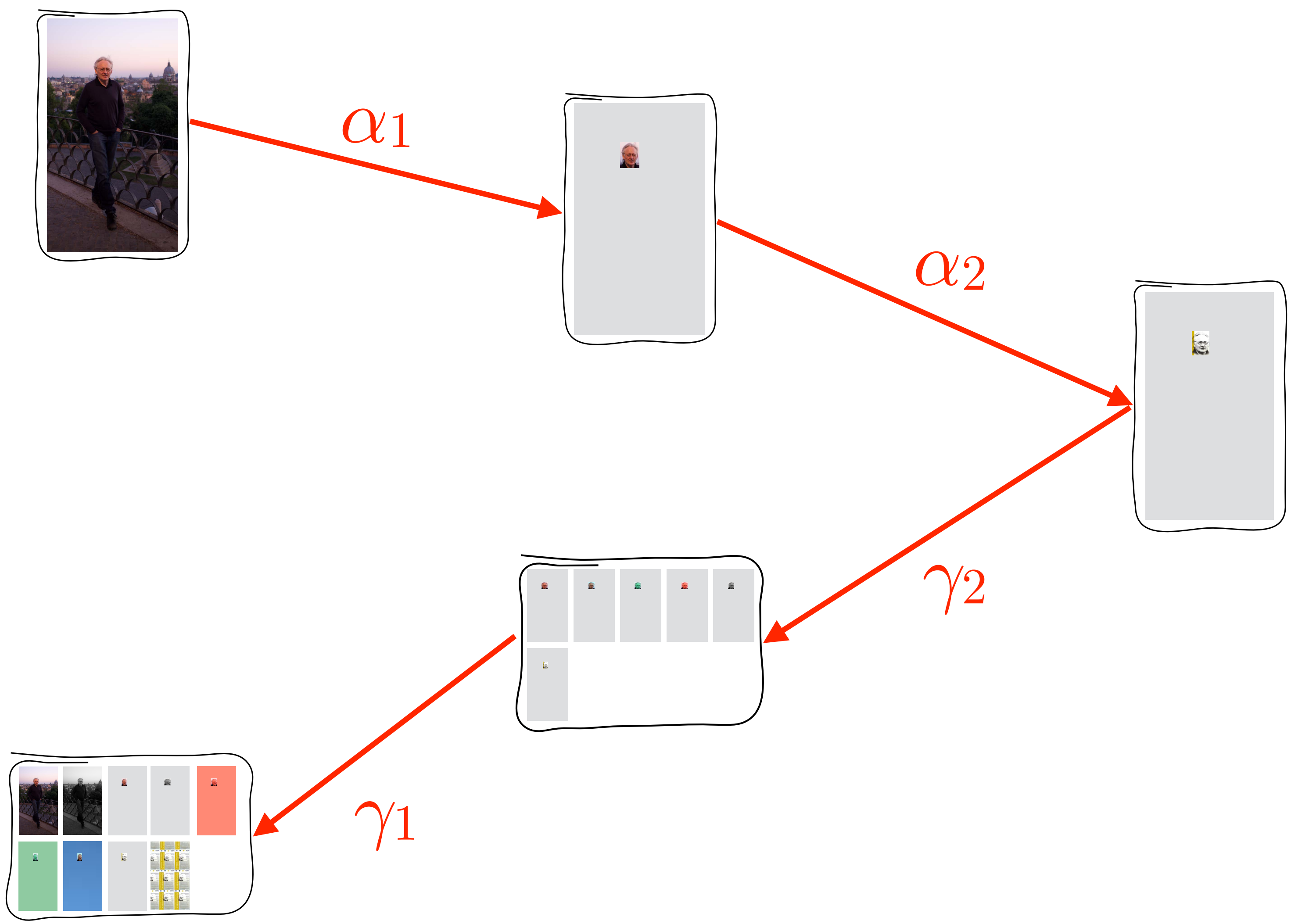
Concretization 2



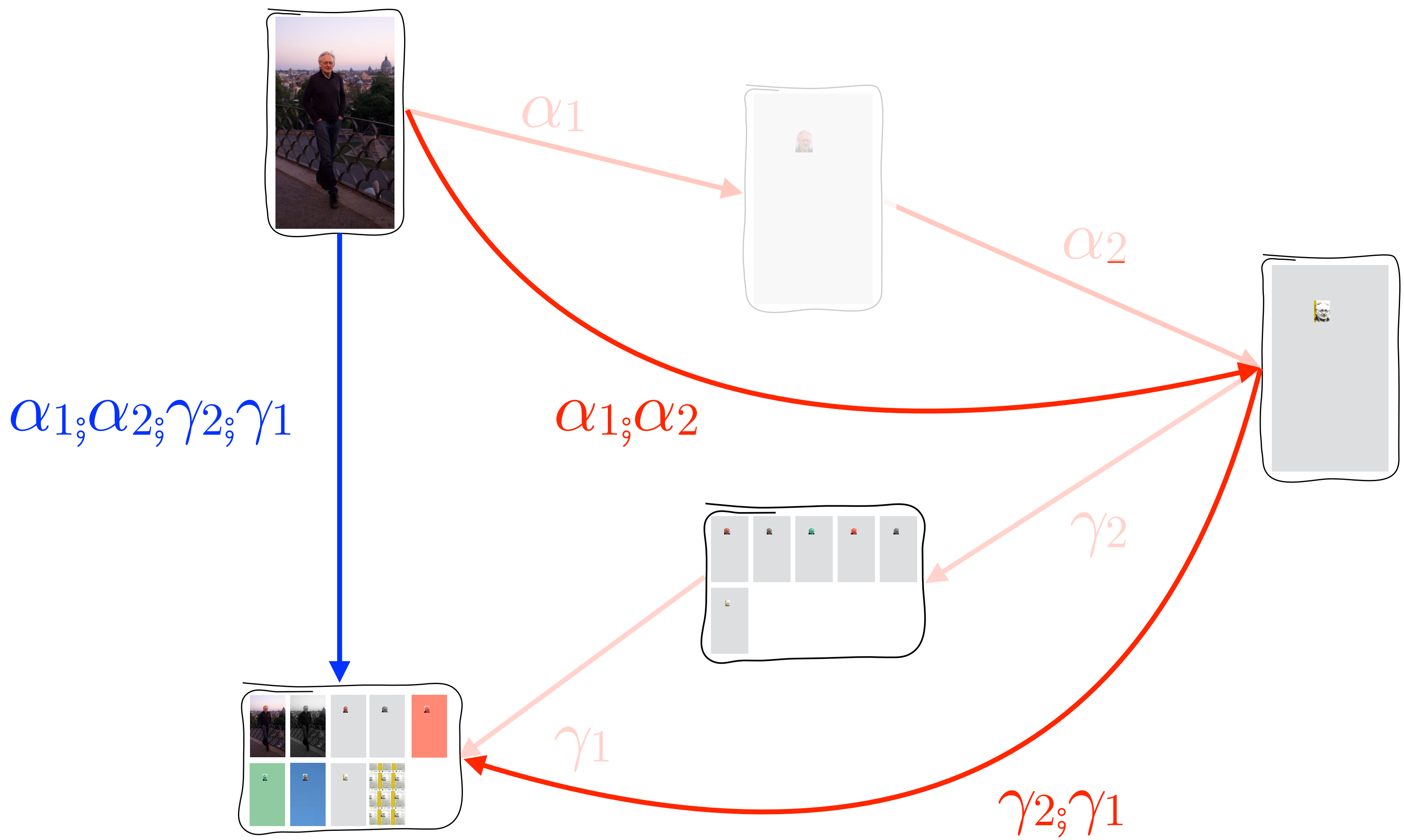
Concretization I



Abstract interpretations

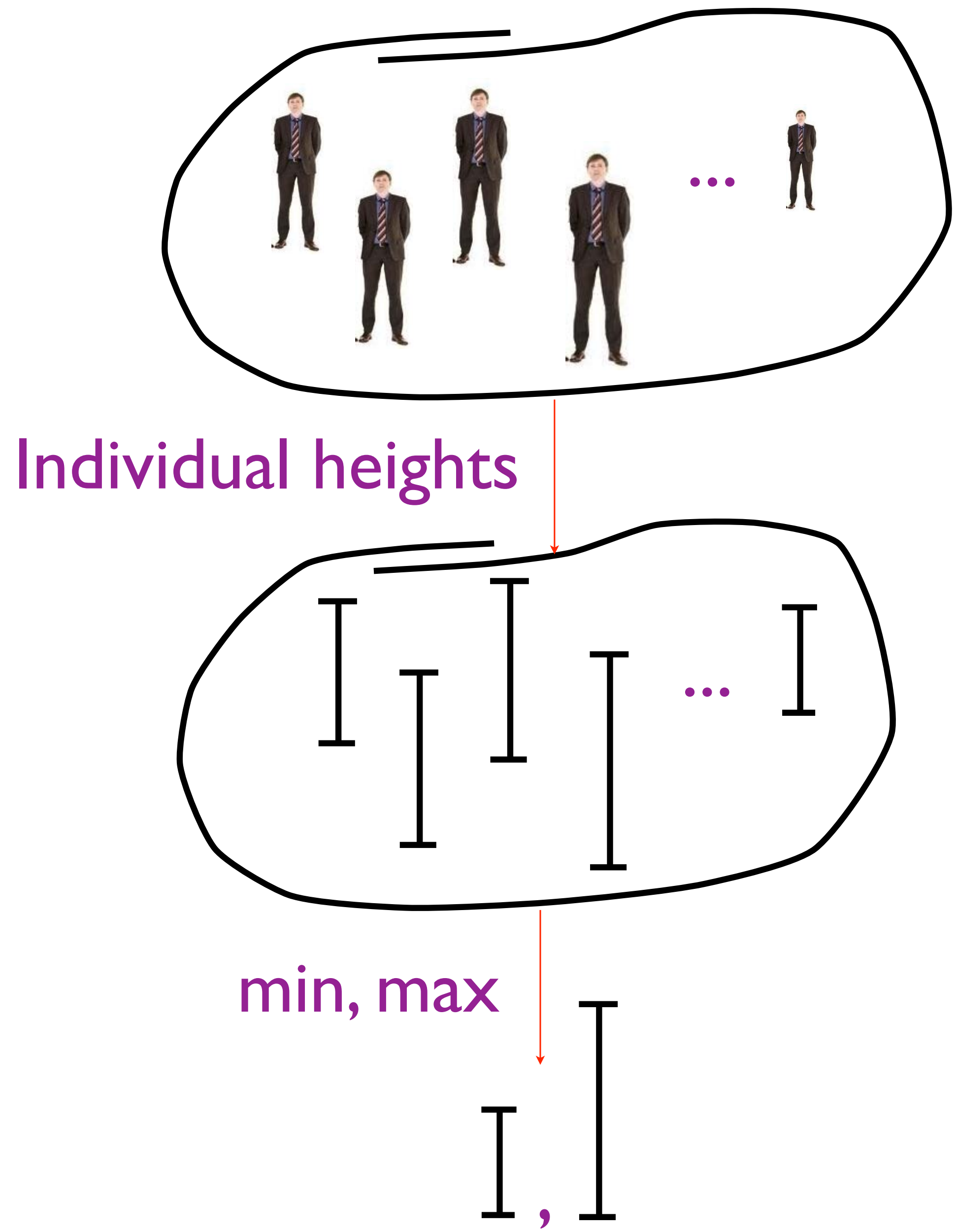


Abstract interpretations



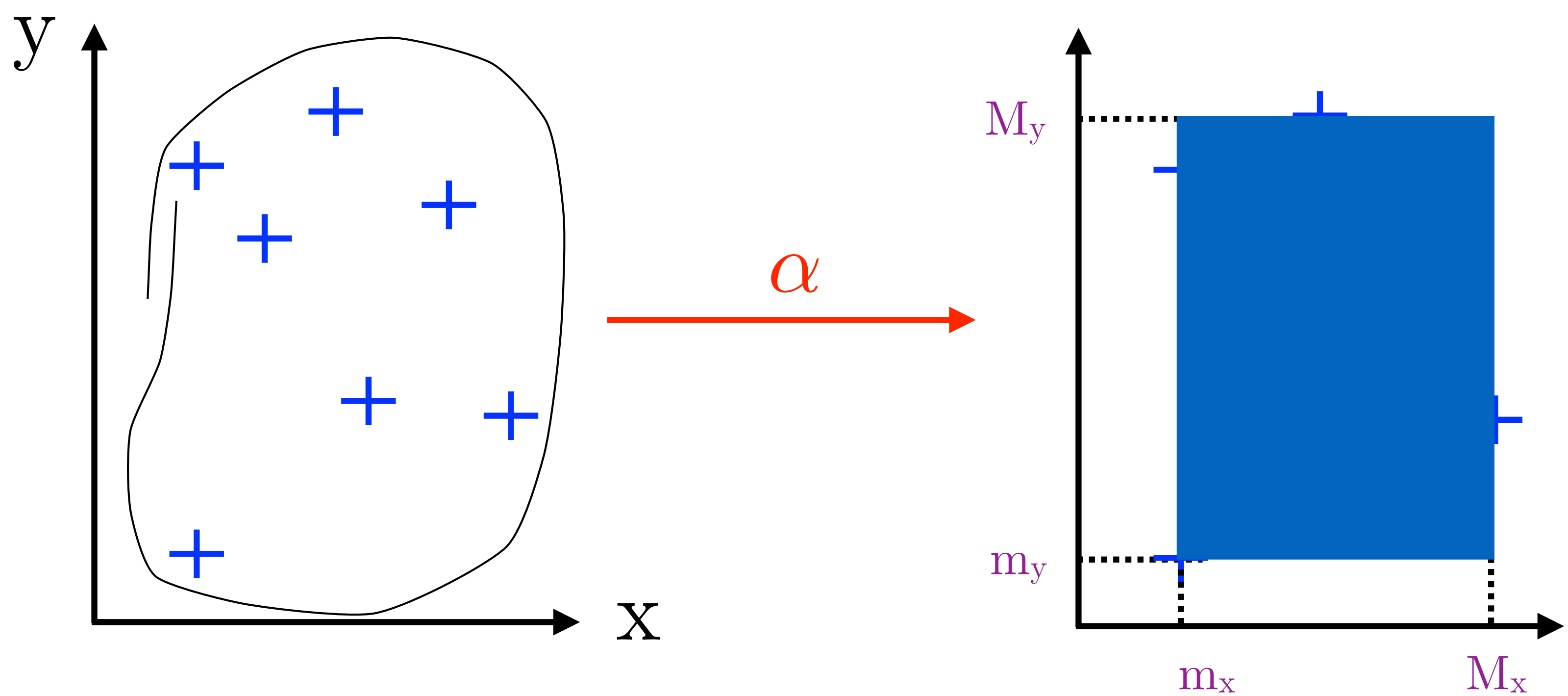
Intuition 2


Height
Fingerprint
Eye color
DNA
Phone metadata


...
Individual heights
...
min, max

Interval abstraction

- Example: interval abstraction (also called *box abstraction*)



