« Software Verification by Abstract Interpretation and the ASTRÉE Static Analyzer »

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Abstract

Abstract interpretation is a theory of sound approximation of the behavior of dynamic systems, in particular the semantics of programming languages. This is the formal basis for automatic correctness proofs by static analysers considering an over-approximation of the set of all possible executions of the program. Contrary to bug-finding methods (e.g. by test, bounded model-checking or error pattern search), no potential error is ever omitted. Hence the proof of satisfaction of a specification is always mathematically valid. Contrary to refinement-based methods, termination is always guaranteed. However, by undecidability of such proofs, the abstraction may yield false alarms whenever a synthesized inductive argument (e.g. a loop invariant) is too weak to make the proof. In this case, some executions considered in the abstract, that is in the over-approximation, might lead to an error while not corresponding to a concrete, that is actual, execution. All the difficulty of the undecidable verification problem is therefore to design safe/sound over-approximations that are coarse enough to be effectively computable by the static analyzer and precise enough to avoid false alarms (the errors leading to true alarms can only be eliminated by correcting the program that does not satisfy the specification).

After a brief introduction to abstract interpretation, we will present the ASTRÉE static analyser (`www.astree.ens.fr`) for proving the absence of runtime errors (such as buffer overrun, dangling pointer, division by zero, float overflow, modular integer arithmetic overflow, ...) in real-time synchronous control/command C applications. The ASTRÉE static analyser uses generalist abstractions (like intervals, octagons, decision trees, symbolic execution, etc) and abstractions for the specific application domain (to cope with filters, integrators, slow divergences due to rounding errors, etc). Since 2003, these domain-specific abstractions allowed for the verification of the absence of runtime errors in several large avionic software, a world première.

Contents

1. The Problem: The Design of Safe and Secure Computer-Based Systems

Bugs Now Show-Up in Everyday Life

– Bugs now appear frequently in everyday life (banks, cars, telephones, . . .)

– Example (HSBC bank ATM [1] at 19 Boulevard Sébastopol in Paris, failure on Nov. 21st 2006 at 8:30 am):

[1] cash machine, cash dispenser, automatic teller machine.

A Strong Need for Software Better Quality

– Poor software quality is not acceptable in safety and mission critical software applications.



– The present state of the art in software engineering does not offer sufficient quality garantees

The Complexity of Software Design

– The design of complex software is difficult and economically critical

– Example (www.designnews.com/article/CA6475332.html):

"Boeing Confirms 787 Delay, Fasteners, Flight Control Software Code Blamed
John Dodge, Editor-in-Chief – Design News, September 5, 2007

> Boeing officials confirmed today that a fastener shortage and problems with flight control software have pushed "first flight" of the Boeing 787 Dreamliner to sometime between mid-November and mid-December [2].
> ...
> The software delays involve Honeywell Aerospace, which is responsible for flight control software. The work on this part of the 787 was simply underestimated, said Bair."

[2] Bill Rigby of Reuters announced that Boeing delays 787 by 3 months on Wed Jan 16, 2008 12:37pm EST.

The Security of Complex Software

– Complex software is subject to security vulnerabilies

– Example (www.wired.com/politics/security/news/2008/01/dreamliner_security)

"FAA: Boeing's New 787 May Be Vulnerable to Hacker Attack

Kim Zetter, freelance journalist in Oakland, CA, Jan. 4, 2008

> Boeing's new 787 Dreamliner passenger jet may have a serious security vulnerability in its onboard computer networks ...
>
> According to the FAA document published in the Federal Register (mirrored at Cryptome.org), the vulnerability exists because the plane's computer systems connect the passenger network with the flight-safety, control and navigation network. It also connects to the airline's business and administrative-support network, which communicates maintenance issues to ground crews.

Tool-Based Software Design Methods

– New tool-based software design methods will have to emerge to face the unprecedented growth and complexification of critical software

– E.g. FCPC (Flight Control Primary Computer)
  - A220: 20 000 LOCs,
  - A340 (V1): 130 000 LOCS
  - A340 (V2): 250 000 LOCS
  - A380: 1.000.000 LOCS
  - A350: static analysis to be integrated in the software production

Validation/Formal Methods

- Bug-finding methods : unit, integration, and system testing, dynamic verification, bounded model-checking, error pattern mining, ...
- Absence of bug proving methods : formally prove that the semantics of a program satisfies a specification
  - theorem-proving & proof checking
  - model-checking
  - static analysis
- In practice : complementary methods are used, very difficult to scale up

Problems with Formal Methods

- Formal specifications (abstract machines, temporal logic, ...) are costly, complex, error-prone, difficult to maintain, not mastered by casual programmers
- Formal semantics of the specification and programming language are inexistant, informal, irrealistic or complex
- Formal proofs are partial (static analysis), do not scale up (model checking) or need human assistance (theorem proving & proof assistants)

  $\Rightarrow$ High costs (for specification, proof assistance, etc).

Avantages of Static Analysis

- Formal specifications are implicit (no need for explicit, user-provided specifications)
- Formal semantics are approximated by the static analyzer (no user-provided models of the program)
- Formal proofs are automatic (no required user-interaction)
- Costs are low (no modification of the software production methodology)
- Scales up to 100.000 to 1.000.000 LOCS
- Rapid and large diffusion in embedded software production industries

Disadvantages of Static Analysis

- Imprecision (acceptable in some applications like WCET or program optimization)
- Incomplete for program verification
- False alarms are due to unsuccessful automatic proofs in 5 to 15% of the cases

For example, 1% of 500.000 potential (true or false) alarms is 5.000, too much to be handled by hand!

Remedies to False Alarms in Astrée

- Astrée is specialized to specific program properties [3]
- Astrée is specialized to real-time synchronous control/command programs written in C
- Astrée offers possibilities of refinement [4]

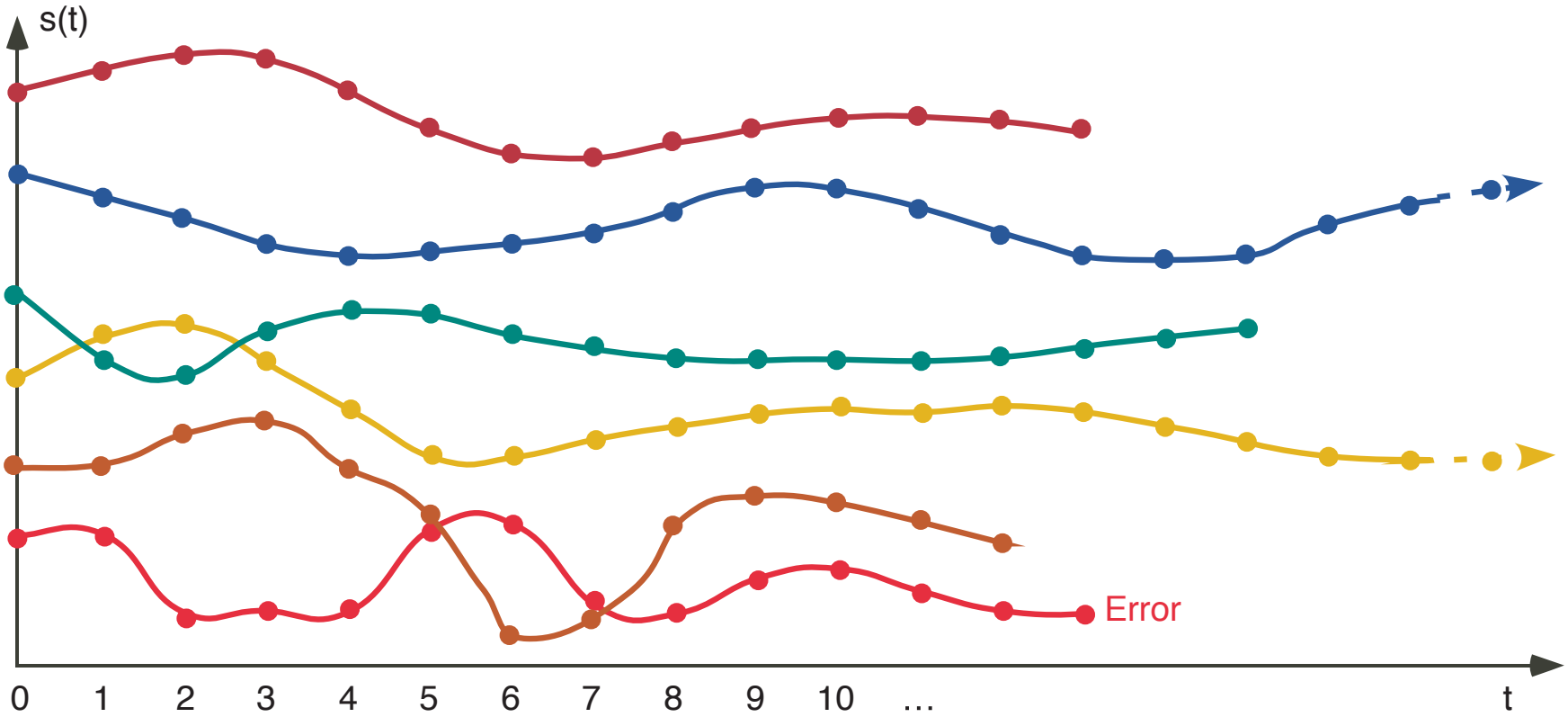
The cost of adapting Astrée to a specific program, should be a small fraction of the cost to test the specific program properties verified by Astrée.