Set of points

Interval abstraction
$[m_x,M_x] \times [m_y,M_y]$

Intuition 3

A C program and one of its executions

```
#include <stdio.h>
int main()
{
            int x, y;
            printf("Enter two integers: ");
            scanf("%d %d",&x, &y);
/* 1: */    while ((x != 6) || ( y != 0)) {
               printf("x = %d, y = %d\n",x,y);
/* 2: */       x = x + 3;
/* 3: */       if (x > 10) x = -x;
/* 4: */       y = y - 2;
/* 5: */       if (y < -5) y = -y;
            }
/* 6: */    printf("x = %d, y = %d\n",x,y);
}
```

```
Enter two integers: x = 0, y = 0
x = 3, y = -2
x = 6, y = -4
x = 9, y = 6
x = -12, y = 4
x = -9, y = 2
x = -6, y = 0
x = -3, y = -2
x = 0, y = -4
x = 3, y = 6
x = 6, y = 4
x = 9, y = 2
x = -12, y = 0
x = -9, y = -2
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x = -3, y = 6
x = 0, y = 4
x = 3, y = 2
x = 6, y = 0
```
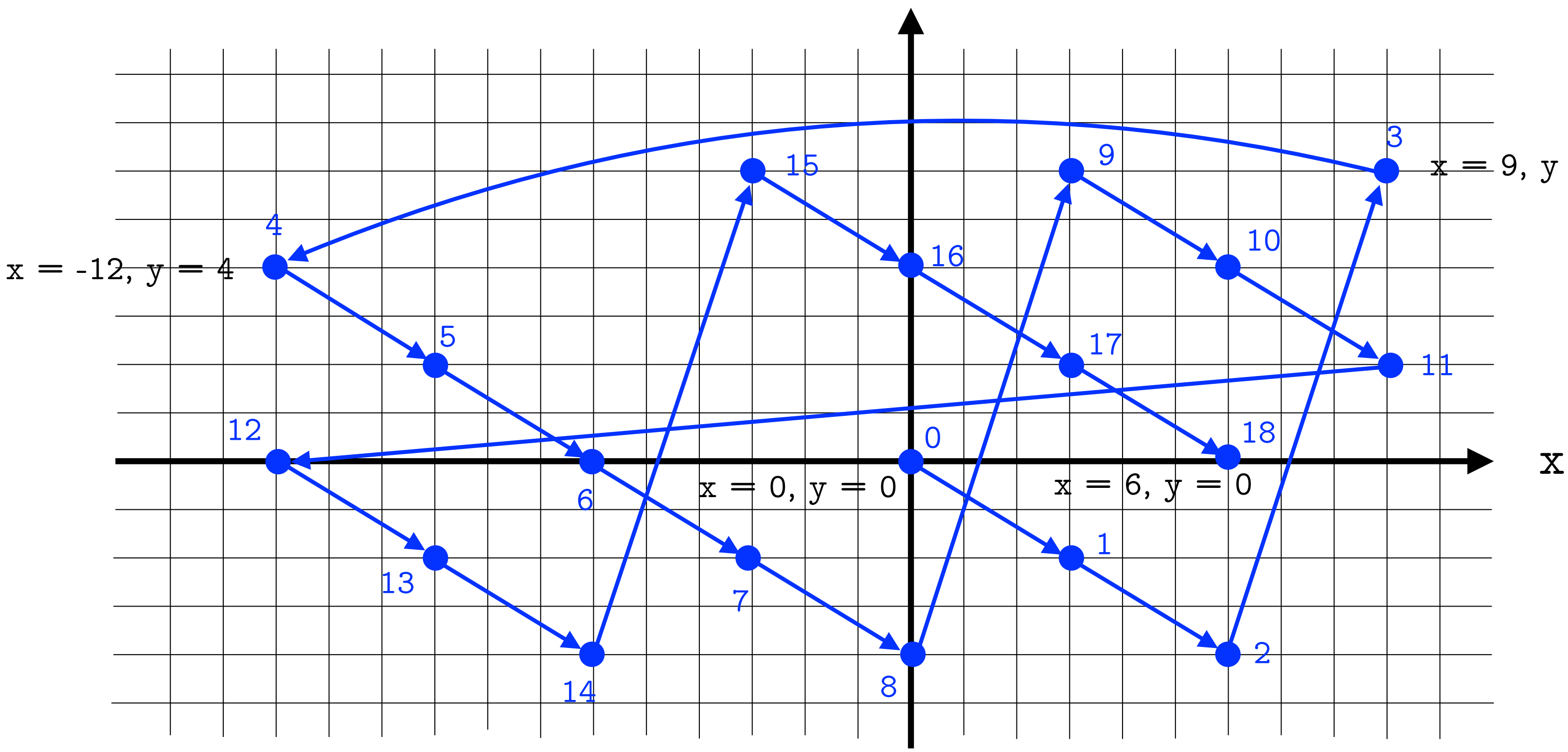
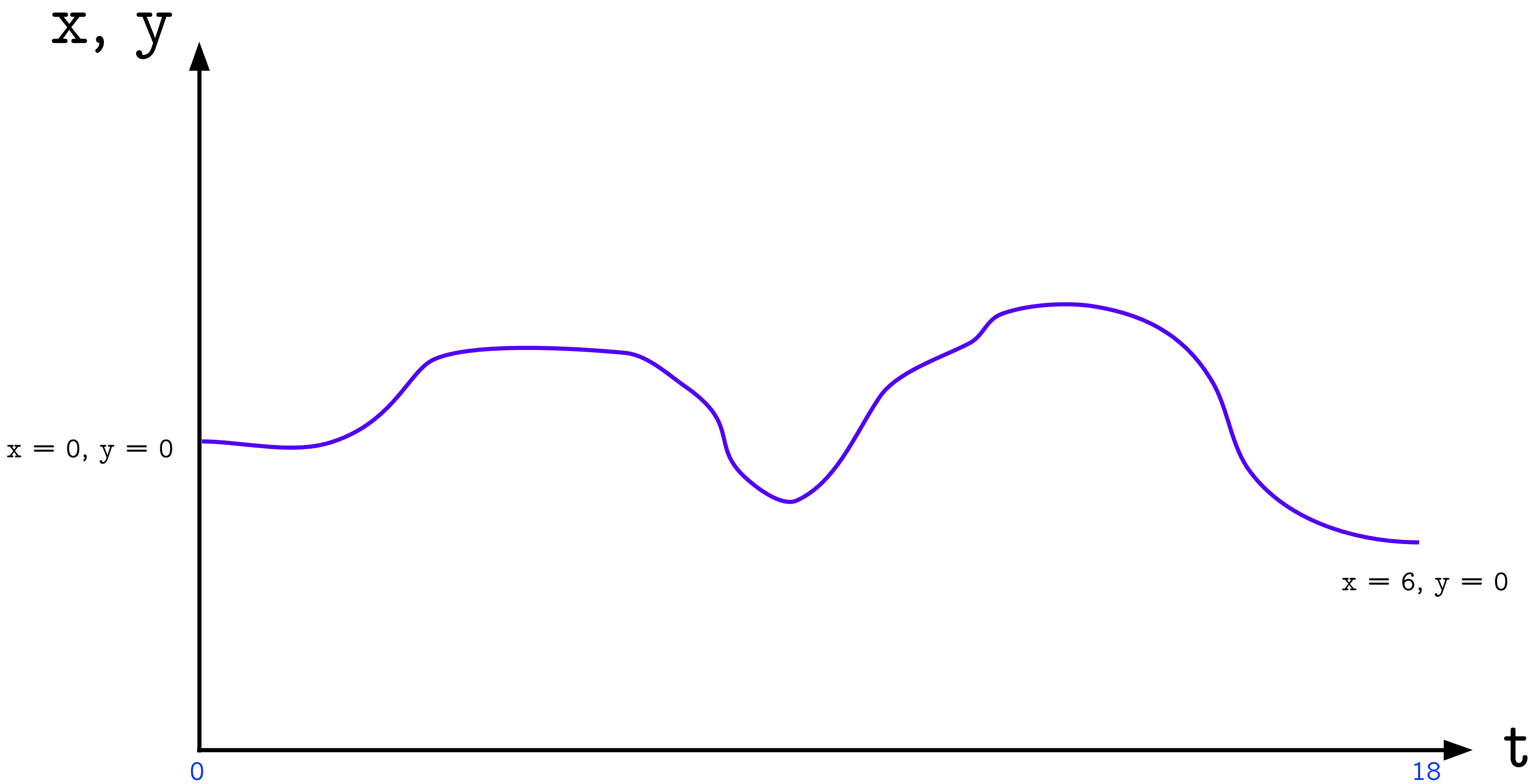

Graphical representation of the execution (I)

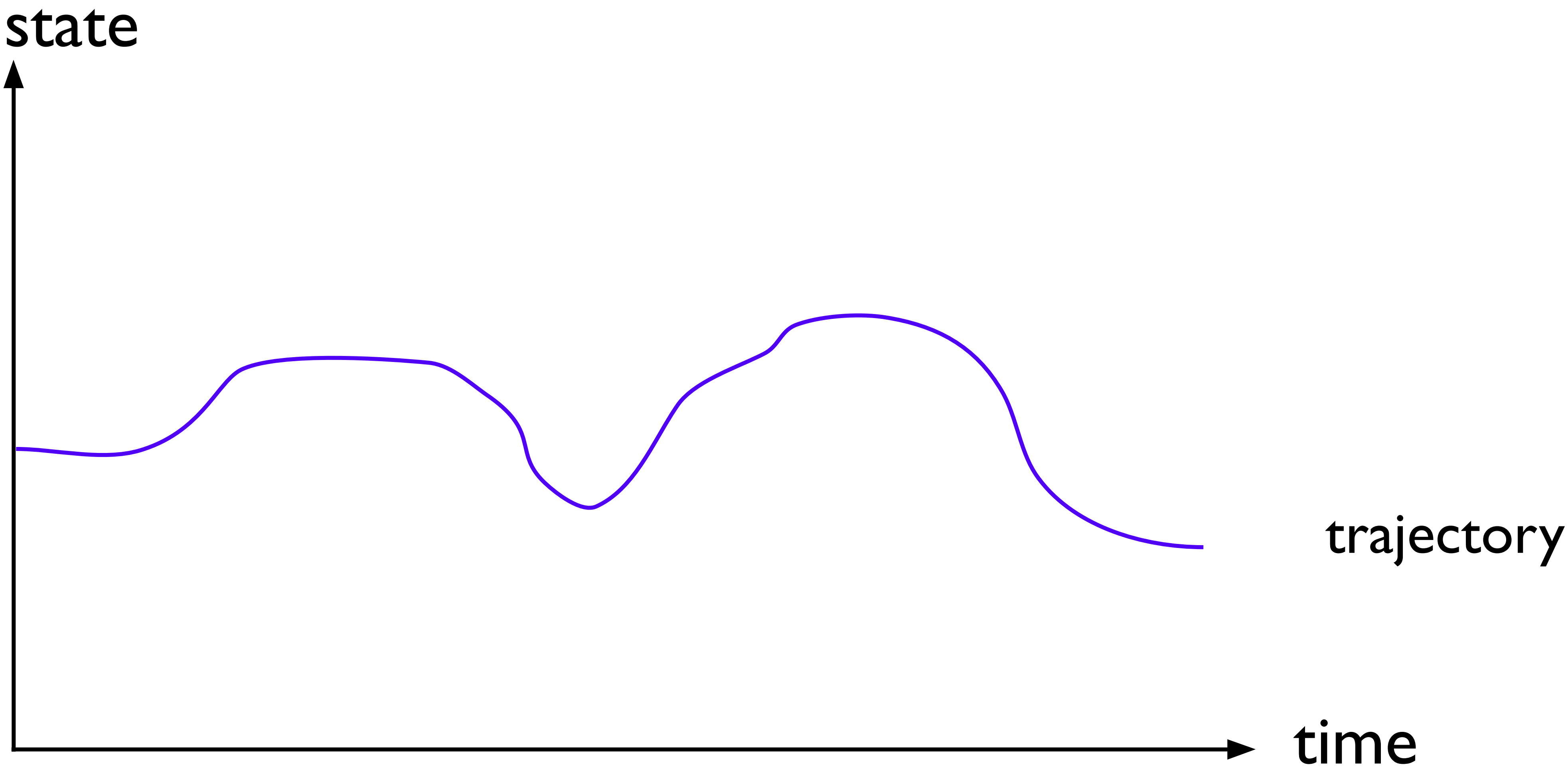


Graphical representation of the execution (2)