[3] proof of absence of runtime errors

[4] parametrizations and analysis directives

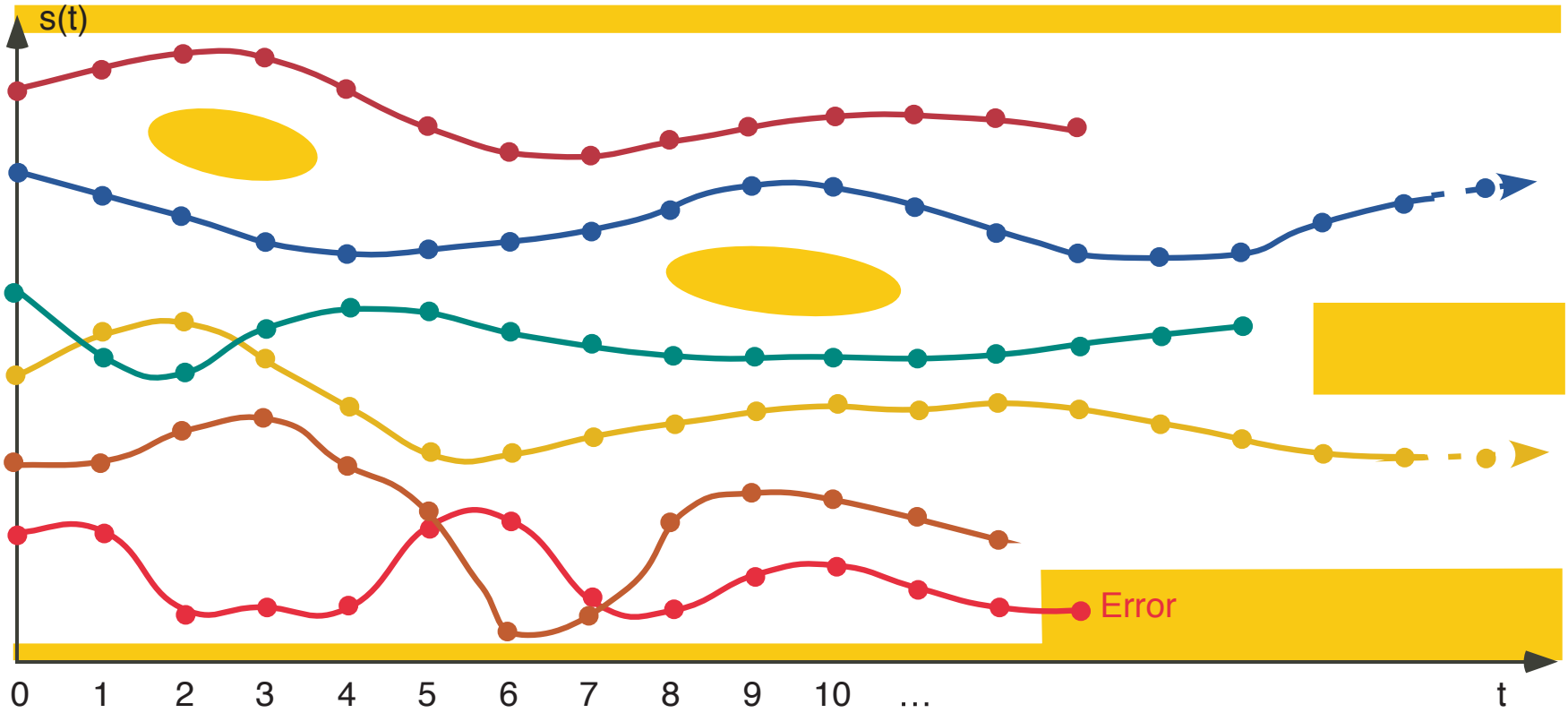
2. Introduction to Static Analysis

Principle of Static Analysis
(1) Concrete Semantics



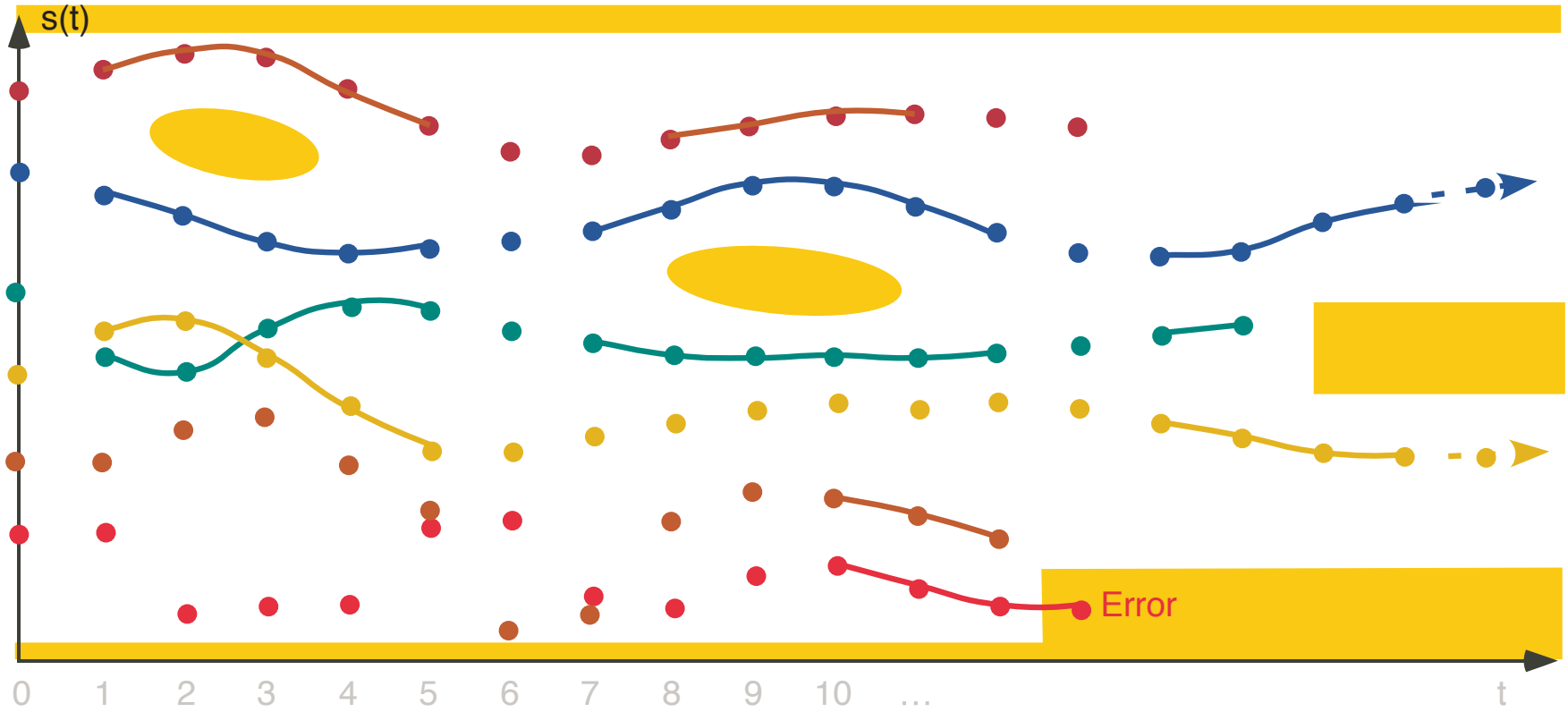
Finite and infinite discrete execution traces

Principle of Static Analysis
(2) Specification



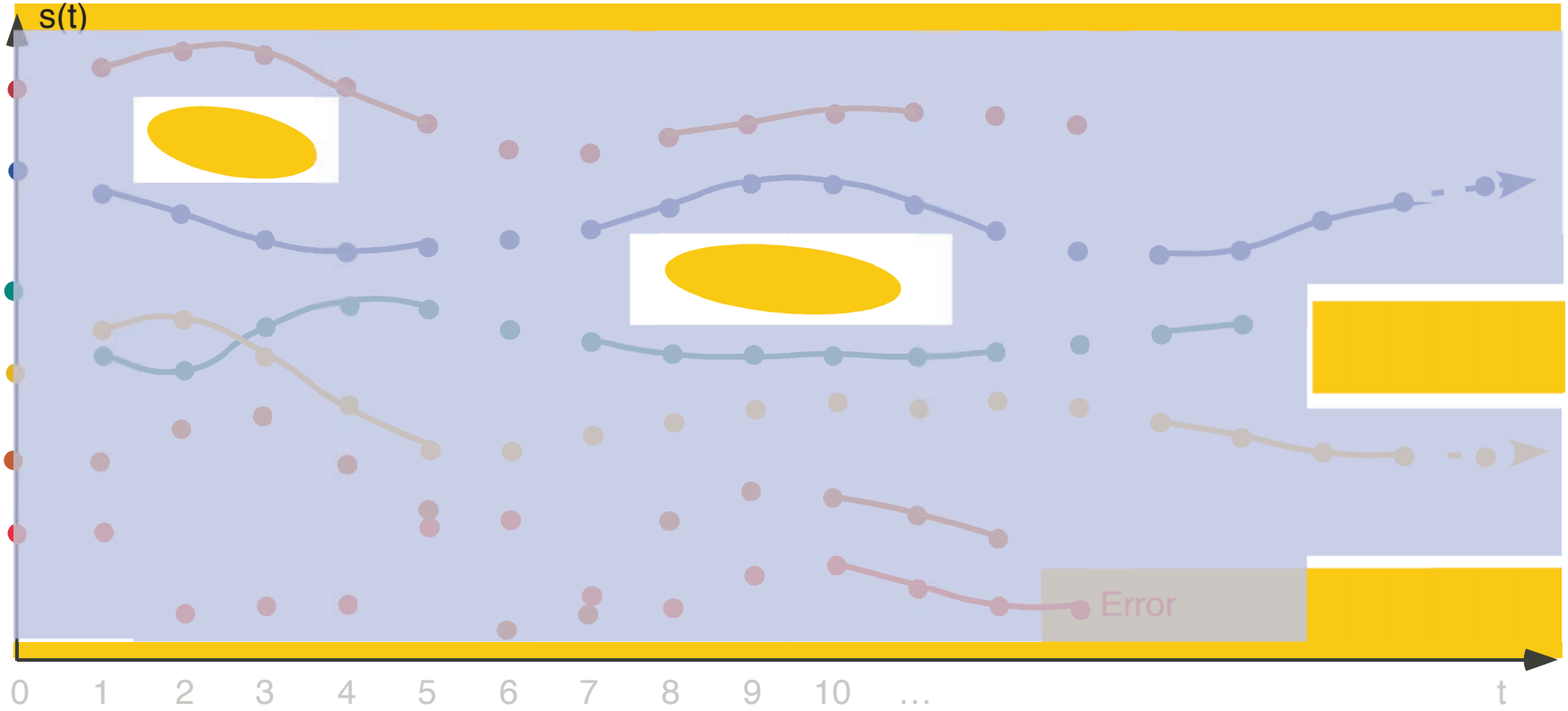
Safety specification of bad states

Principle of Static Analysis
(3.1) Abstract Semantics



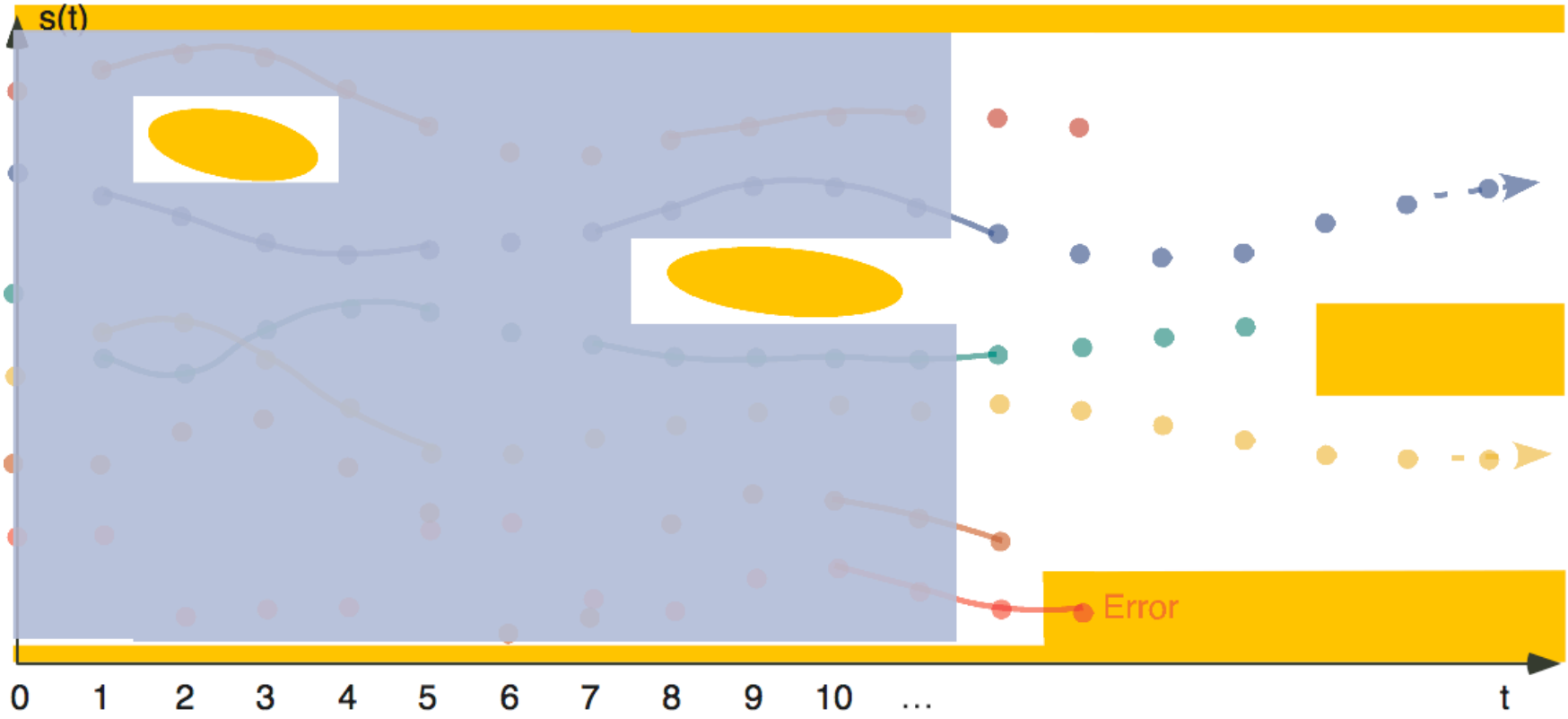
Abstraction to reachable partial traces

Principle of Static Analysis
(3.2) Abstract Semantics



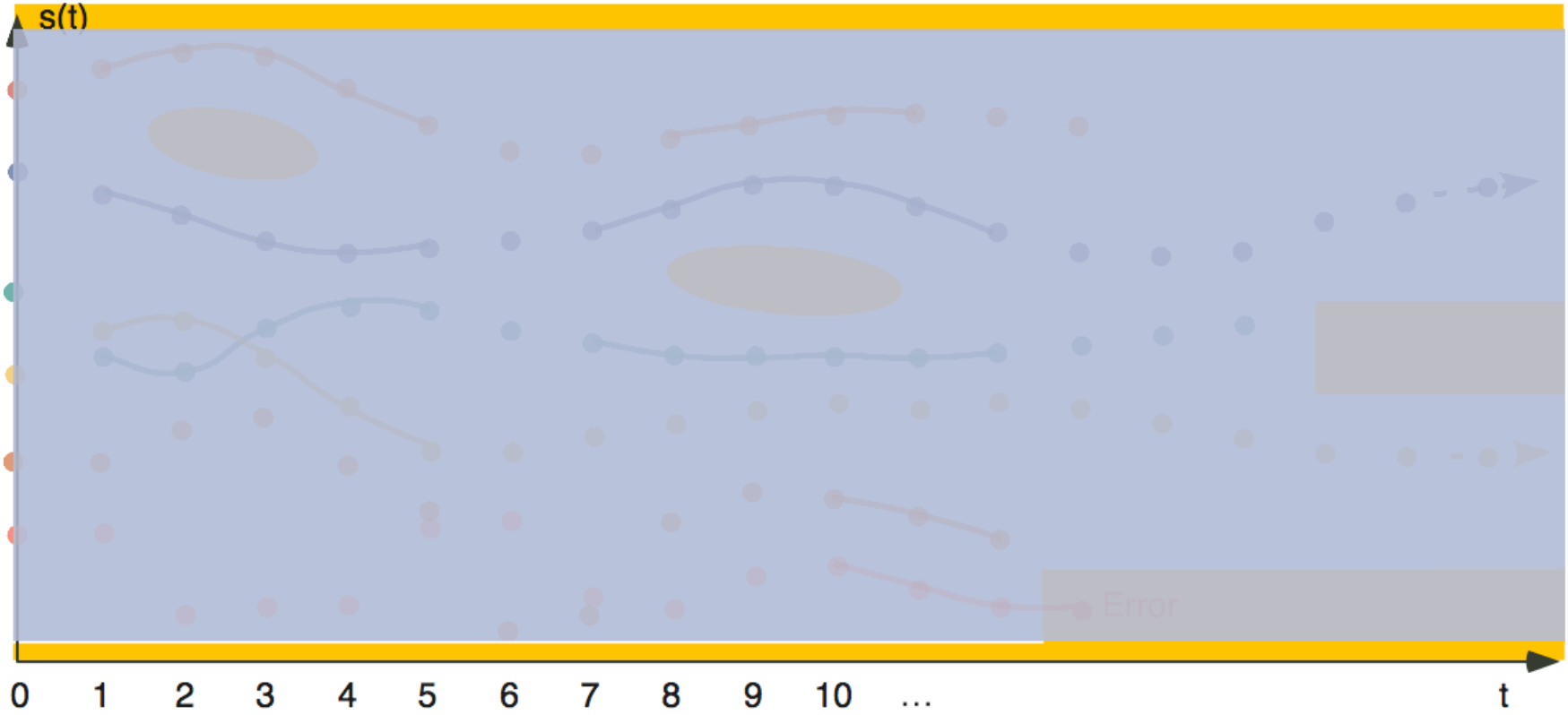
Further abstraction to a pavage of intervals

Unsoundness
(False Negatives)



Some states are omitted (e.g. bounded model checking)

Incomplete
(False Positive/Alarms)



Over-approximation containing unreachable states

3. Abstract Interpretation

The Theory of Abstract Interpretation