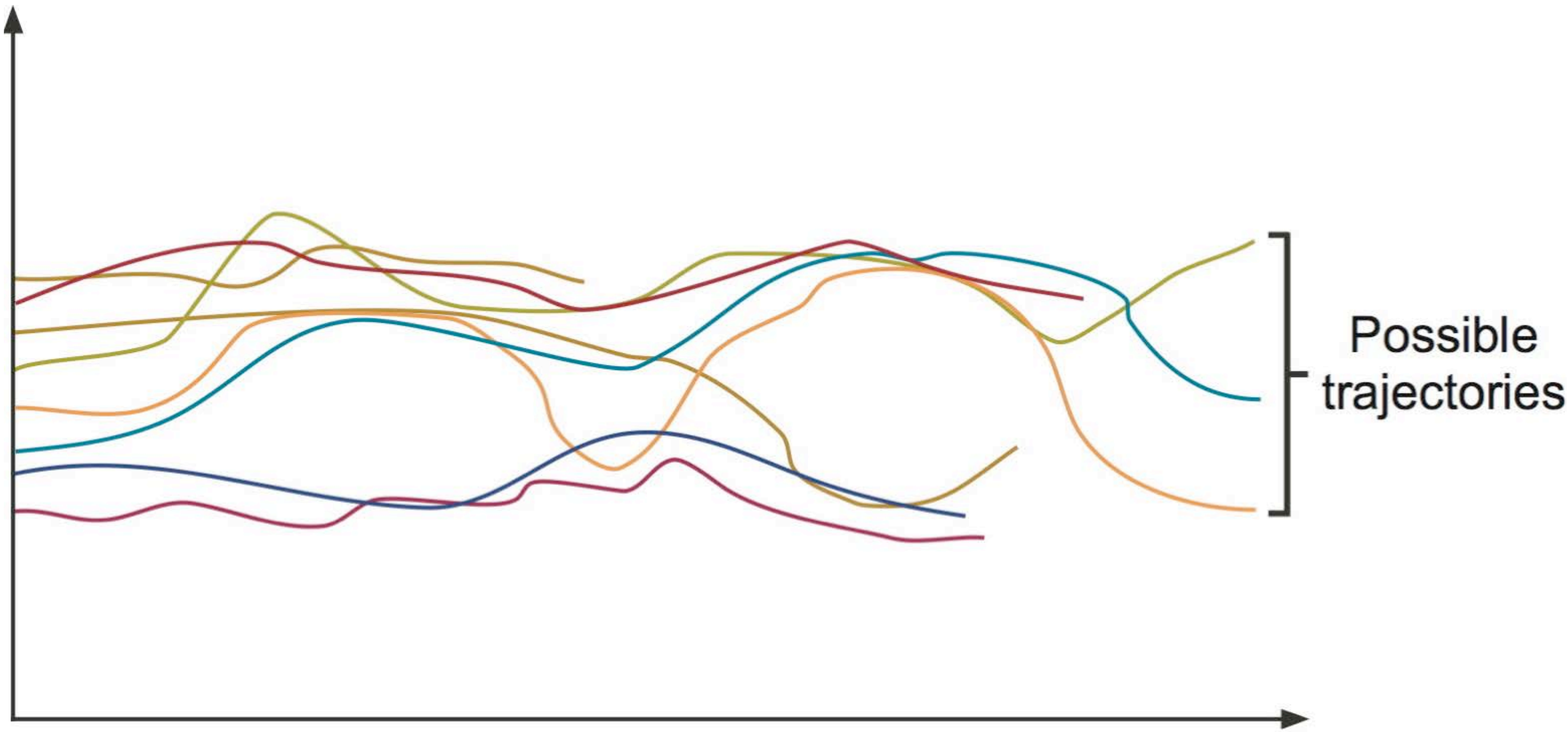
Semantics

Formalize what it means to run a program



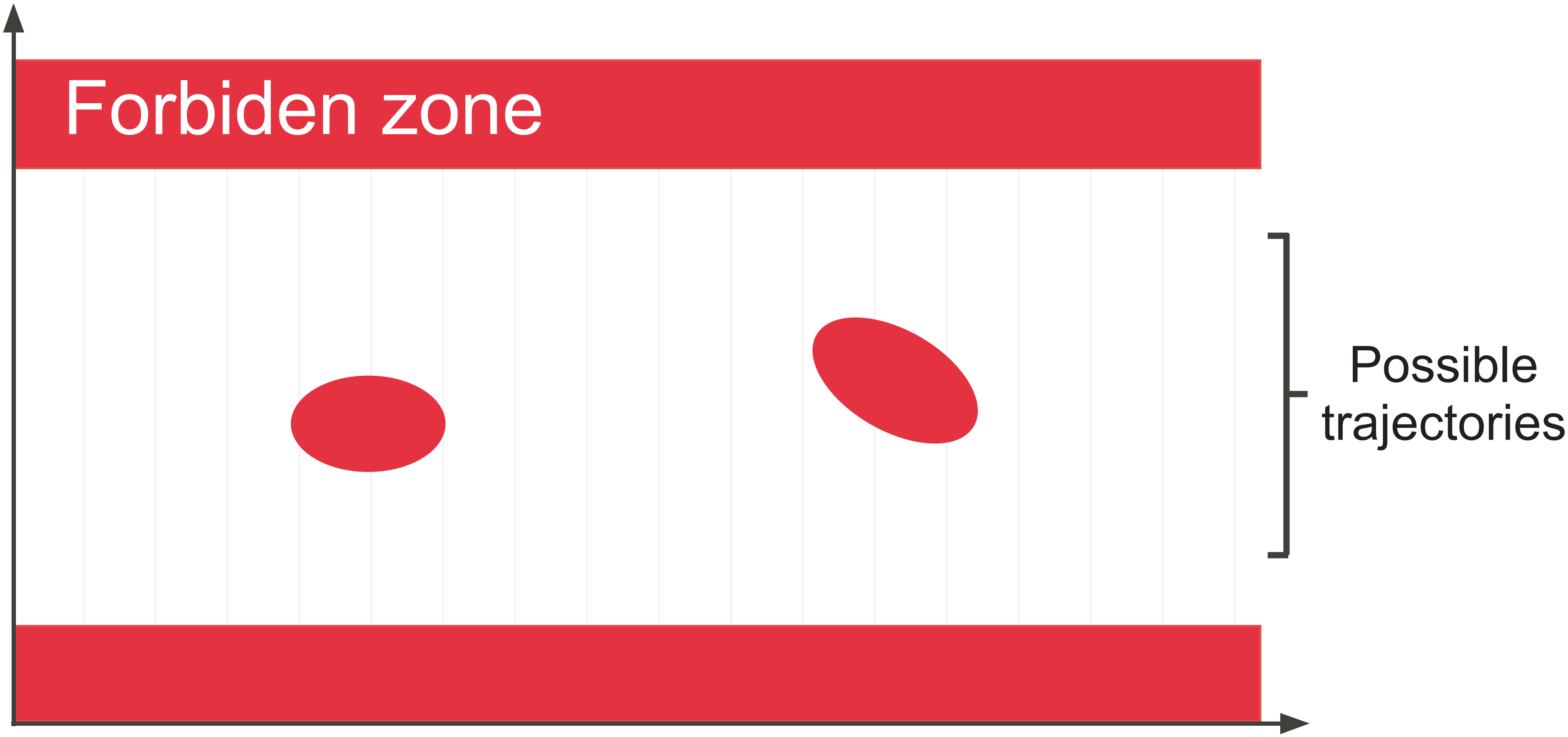
Properties (Collecting semantics)

*Formalize what you are interested to **know** about program behaviors*



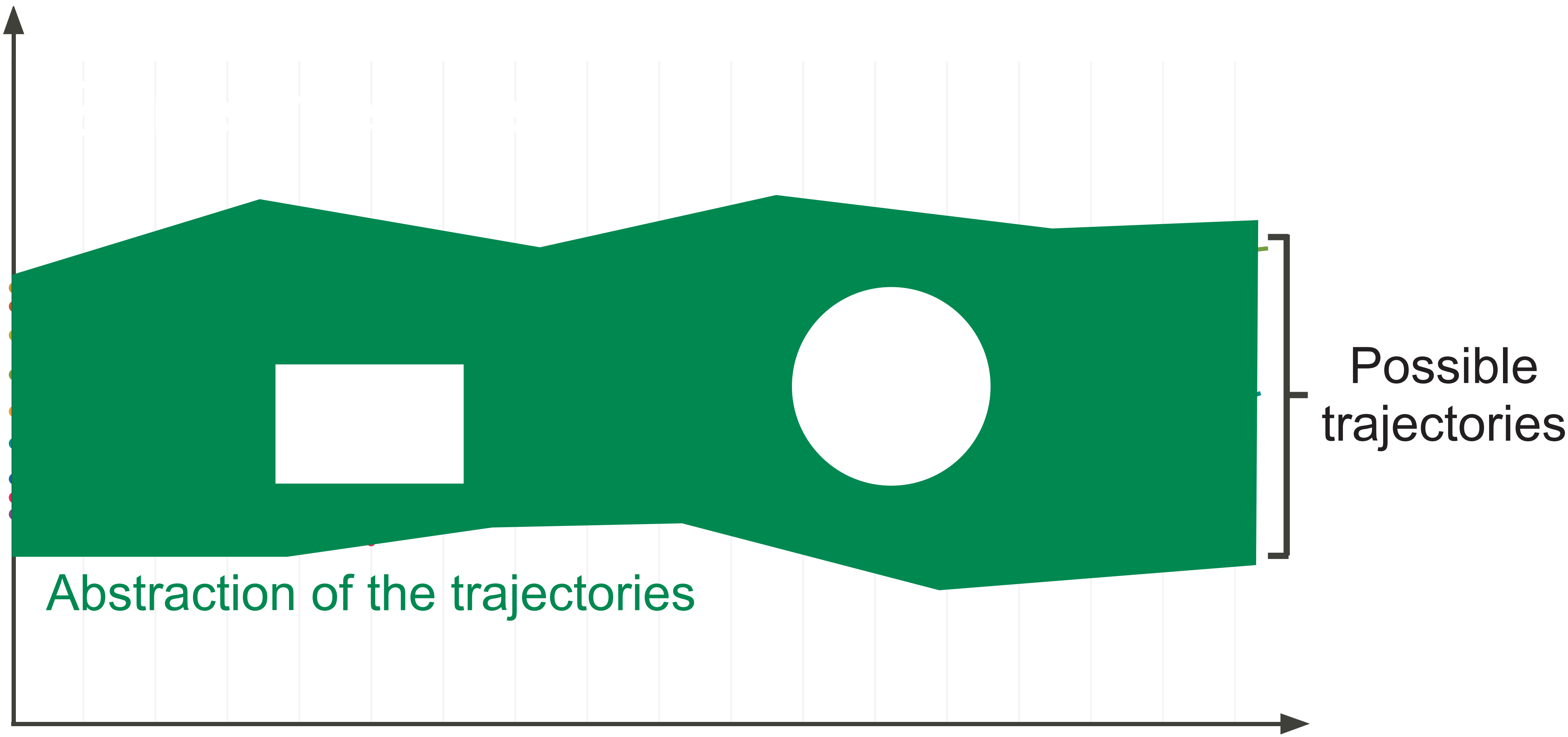
Specification

Formalize what you are interested to ***prove*** *about program behaviors*



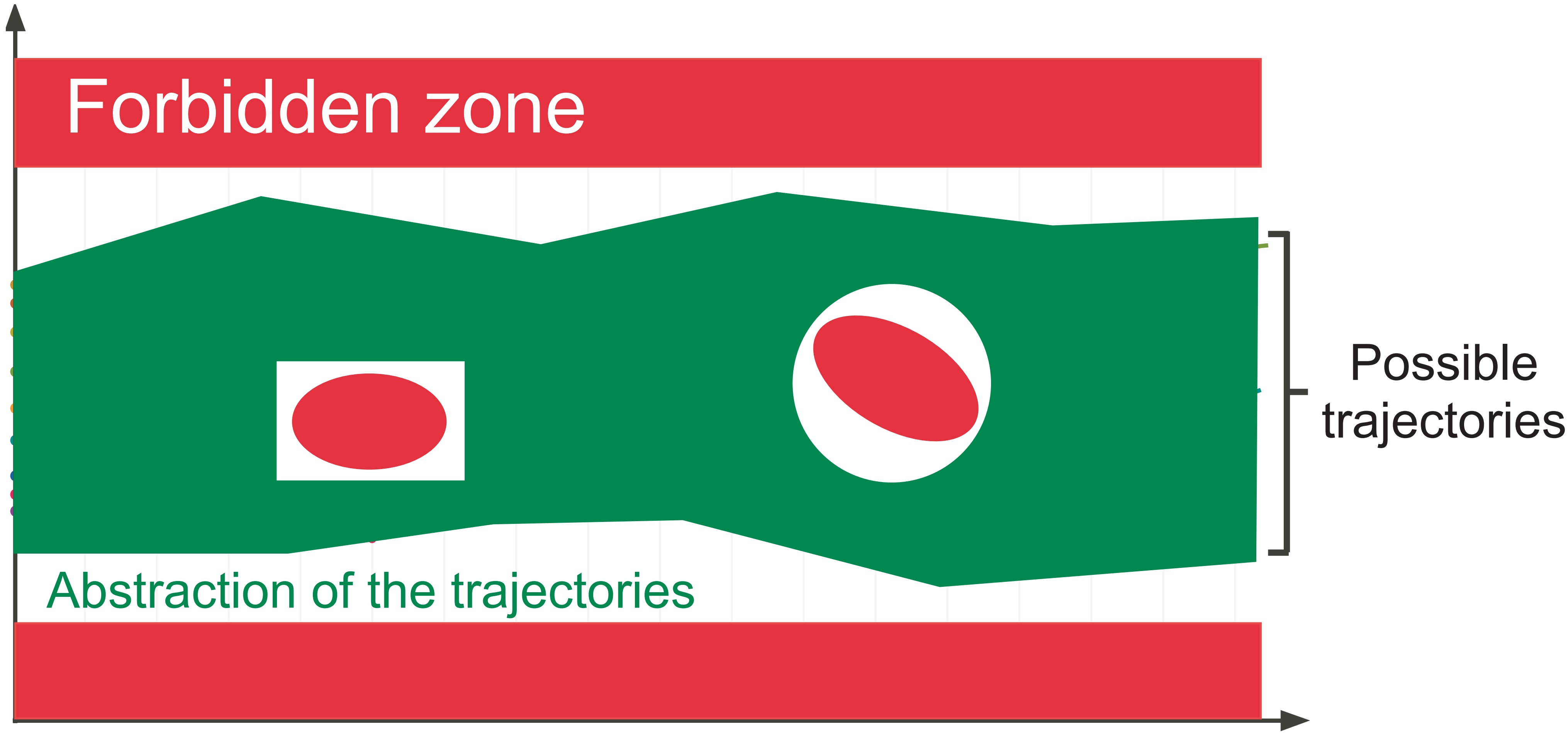
Abstraction

Abstract *away all information on program behaviors irrelevant to the proof*



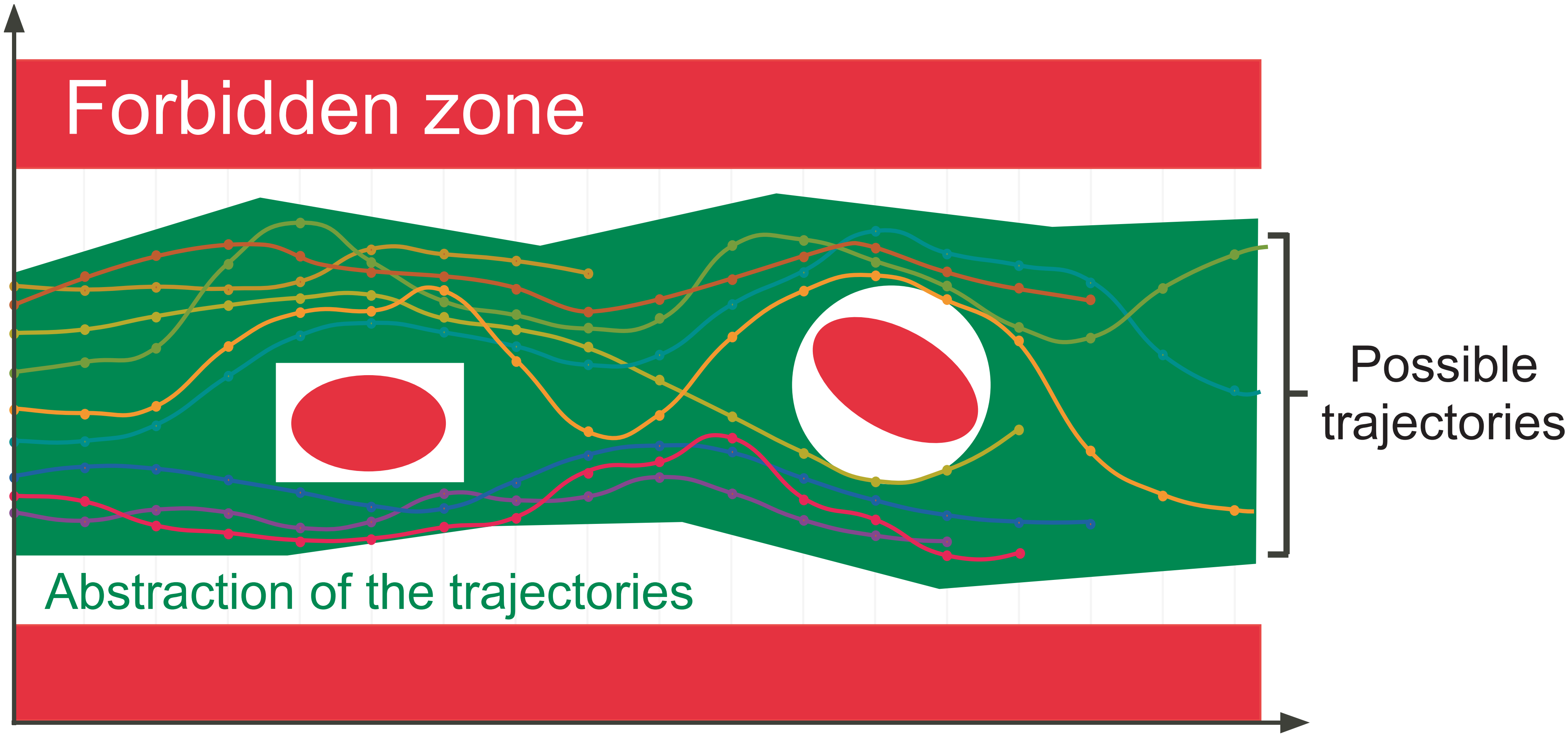
Verification

*The proof is fully **automatic***



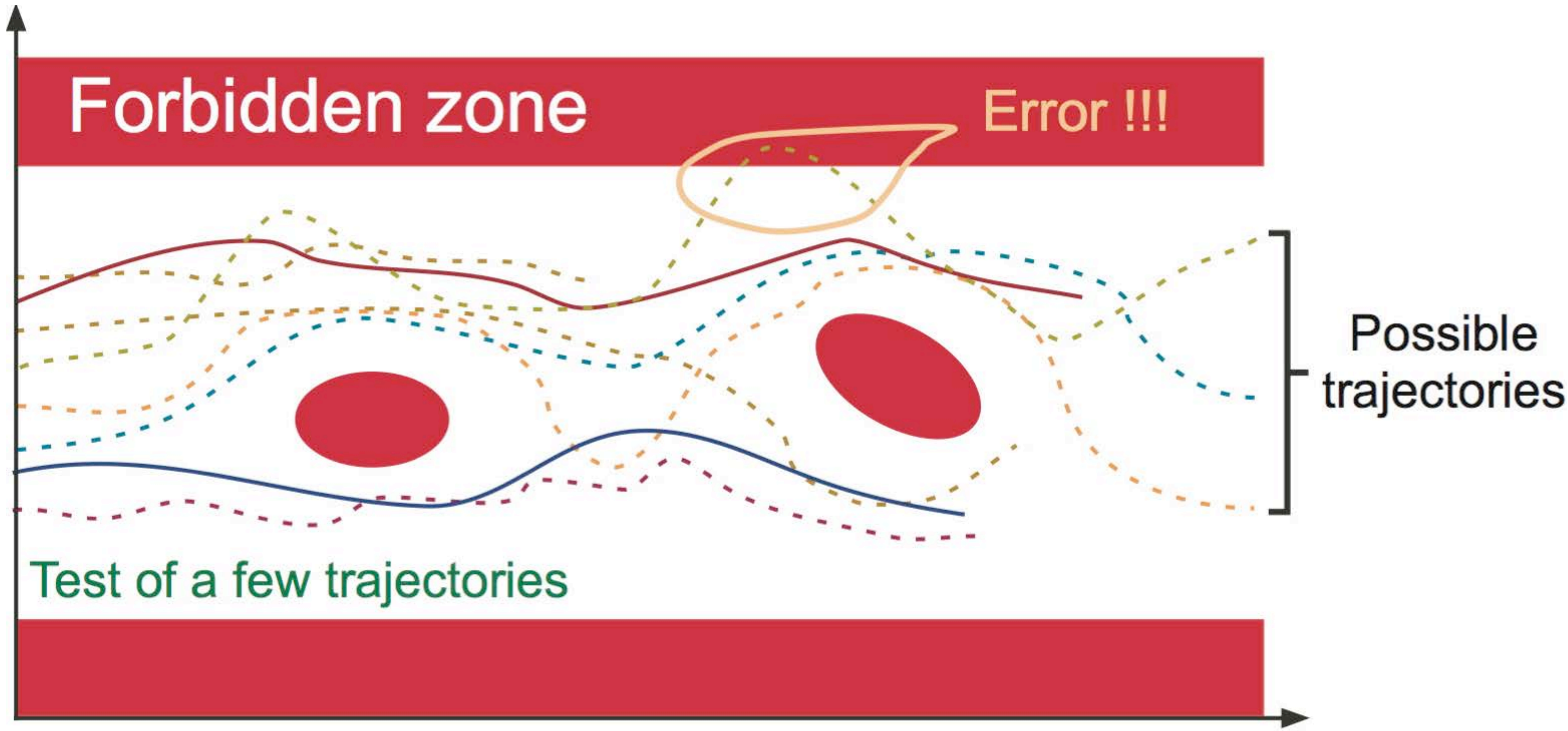
Soundness

*Never forget any possible case so the **abstract proof is correct in the concrete***



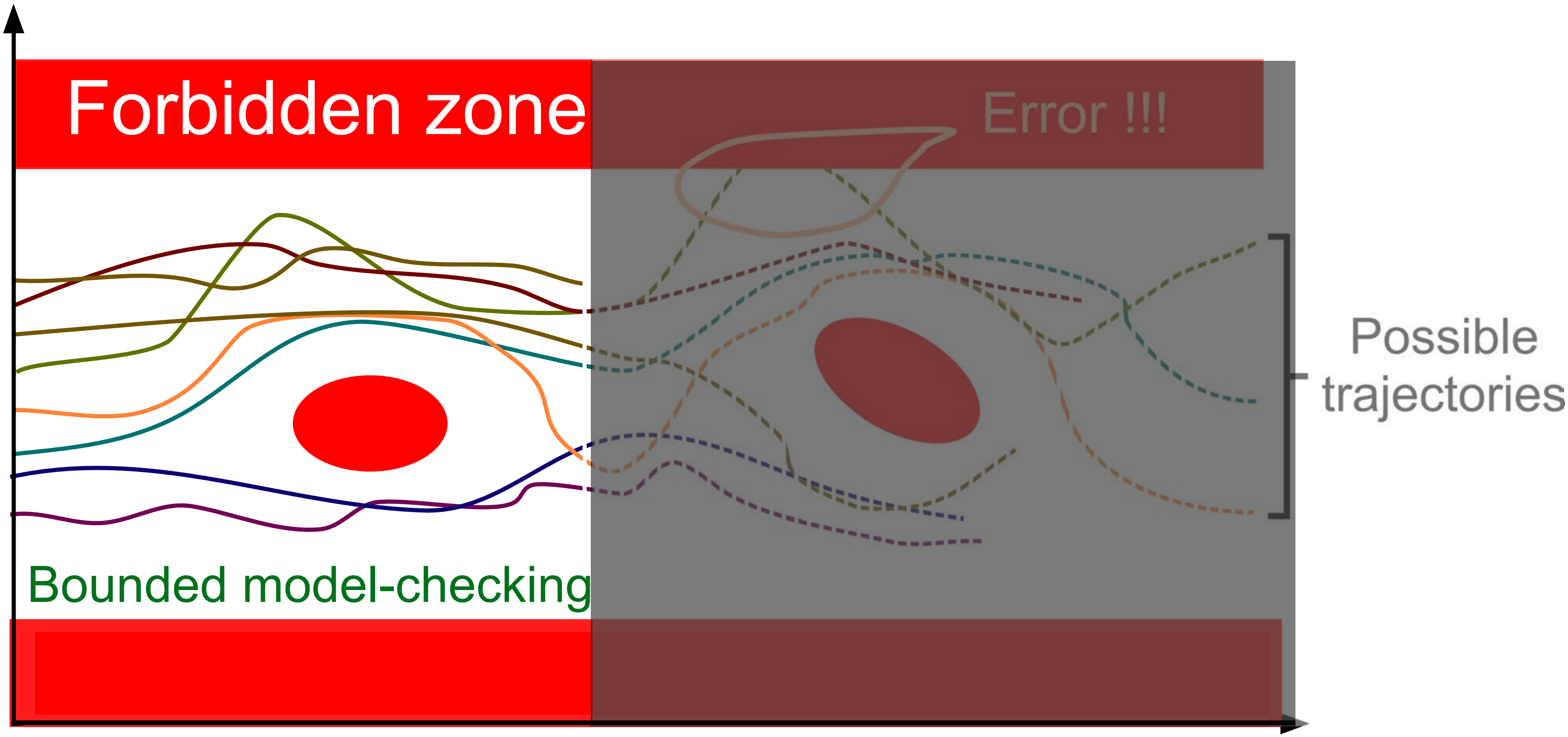
Unsound methods: testing

Try a few cases



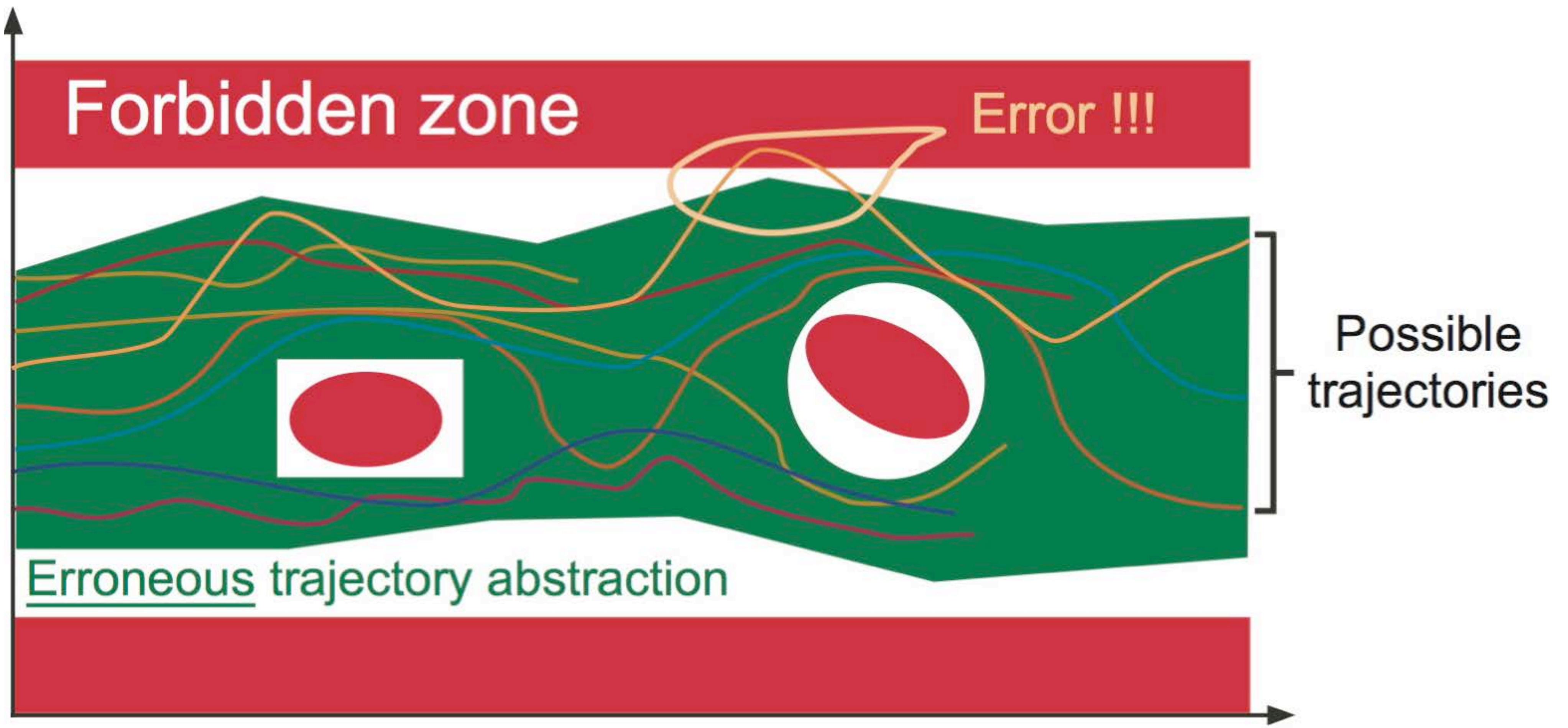
Unsound methods: bounded model checking

Simulate the beginning of all executions (so called bounded model-checking)



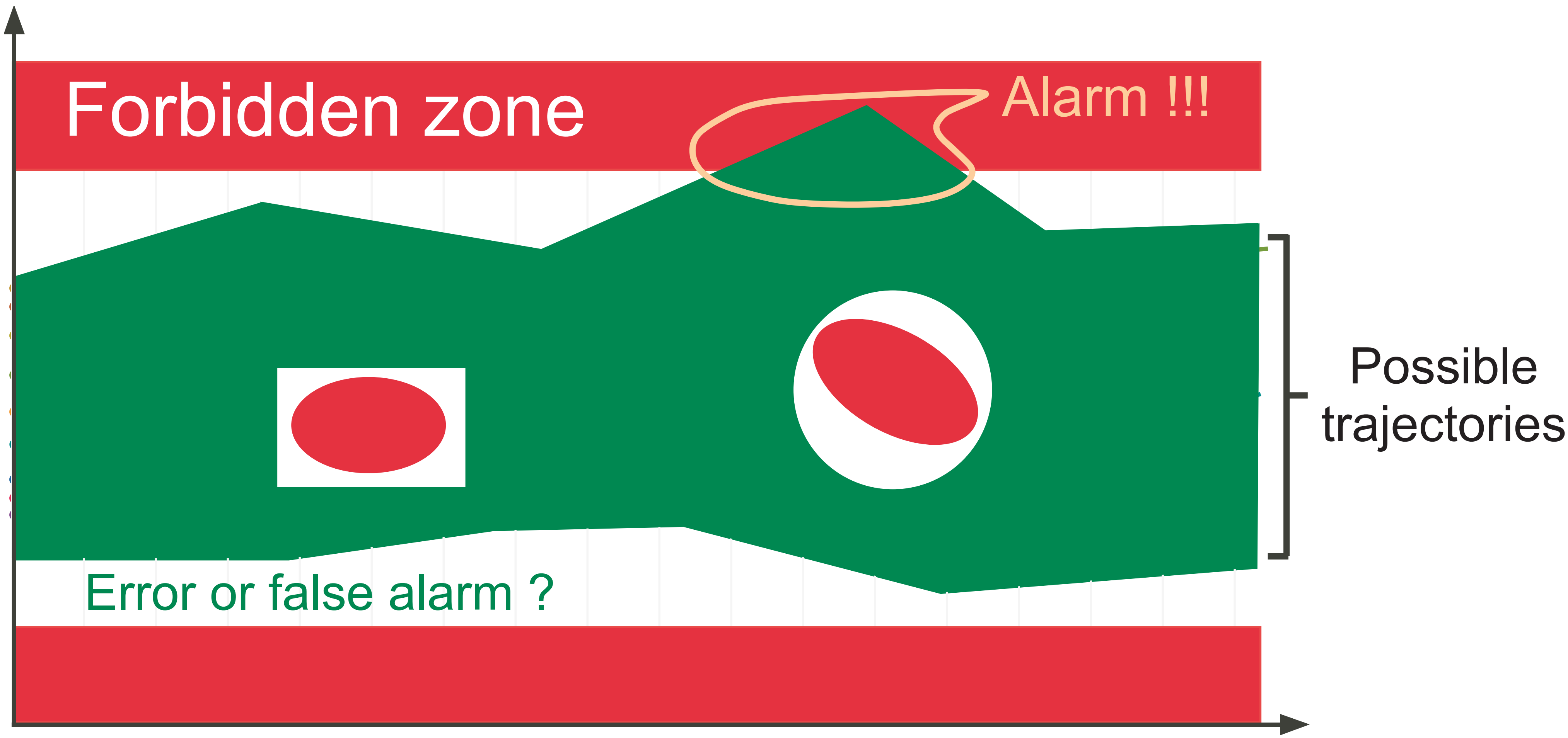
Unsound methods: soundiness

Many static analysis tools are ***unsound*** *(e.g. Coverity, etc.) so inconclusive*