- A theory of sound approximation of mathematical structures, in particular those involved in the behavior of computer systems
- Systematic derivation of sound methods and algorithms for approximating undecidable or highly complex problems in various areas of computer science
- Main practical application is on the safety and security of complex hardware and software computer systems
- Abstraction: extracting information from a system description that is relevant to proving a property

Applications of Abstract Interpretation

- Static Program Analysis [CC77], [CH78], [CC79] including Dataflow Analysis; [CC79], [CC00], Set-based Analysis [CC95], Predicate Abstraction [Cou03], ...
- Grammar Analysis and Parsing [CC03];
- Hierarchies of Semantics and Proof Methods [CC92], [Cou02];
- Typing & Type Inference [Cou97];
- (Abstract) Model Checking [CC00];
- Program Transformation (including program optimization, partial evaluation, etc) [CC02];

Applications of Abstract Interpretation (Cont'd)

- Software Watermarking [CC04];
- Bisimulations [RT04, RT06];
- Language-based security [GM04];
- Semantics-based obfuscated malware detection [PCJD07].
- Databases [AGM93, BPC01, BS97]
- Computational biology [Dan07]
- Quantum computing [JP06, Per06]

All these techniques involve sound approximations that can be formalized by abstract interpretation

4. Principle of Abstract Interpretation

References

[POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In 4^{th} *ACM POPL*.