Alarms

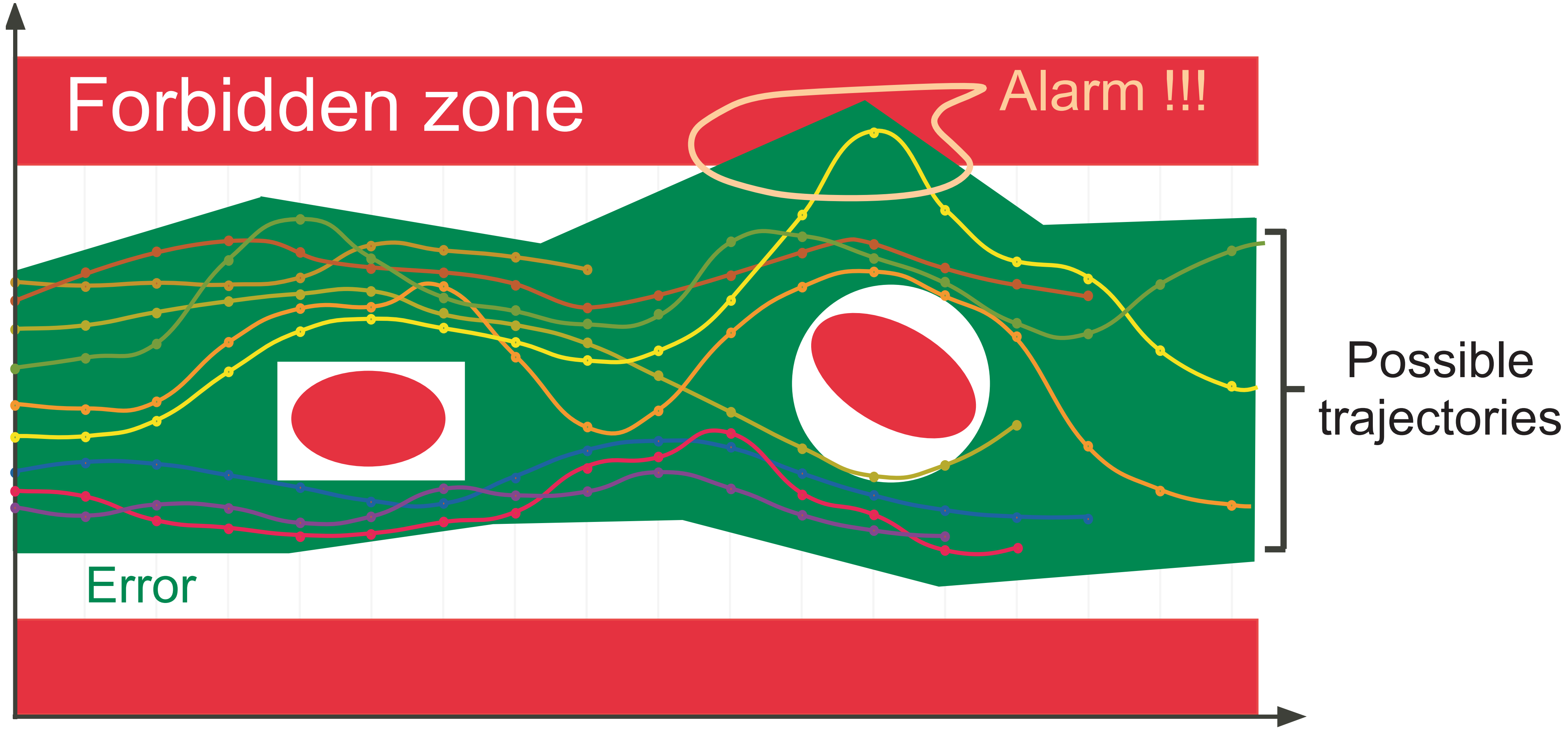
When abstract proofs may fail while concrete proofs would succeed



By soundness an alarm must be raised for this over-approximation!

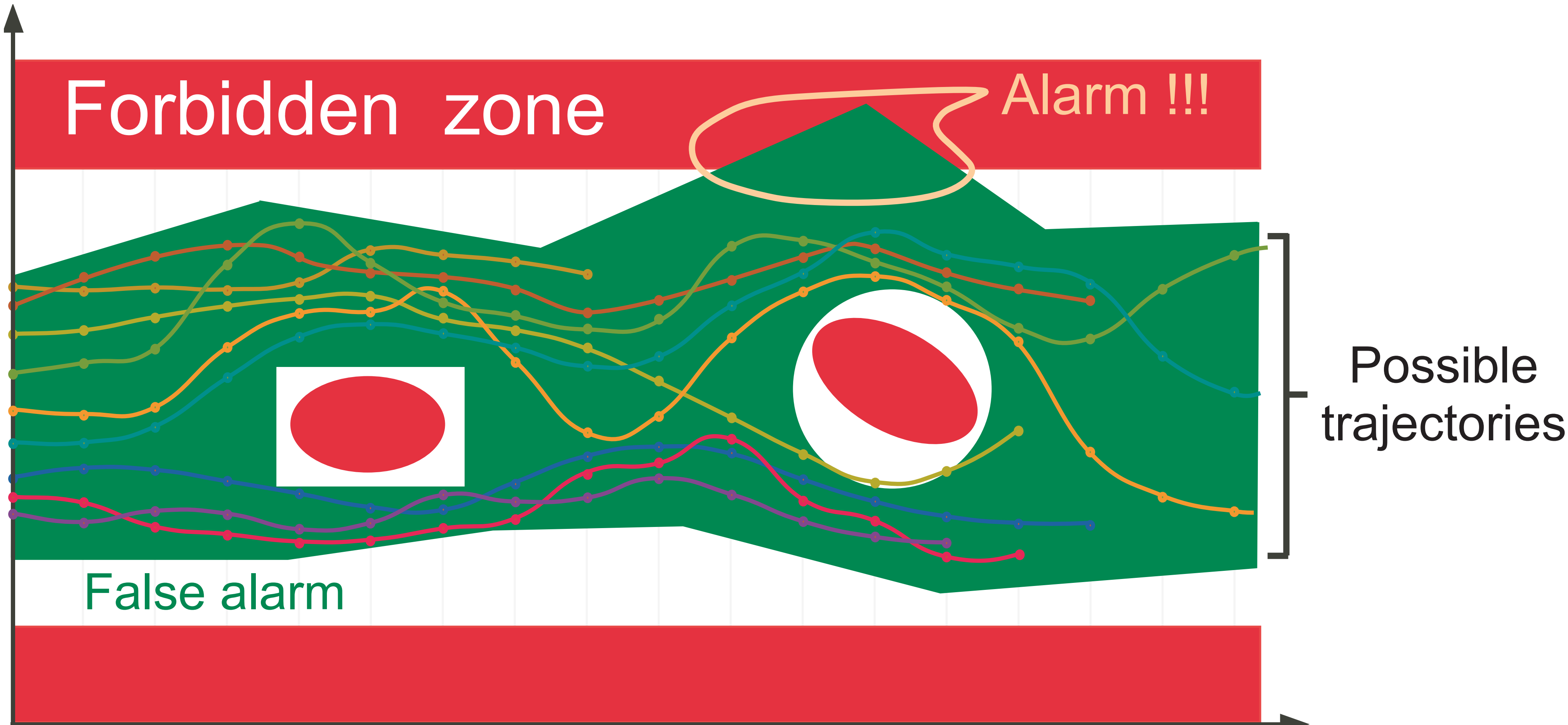
True alarm

The abstract alarm may correspond to a concrete error



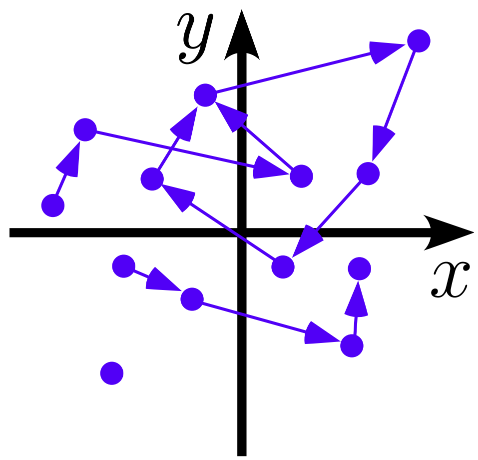
False alarm

The abstract alarm may correspond to no concrete error (false negative)

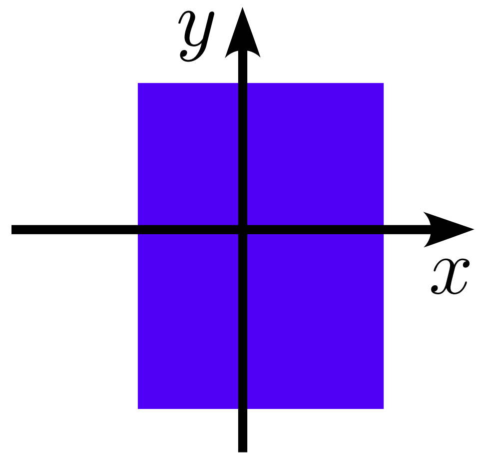


What to do in presence of false alarms

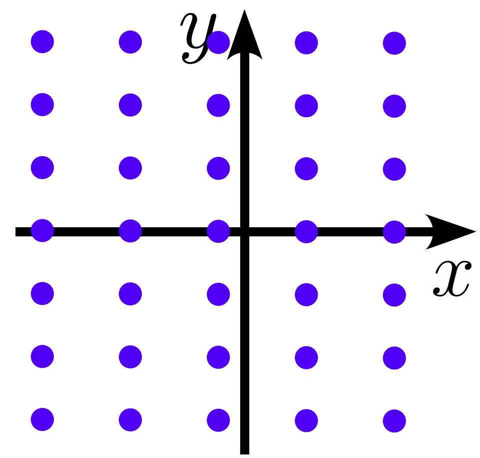
- False alarms are ultimately unavoidable (Gödel's incompleteness)
- Consider finite cases or decidable cases only (model-checking, *does not scale*)
- Ask for human help by providing information on the program behavior (theorem provers, SMT solvers), *program specific and labor costly*
- Have specialists refine the abstract interpretation (e.g. Astrée, http://www.absint.com/astree/index.htm), *shared cost*



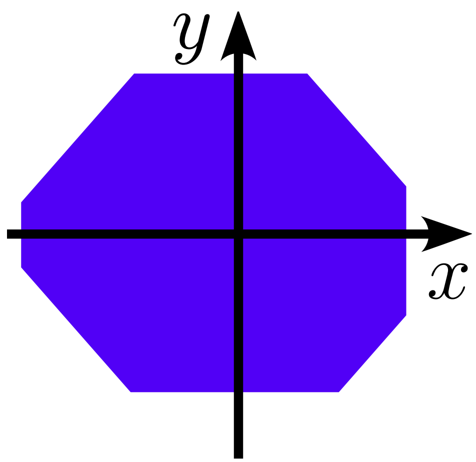
Collecting semantics: partial traces



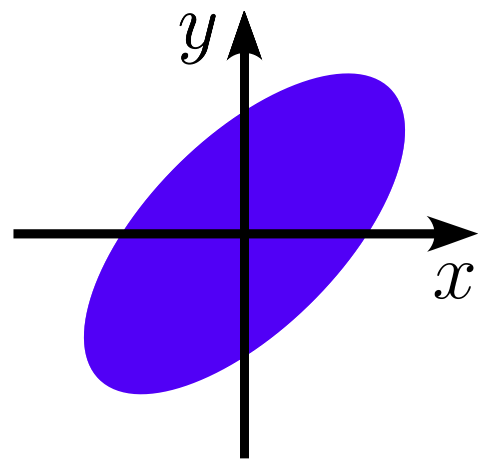
Intervals: $\mathtt{x} \in [a, b]$



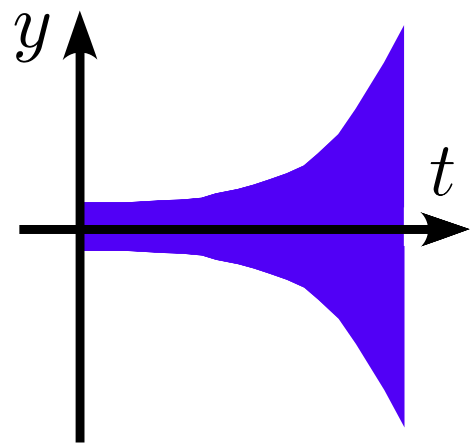
Simple congruences: $\mathtt{x} \equiv a[b]$



Octagons: $\pm\mathtt{x} \pm \mathtt{y} \leqslant a$



Ellipses: $\mathtt{x}^2 + b\mathtt{y}^2 - a\mathtt{x}\mathtt{y} \leqslant d$



Exponentials: $-a^{bt} \leqslant \mathtt{y}(t) \leqslant a^{bt}$

The very first static analysis

Brahmagupta

Brahmagupta (Sanskrit: ब्रह्मगुप्त; (598–c.670 CE) was an Indian mathematician and astronomer who wrote two important works on Mathematics and Astronomy: the *Brāhmasphuṭasiddhānta* (Extensive Treatise of Brahma) (628), a theoretical treatise, and the *Khaṇḍakhādyaka*, a more practical text.

Brahmagupta

Born	598 CE
Died	c.670 CE
Fields	Mathematics, Astronomy
Known for	Zero, modern Number system

The rule of signs by Brahmagupta (628)

18.30. [The sum] of two positives is positives, of two negatives negative;

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- Sometimes imprecise (don't know the sign of the sum of a positive and a negative)

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- Sometimes imprecise (don't know the sign of the sum of a positive and a negative)
- Useful in practice (if you know what to do when you don't know the sign)
- e.g. in compilation: do not optimize (a division by 2 into a shift when positive(*))

(*) Unless processor uses 2's complement and can shift the sign.

The rule of signs by Brahmagupta (628)

18.30. [The sum] of two positives is positives, of two negatives negative; [...]

18.32. A negative minus zero is negative, a positive [minus zero] positive; zero [minus zero] is zero. When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

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18.33. The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero.

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18.34. A positive divided by a positive or a negative divided by a negative is positive; a zero divided by a zero is zero; a positive divided by a negative is negative; a negative divided by a positive is [also] negative.

wrong

The rule of signs by Michel Sintzoff (1972)

For example, a×a+b×b yields the value 25 when a is 3 and b is −4, and when + and × are the arithmetic multiplication and addition.
But a×a+b×b yields always the object "pos" when a and b are the objects "pos" or "neg", and when the valuation is defined as follows :

```
pos+pos=pos                    pos×pos=pos
pos+neg=pos,neg                pos×neg=neg
neg+pos=pos,neg                neg×pos=neg
neg+neg=neg                    neg×neg=pos
V(p+q)=V(p)+V(q)               V(p×q)=V(p)×V(q)
V(0)=V(1)=...=pos
V(-1)=V(-2)=...=neg
```

The valuation of a×a+b×b yields "pos" by the following computations :

```
V(a)=pos,neg                V(b)=pos,neg
V(a×a)=pos×pos,neg×neg      V(b×b)=pos×pos,neg×neg
      =pos,pos=pos                =pos,pos=pos
V(a×a+b×b)=V(a×a)+V(b×b)=pos+pos=pos
```

This valuation proves that the result of a×a+b×b is always positive and hence allows to compute its square root without any preliminary dynamic test on its sign. On the other hand, the

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neg+neg=neg                 neg×neg=pos
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neg+pos=pos,neg
neg+neg=neg
V(p+q)=V(p)+V(q)
V(0)=V(1)=...=pos
V(-1)=V(-2)=...=neg

pos×pos=pos
pos×neg=neg
neg×pos=neg
neg×neg=pos
V(p×q)=V(p)×V(q)

wrong

0∈pos x -1∈neg
= 0∉neg

The valuation of a×a+b×b yields "pos" by the following computations :

V(a)=pos,neg
V(a×a)=pos×pos,neg×neg
=pos,pos=pos

V(b)=pos,neg
V(b×b)=pos×pos,neg×neg
=pos,pos=pos

V(a×a+b×b)=V(a×a)+V(b×b)=pos+pos=pos

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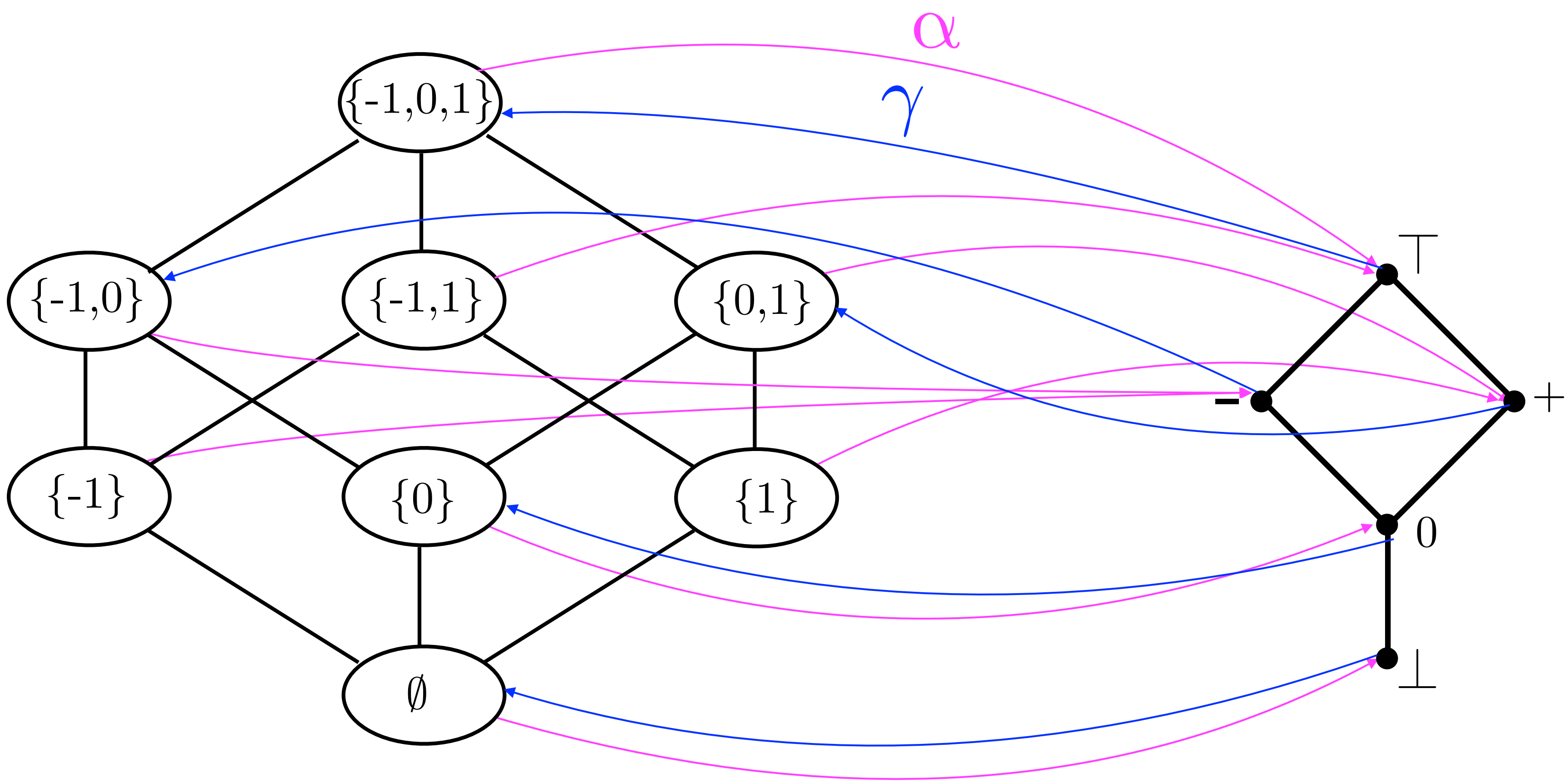
The rule of signs Cousot & Cousot (1979)



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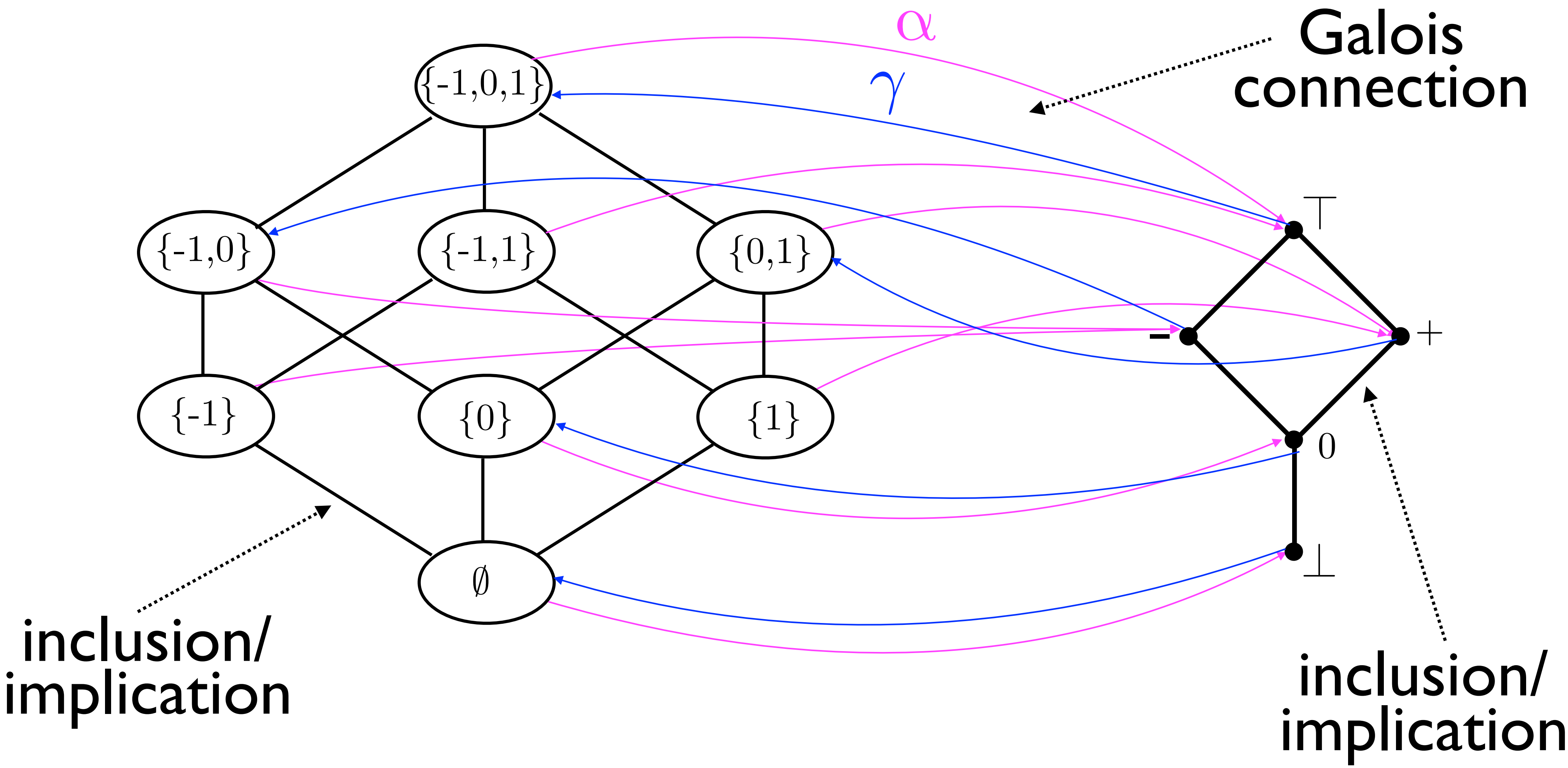

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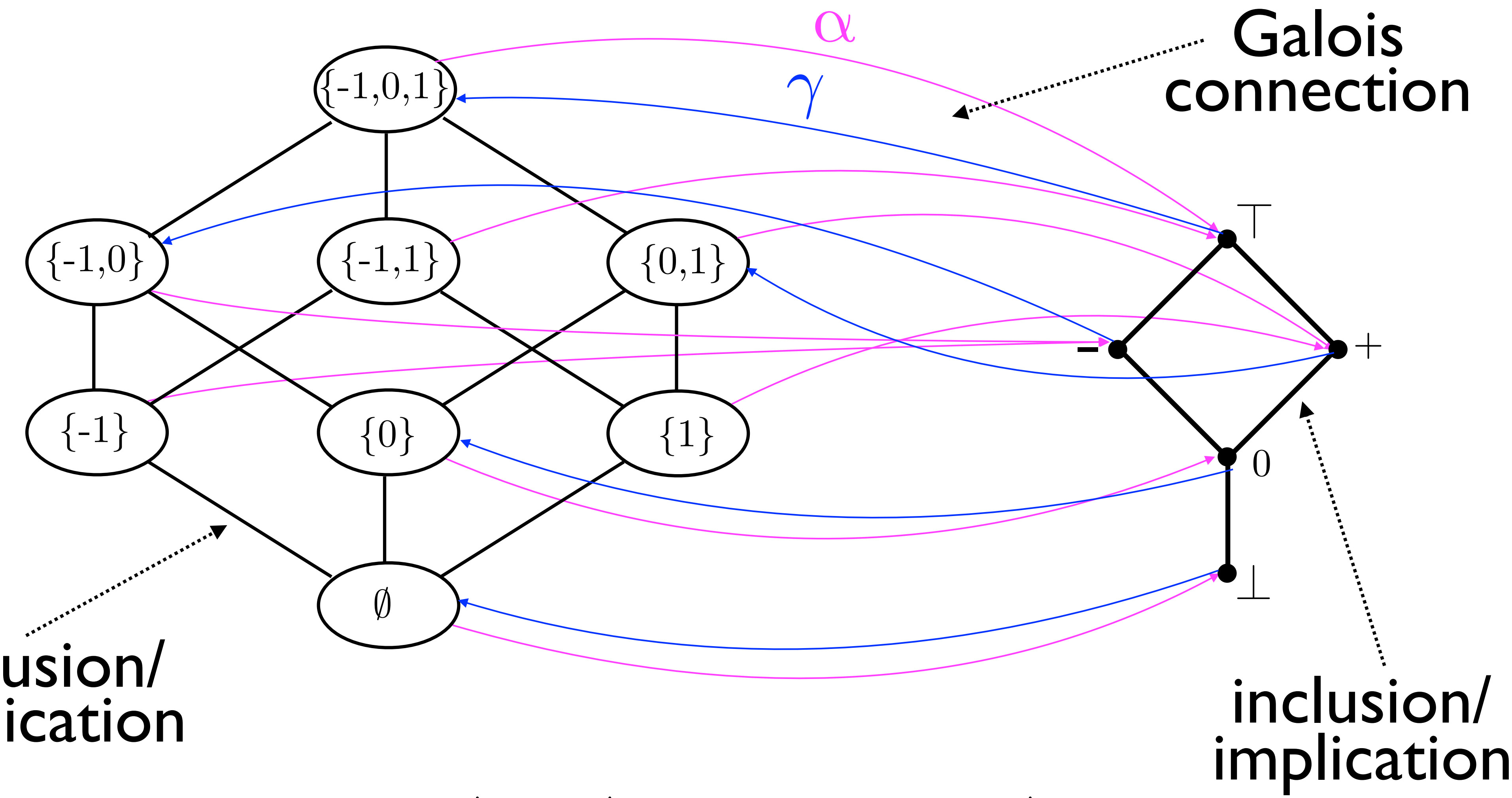


calculational design method

$$+ \; + \; + \quad = \quad +$$

$$\gamma \downarrow \qquad \downarrow \gamma \qquad\qquad \uparrow \alpha$$

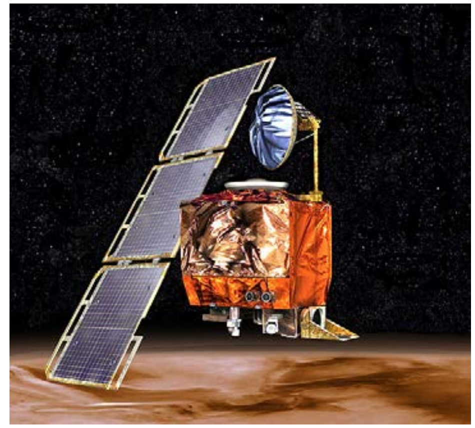
$$\{0,1\} + \{0,1\} = \{0,1,2[2]\} = \{0,1\}$$

Application of abstract interpretation to static analysis

All computer scientists have experienced bugs

Ariane 5.01 failure
(overflow)

Patriot failure
(float rounding)



Mars orbiter loss
(unit error)

Heartbleed
(buffer overrun)

- Checking the presence of bugs by debugging is great
- Proving their absence by static analysis is even better!

Static analysis

- Check program properties (automatically, using the program text only, without running the program)
- Difficulties:
  - Undecidability / complexity:
    - Precision
    - Scalability
  - Soundness (correctness)
  - Induction: widening/narrowing

Fixpoint

Fixpoint equation

$\{y \geqslant 0\}$ $\leftarrow$ hypothesis

```
x = y
```

$\{I(x, y)\}$ $\leftarrow$ loop invariant

```
while (x > 0) {
  x = x - 1;
}
```

Floyd-Naur-Hoare verification conditions:

$$(y \geqslant 0 \wedge x = y) \Longrightarrow I(x, y) \qquad \textit{initialisation}$$

$$(I(x, y) \wedge x > 0 \wedge x' = x - 1) \Longrightarrow I(x', y) \qquad \textit{iteration}$$

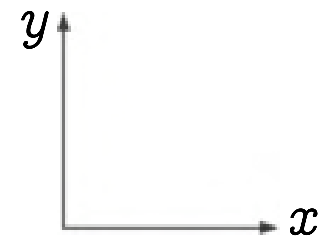
Equivalent fixpoint equation:

$$I(x, y) \;=\; x \geqslant 0 \wedge (x = y \vee I(x + 1, y)) \qquad (\textit{i.e.}\ I = F(I)\,^{(5)})$$

(5) We look for the most precise invariant I, implying all others, that is $\mathsf{lfp}^{\Longrightarrow} F$.