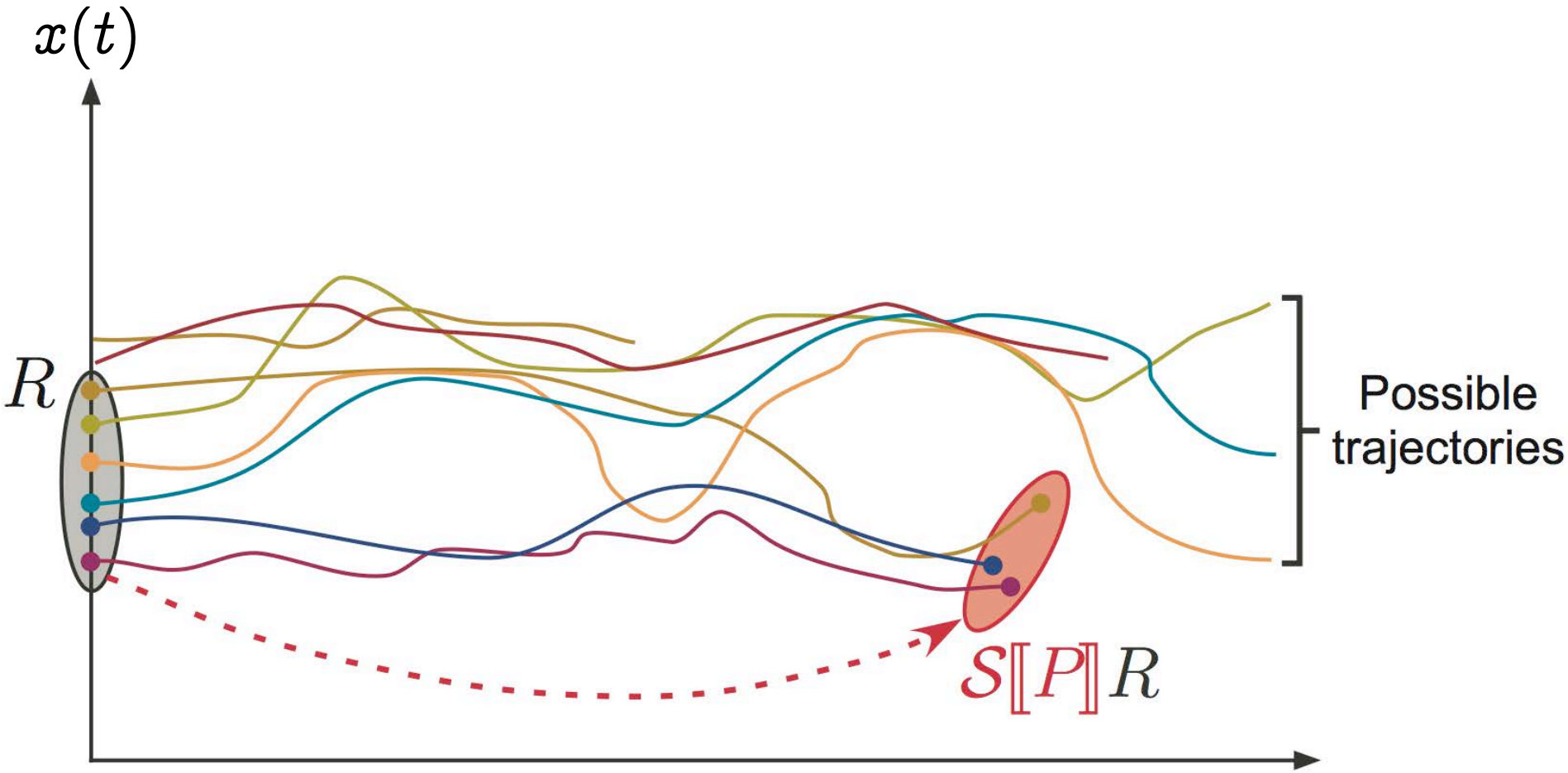
[Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In 6^{th} *ACM POPL*.

Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T\ X\ ;$	
$\quad \mid\ T\ X\ ;\ D'$	
$C ::= X = E;$	commands $C \in \mathbb{C}$
$\quad \mid\ \texttt{while}\ B\ C'$	
$\quad \mid\ \texttt{if}\ B\ C'\ \texttt{else}\ C''$	
$\quad \mid\ \{\ \mathrm{C}_1\ \ldots\ \mathrm{C}_n\ \}, (n \geq 0)$	
$P ::= D\ C$	program $P \in \mathbb{P}$

Postcondition semantics



States

Values of given type:

$$\mathcal{V}[\![T]\!] \ : \ \text{values of type } T \in \mathbb{T}$$
$$\mathcal{V}[\![\texttt{int}]\!] \triangleq \{z \in \mathbb{Z} \mid \texttt{min\_int} \leq z \leq \texttt{max\_int}\}$$

Program states $\Sigma[\![P]\!]$ [5]:

$$\Sigma[\![D\ C]\!] \triangleq \Sigma[\![D]\!]$$
$$\Sigma[\![T\ X\ ;]\!] \triangleq \{X\} \mapsto \mathcal{V}[\![T]\!]$$
$$\Sigma[\![T\ X\ ;\ D]\!] \triangleq (\{X\} \mapsto \mathcal{V}[\![T]\!]) \cup \Sigma[\![D]\!]$$

[5] States $\rho \in \Sigma[\![P]\!]$ of a program P map program variables X to their values $\rho(X)$

Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[\![P]\!] \triangleq \wp(\Sigma[\![P]\!]) \qquad \text{sets of states}$$

i.e. program properties where $\subseteq$ is implication, $\varnothing$ is false, $\cup$ is disjunction.