Iterates

Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

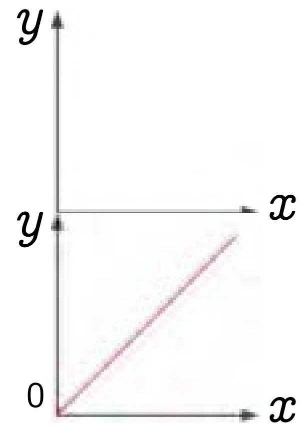


$$I^0(x, y) = \text{false}$$

Iterates

Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

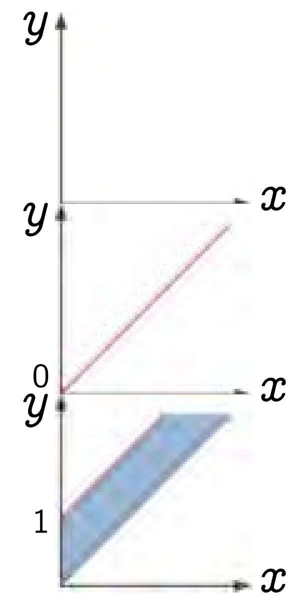
$$
\begin{aligned}
I^0(x,y) &= \text{false} \\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\
&= 0 \leqslant x = y
\end{aligned}
$$



Iterates

$$\text{Iterates } I = \lim_{n\to\infty} F^n(\text{false})$$

$$\begin{aligned}
I^0(x,y) &= \text{false} \\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\
&= 0 \leqslant x = y \\
I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\
&= 0 \leqslant x \leqslant y \leqslant x+1
\end{aligned}$$



Iterates

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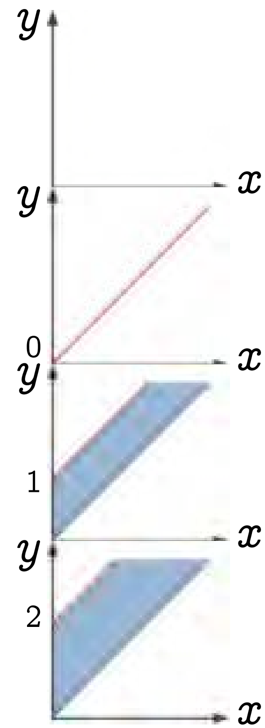
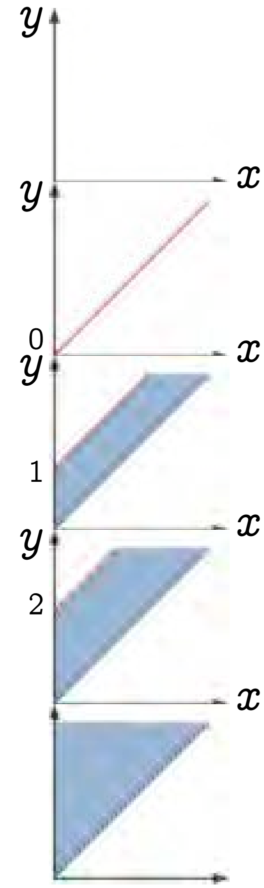
I^2(x, y) &= x \geqslant 0 \wedge (x = y \vee I^1(x + 1, y)) \\
&= 0 \leqslant x \leqslant y \leqslant x + 1 \\
I^3(x, y) &= x \geqslant 0 \wedge (x = y \vee I^2(x + 1, y)) \\
&= 0 \leqslant x \leqslant y \leqslant x + 2
\end{aligned}
$$



Convergence acceleration: widening

Accelerated Iterates $I = \lim_{n\to\infty} F^n(\text{false})$

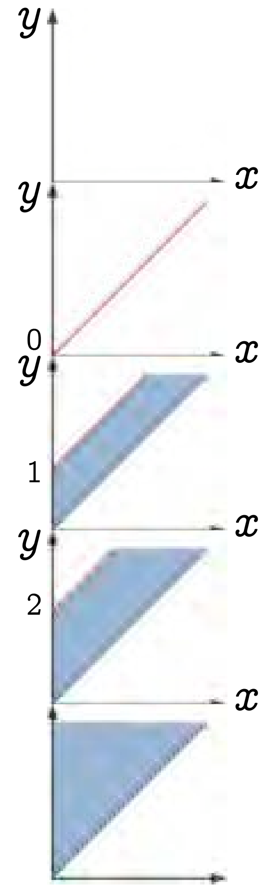
$$\begin{aligned}
I^0(x,y) &= \text{false}\\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y))\\
&= 0 \leqslant x = y\\
I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y))\\
&= 0 \leqslant x \leqslant y \leqslant x+1\\
I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y))\\
&= 0 \leqslant x \leqslant y \leqslant x+2\\
I^4(x,y) &= I^2(x,y) \mathbin{\nabla} I^3(x,y) \leftarrow \text{widening}\\
&= 0 \leqslant x \leqslant y
\end{aligned}$$



Fixed point

Accelerated Iterates $I = \lim_{n \to \infty} F^n(\text{false})$

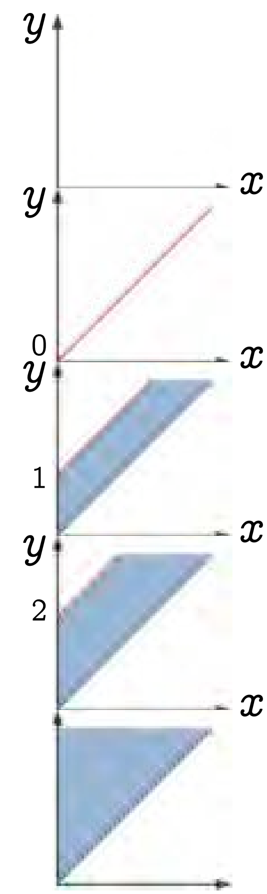
$$
\begin{aligned}
I^0(x,y) &= \text{false} \\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y)) \\
&= 0 \leqslant x = y \\
I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y)) \\
&= 0 \leqslant x \leqslant y \leqslant x+1 \\
I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y)) \\
&= 0 \leqslant x \leqslant y \leqslant x+2 \\
I^4(x,y) &= I^2(x,y) \mathbin{\nabla} I^3(x,y) \leftarrow \text{widening} \\
&= 0 \leqslant x \leqslant y \\
I^5(x,y) &= x \geqslant 0 \wedge (x = y \vee I^4(x+1,y)) \\
&= I^4(x,y) \quad \text{fixed point!}
\end{aligned}
$$



Octagons

Accelerated Iterates $I = \lim_{n\to\infty} F^n(\text{false})$

$$
\begin{aligned}
I^0(x,y) &= \text{false}\\
I^1(x,y) &= x \geqslant 0 \wedge (x = y \vee I^0(x+1,y))\\
&= 0 \leqslant x = y\\
I^2(x,y) &= x \geqslant 0 \wedge (x = y \vee I^1(x+1,y))\\
&= 0 \leqslant x \leqslant y \leqslant x+1\\
I^3(x,y) &= x \geqslant 0 \wedge (x = y \vee I^2(x+1,y))\\
&= 0 \leqslant x \leqslant y \leqslant x+2\\
I^4(x,y) &= I^2(x,y) \,\triangledown\, I^3(x,y) \leftarrow \text{widening}\\
&= 0 \leqslant x \leqslant y\\
I^5(x,y) &= x \geqslant 0 \wedge (x = y \vee I^4(x+1,y))\\
&= I^4(x,y) \quad \text{fixed point!}
\end{aligned}
$$

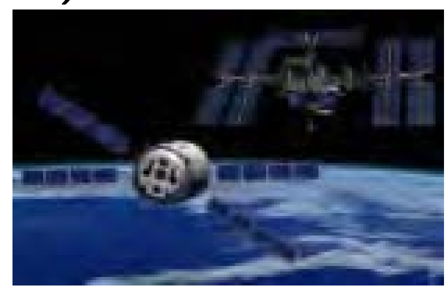


The invariants are computer representable with octagons!

Industrialisation: Development in cooperation with Airbus France

- Automatic proofs of absence of runtime errors in Electric Flight Control Software:
  - A340/600: 132.000 lines of C, 40mn on a PC 2.8 GHz, 300 Mb (Nov. 2003)
  - A380: 1.000.000 lines of C, 34h, 8 Gb (Nov. 2005)

  no false alarm, **World premières !**



- Automatic proofs of absence of runtime errors in the ATV software[2]:
  - C version of the automatic docking software: 102.000 lines of C, 23s on a Quad-Core AMD Opteron™ processor, 16 Gb (Apr. 2008)

---

[2] the Jules Vernes Automated Transfer Vehicle (ATV) enabling ESA to transport payloads to the International Space Station.

Application of abstract interpretation to program proof methods

Maximal execution trace

```
#include <stdio.h>
int main() {
          int x,y;
          printf("Enter an integer: ");
          scanf("%d",&x); y = x;
/* 1: */  while (x != 0) {
             printf("x = %d, y = %d\n",x,y);
/* 2: */     x = x - 1;
/* 3: */     y = y + 2;
          }
/* 4: */  printf("x = %d, y = %d\n",x,y); }
```

```
Enter an integer: 3
x = 3, y = 3
x = 2, y = 5
x = 1, y = 7
x = 0, y = 9
```

```
Enter an integer: -1
x = -1, y = -1
x = -2, y = 1
x = -3, y = 3
x = -4, y = 5
…
x = -738245, y = 1476487
…
```

$\langle 1{:},3,3,3\rangle \longrightarrow \langle 2{:},3,3,3\rangle \longrightarrow \langle 3{:},3,2,3\rangle \longrightarrow \langle 1{:},3,2,5\rangle \longrightarrow \langle 2{:},3,2,5\rangle \longrightarrow \langle 3{:},3,1,5\rangle \longrightarrow \langle 1{:},3,1,7\rangle \longrightarrow \langle 2{:},3,1,7\rangle \longrightarrow \langle 3{:},3,0,7\rangle \longrightarrow \langle 1{:},3,0,9\rangle \longrightarrow \langle 6{:},3,0,9\rangle$

Maximal execution trace

```
#include <stdio.h>
int main() {
          int x,y;
          printf("Enter an integer: ");
          scanf("%d",&x); y = x;
/* 1: */  while (x != 0) {
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/* 2: */     x = x - 1;
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          }
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```

```
Enter an integer: 3
x = 3, y = 3
x = 2, y = 5
x = 1, y = 7
x = 0, y = 9
```

```
Enter an integer: -1
x = -1, y = -1
x = -2, y = 1
x = -3, y = 3
x = -4, y = 5
…
x = -738245, y = 1476487
```

state — memory state — value y of y, value x of x, initial value x_0 of x; control point

initial state $\in$ *init* $[\![\mathrm{P}]\!]$

transition $\in$ *trans* $[\![\mathrm{P}]\!]$

$\underbrace{\langle 1:,3,3,3\rangle}_{\text{initial state}} \longrightarrow \langle 2:,3,3,3\rangle \longrightarrow \underbrace{\langle 3:,3,2,3\rangle \longrightarrow \langle 1:,3,2,5\rangle}_{\text{transition}} \longrightarrow \langle 2:,3,2,5\rangle$
$\longrightarrow \langle 3:,3,1,5\rangle \longrightarrow \langle 1:,3,1,7\rangle \longrightarrow \langle 2:,3,1,7\rangle \longrightarrow \langle 3:,3,0,7\rangle \longrightarrow$
$\langle 1:,3,0,9\rangle \longrightarrow \langle 6:,3,0,9\rangle$

Maximal trace semantics

- The trace semantics of a program is the set of all possible maximal finite or infinite execution traces for that program
- The trace semantics of a programing language maps programs to their trace semantics

Inductive definition

- Partial traces:
  - A trace with one initial state is a partial trace
  - A partial trace extended by a transition is a partial trace
- Maximal traces:
  - Finite traces with no extension by a transition
  - Infinite traces which prefixes are all partial traces

Fixpoint partial trace semantics

- initial states of program P: $\mathit{init}\llbracket \mathrm{P} \rrbracket$
- transitions of programs P: $\mathit{trans}\llbracket \mathrm{P} \rrbracket$
- $\mathrm{F}^{\mathrm{t}}\llbracket \mathrm{P} \rrbracket X = \{\, \mathrm{s} \mid \mathrm{s} \in \mathit{init}\llbracket \mathrm{P} \rrbracket \,\} \cup$
  $\{\, \sigma \mathrm{s}\mathrm{s}' \mid \sigma \mathrm{s} \in X \wedge \mathrm{s}\mathrm{s}' \in \mathit{trans}\llbracket \mathrm{P} \rrbracket \,\}$
- $\mathrm{S}^{\mathrm{t}}\llbracket \mathrm{P} \rrbracket = \mathsf{lfp}^{\subseteq}\, \mathrm{F}^{\mathrm{t}}\llbracket \mathrm{P} \rrbracket$

Invariance abstraction

- Collect at each control point the possible values of variables when execution reaches that control point
- $\alpha(X)\mathrm{c} = \{\mathrm{m} \mid \exists\sigma,\sigma'.\ \sigma\langle\mathrm{c},\mathrm{m}\rangle\sigma' \in X\}$
- Invariance semantics: $\mathrm{S^i}[\![\mathrm{P}]\!] = \alpha(\mathrm{S^t}[\![\mathrm{P}]\!])$

Invariance abstraction

- Collect at each control point the possible values of variables when execution reaches that control point
- $\mathrm{S^i}[\![\mathrm{P}]\!] = \alpha(\mathrm{S^t}[\![\mathrm{P}]\!])c = \{m \mid \exists\sigma,\sigma'.\ \sigma\langle c,m\rangle\sigma' \in \mathrm{S^t}[\![\mathrm{P}]\!]\}$

```c
#include <stdio.h>
int main() {
              int x,y;
              printf("Enter an integer: ");
              scanf("%d",&x); y = x;
/* 1: */      while (x != 0) {
                 printf("x = %d, y = %d\n",x,y);
/* 2: */         x = x - 1;
/* 3: */         y = y + 2;
              }
/* 4: */      printf("x = %d, y = %d\n",x,y); }
```

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x\}$ ← /* 1: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x\}$ ← /* 2: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 - 2x - 2\}$ ← /* 3: */

$\{\langle x_0, x, \mathrm{y}\rangle \mid \mathrm{y} = 3x_0 \wedge x = 0\}$ ← /* 4: */

Calculations design of the verification conditions

- $\alpha(\mathrm{F}^{\mathrm{t}}[\![\mathrm{P}]\!]X)$
  $= \boldsymbol{\lambda}\,\mathrm{c}.\{\mathrm{m} \mid \exists \sigma,\sigma'.\ \sigma\langle \mathrm{c},\mathrm{m}\rangle\sigma' \in X\}$
  $= \ldots$
  $= \mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!](\alpha(X))$
  where $\mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!]$ are the Turing/Floyd/Naur/Hoare verification conditions
- It follows that $\mathrm{S}^{\mathrm{i}}[\![\mathrm{P}]\!] = \mathsf{lfp}^{\dot\subseteq}\ \mathrm{F}^{\mathrm{i}}[\![\mathrm{P}]\!]$
- The proof method is then by fixpoint induction (Tarski 1955)

Application to the semantics of programming languages

General idea

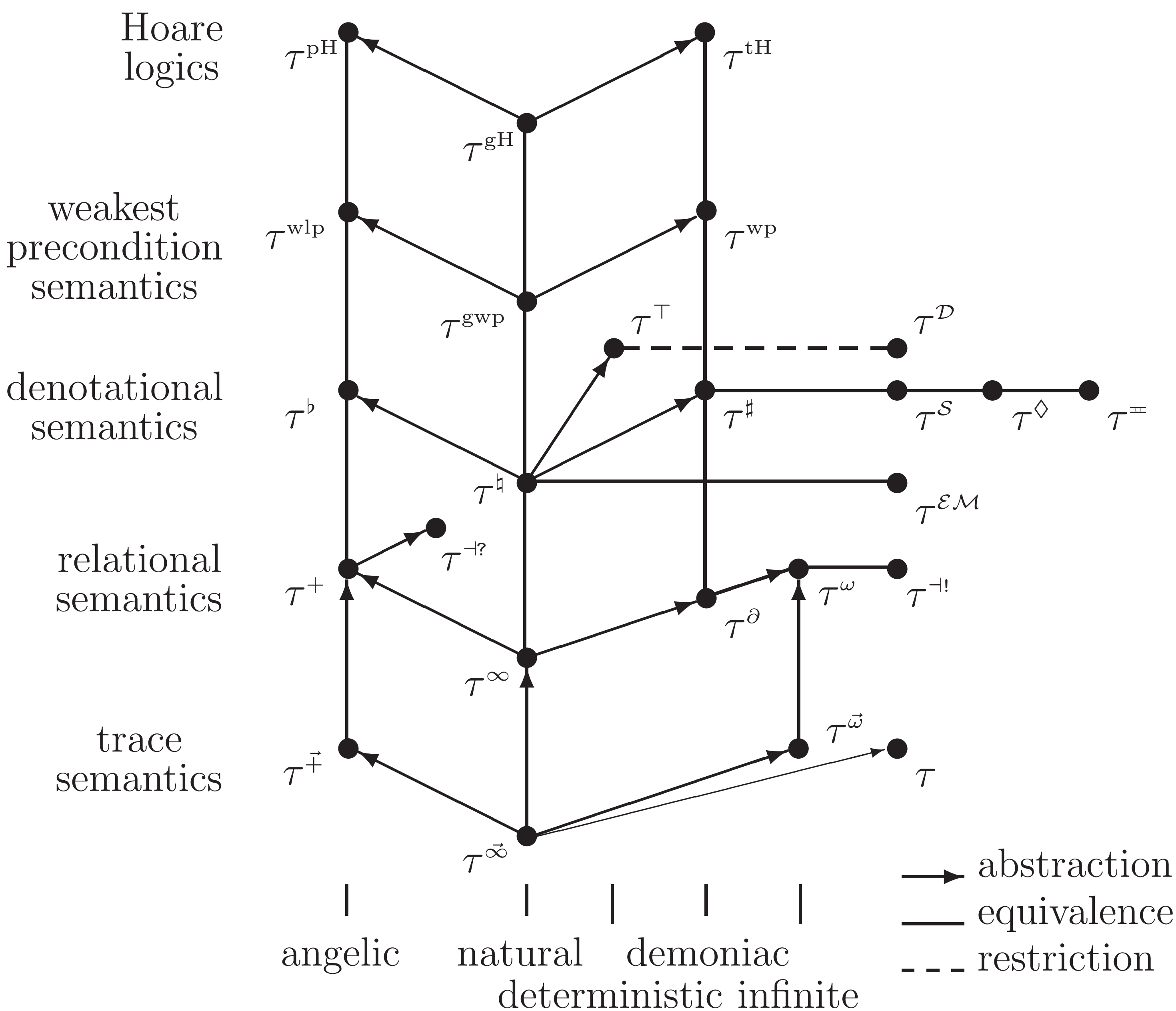
- All known semantics are abstractions of a most precise semantics

Abstraction to denotational semantics

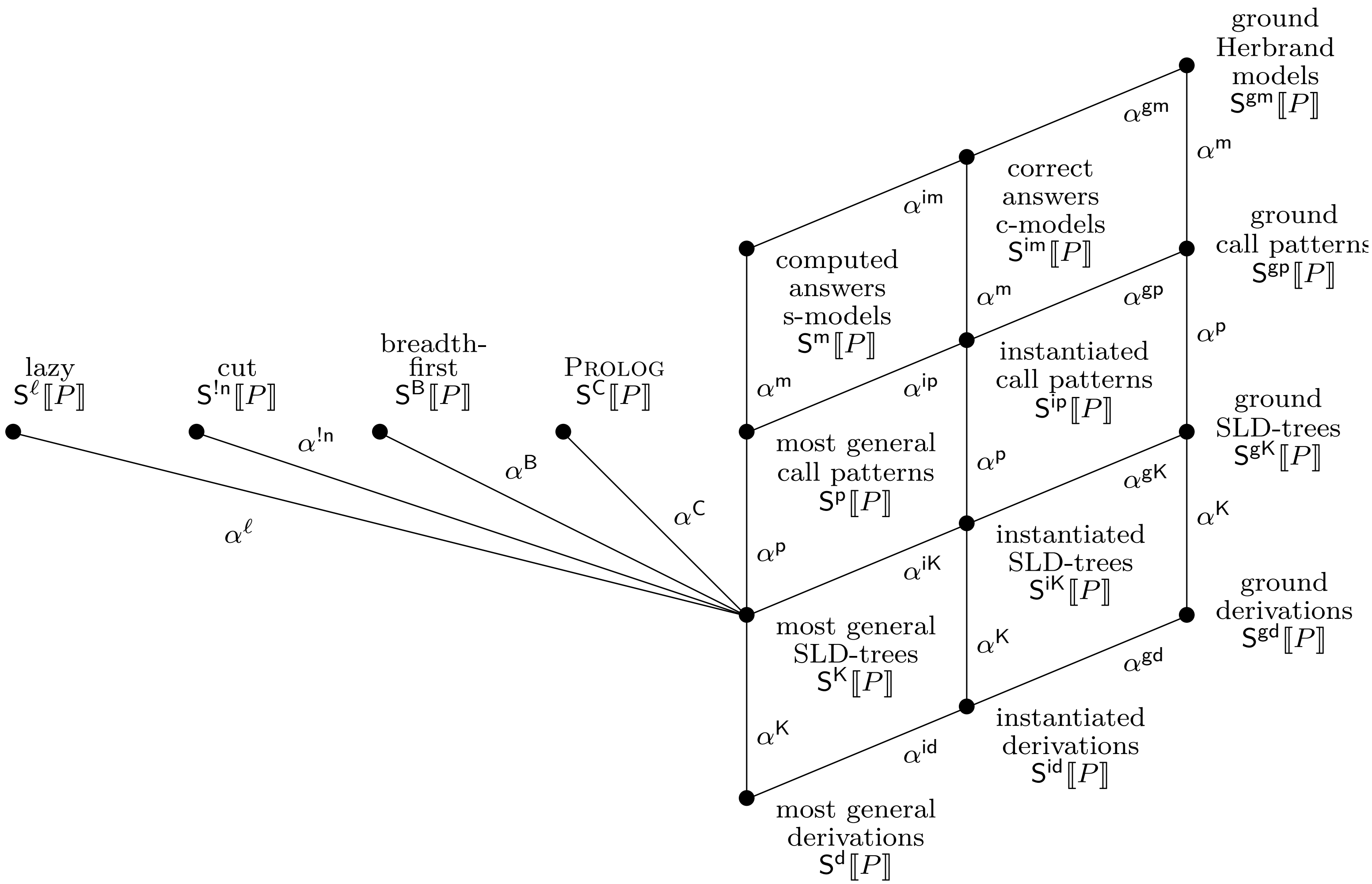
- The maximal trace semantics $\mathrm{S}^{\mathrm{m}}[\![\mathrm{P}]\!]$ (maximal finite and infinite execution traces
- Denotational semantics abstraction:
  - $\mathrm{S}^{\mathrm{d}}[\![\mathrm{P}]\!] = \alpha(\mathrm{S}^{\mathrm{m}}[\![\mathrm{P}]\!])$
  - $\alpha(X) = \boldsymbol{\lambda}\mathrm{s}.\{\mathrm{s}' \mid \exists\sigma.\ \mathrm{s}\sigma\mathrm{s}' \in X\} \cup \{\perp \mid \exists\sigma.\ \mathrm{s}\sigma\ldots \in X\}$

  i.e. a map of initial states to the set of final states plus $\perp$ in case of non-termination

Hierarchy of abstractions



idem for Prolog



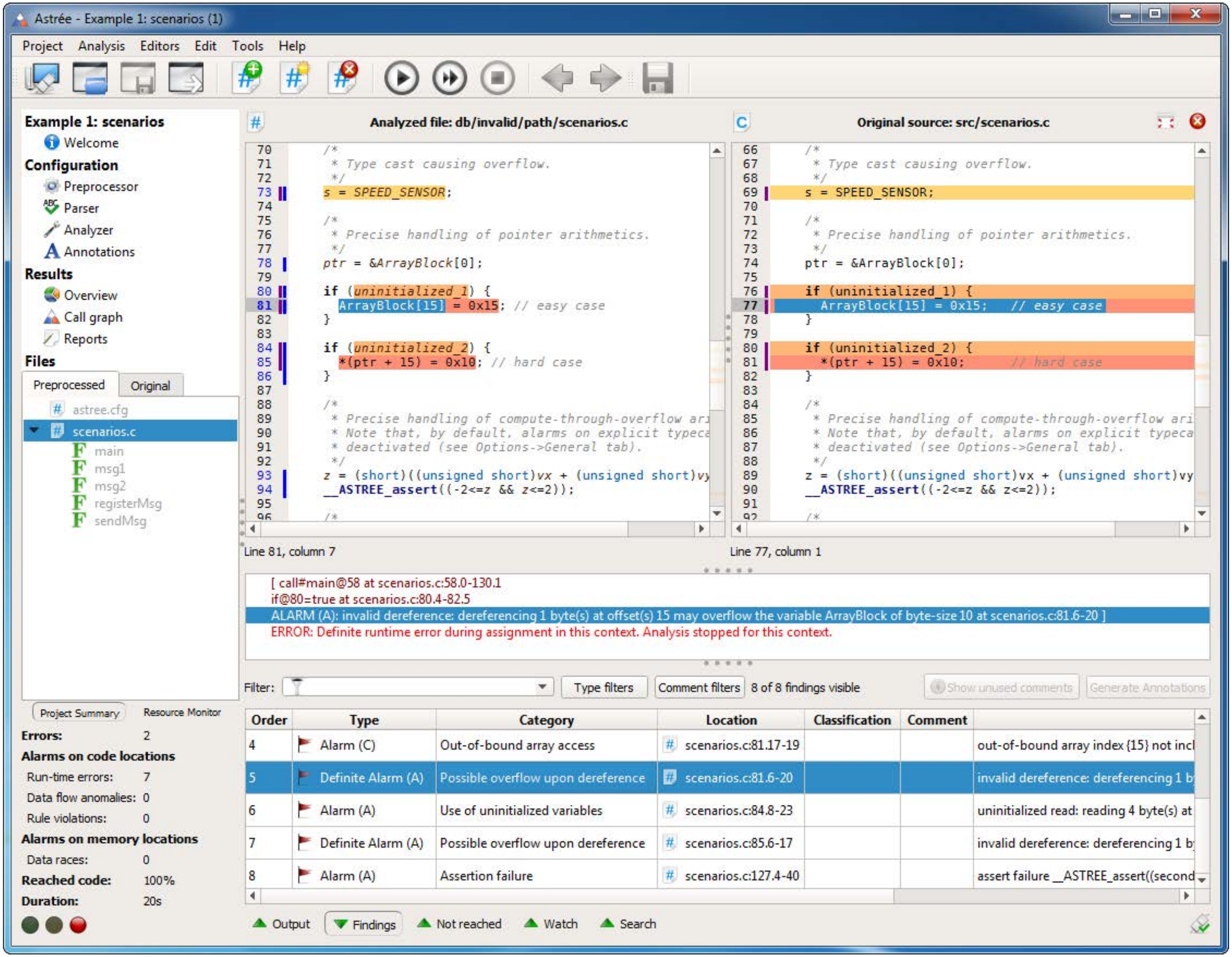
- all semantics are abstractions of $S^{d}[\![P]\!]$

Conclusion

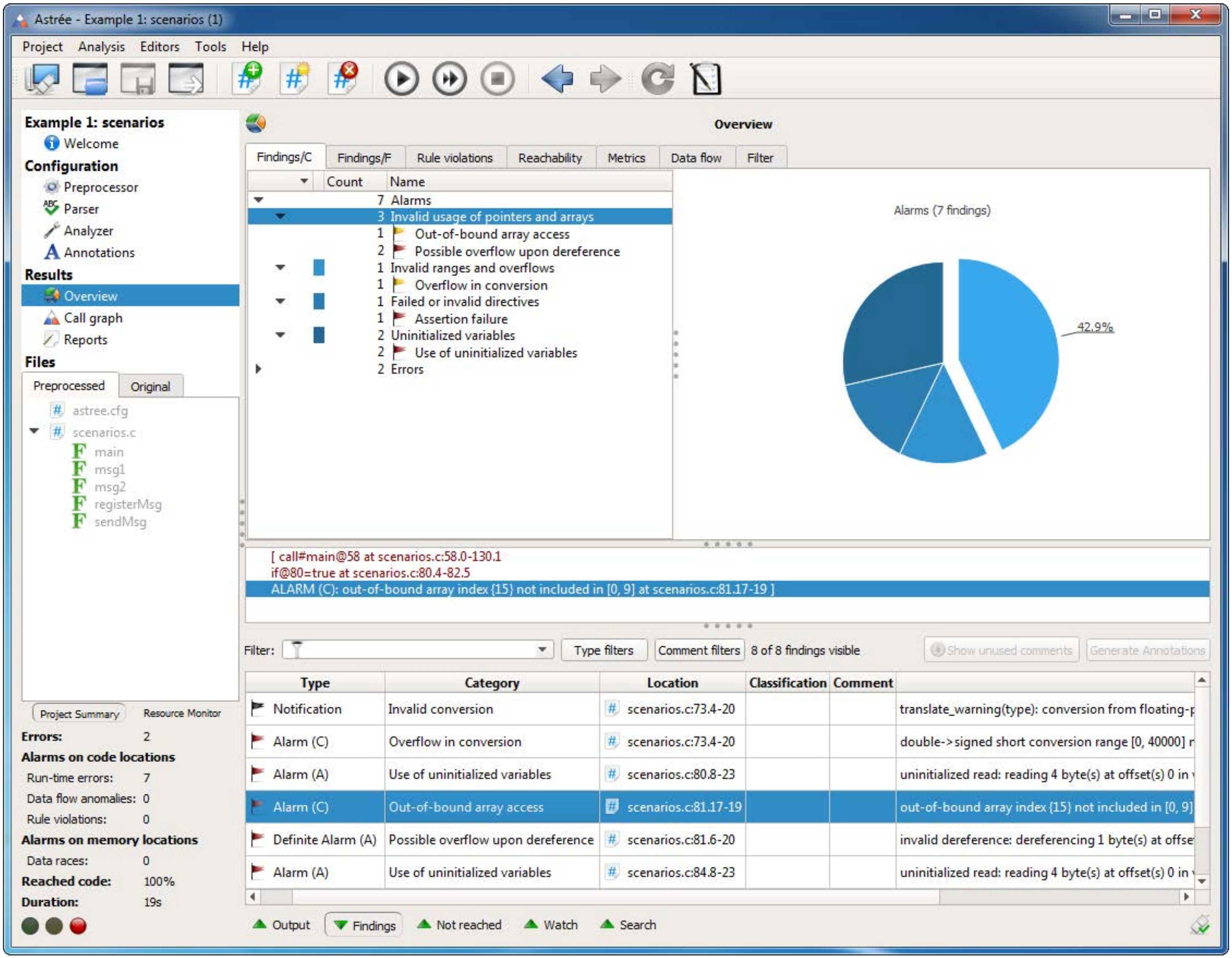
Abstract interpretation

- A well-developed theory, still in progress
- Active research e.g.
    - abstract domains to handle e.g. complex data structures
    - abstraction of parallelism with weak memory models
    - applications to biology, …
- Industrial-quality static analyzers

Industrialisation: Astrée



Industrialisation: Astrée



Many other static analyzers

- Julia (Java) http://www.juliasoft.com
- Ikos, NASA https://ti.arc.nasa.gov/opensource/ikos/
- Clousot for code contract, Microsoft, https://github.com/Microsoft/CodeContracts
- Infer (Facebook) http://fbinfer.com
- Zoncolan (Facebook)
- Google
- …

Static analysis for software development

- Users of Astrée:

  …

- Why not all software developers use static analysis tools?

Irresponsibility

- Computer engineering is the only technology where developers are not responsible for their errors, even the trivial ones:

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The future

- Safety and security does matter to the general public
- Computer scientists will ultimately be held responsible for there errors
- At least the automatically discoverable ones
- Since this is now part of the state of the art
- Automatic static analysis, verification, etc has a brilliant future.

Francesco Logozzo, designer of the Zoncolan static analyzer at Facebook wrote me on 09/12/2016:

``Finding people who really know static analysis is very hard, you should tell your students that if they want a great job in a Silicon Valley company they should study abstract interpretation not JavaScript. Feel free to quote me on that ;-)”

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The End, Thank You