Concrete Reachability Semantics of Programs

$$\mathcal{S}[\![X = E;]\!]R \triangleq \{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in R \cap \mathrm{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \triangleq v, \qquad \rho[X \leftarrow v](Y) \triangleq \rho(Y)$$

$$\mathcal{S}[\![\texttt{if } B \ C']\!]R \triangleq \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{B}[\![\neg B]\!]R$$

$$\mathcal{B}[\![B]\!]R \triangleq \{\rho \in R \cap \mathrm{dom}(B) \mid B \text{ holds in } \rho\}$$

$$\mathcal{S}[\![\texttt{if } B \ C' \texttt{ else } C'']\!]R \triangleq \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{S}[\![C'']\!](\mathcal{B}[\![\neg B]\!]R)$$

$$\mathcal{S}[\![\texttt{while } B \ C']\!]R \triangleq \text{let } \mathcal{W} = \mathbf{lfp}^{\subseteq}_{\varnothing} \ \boldsymbol{\lambda}\, \mathcal{X} \cdot R \cup \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}[\![\{\}]\!]R \triangleq R$$

$$\mathcal{S}[\![\{C_1 \ldots C_n\}]\!]R \triangleq \mathcal{S}[\![C_n]\!] \circ \ldots \circ \mathcal{S}[\![C_1]\!] \quad n > 0$$

$$\mathcal{S}[\![D \ C]\!]R \triangleq \mathcal{S}[\![C]\!](\Sigma[\![D]\!]) \quad \text{(uninitialized variables)}$$

Not computable (undecidability).

Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\sharp[\![P]\!], \sqsubseteq, \bot, \sqcup \rangle$$

such that:

$$\langle \mathcal{D}[\![P]\!], \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \mathcal{D}^\sharp[\![P]\!], \sqsubseteq \rangle$$

i.e.

$$\forall X \in \mathcal{D}[\![P]\!], Y \in \mathcal{D}^\sharp[\![P]\!] : \alpha(X) \sqsubseteq Y \iff X \subseteq \gamma(Y)$$

hence $\langle \mathcal{D}^\sharp[\![P]\!], \sqsubseteq, \bot, \sqcup \rangle$ is a complete lattice such that $\bot = \alpha(\varnothing)$ and $\sqcup X = \alpha(\cup\, \gamma(X))$

Example 1 of Abstraction

Traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\xrightarrow{\alpha}$ Strongest liberal postcondition: final states s reachable from a given precondition P

$$\alpha(X) = \boldsymbol{\lambda}\, P \cdot \{s \mid \exists \sigma_0 \sigma_1 \ldots \sigma_n \in X : \sigma_0 \in P \wedge s = \sigma_n\}$$

We have (Σ: set of states, $\dot{\subseteq}$ pointwise):

$$\langle \wp(\Sigma^\infty), \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \wp(\Sigma) \overset{\cup}{\longmapsto} \wp(\Sigma), \dot{\subseteq} \rangle$$

Example 2 of Abstraction

Traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\xrightarrow{\alpha_1}$ Set of reachable states: set of states appearing at least once along one of these traces (global invariant)

$$\alpha_1(X) = \{\sigma_i \mid \sigma \in X \wedge 0 \leq i < |\sigma|\}$$

$\xrightarrow{\alpha_2}$ Partitionned set of reachable states: project along each control point (local invariant)

$$\alpha_2(\{\langle c_i,\ \rho_i\rangle \mid i \in \Delta\}) = \boldsymbol{\lambda}\, c \cdot \{\rho_i \mid i \in \Delta \wedge c = c_i\}$$

$\xrightarrow{\alpha_3}$ Partitionned cartesian set of reachable states: project along each program variable (relationships between variables are now lost)

$$\alpha_3(\boldsymbol{\lambda}\, c \cdot \{\rho_i \mid i \in \Delta_c\}) = \boldsymbol{\lambda}\, c \cdot \boldsymbol{\lambda}\, \mathtt{X} \cdot \{\rho_i(\mathtt{X}) \mid i \in \Delta_c\}$$

$\xrightarrow{\alpha_4}$ Partitionned cartesian interval of reachable states: take min and max of the values of the variables [6]

$$\alpha_4(\boldsymbol{\lambda}\, c \cdot \boldsymbol{\lambda}\, \mathtt{X} \cdot \{v_i \mid i \in \Delta_{c,\mathtt{X}}\} =$$
$$\boldsymbol{\lambda}\, c \cdot \boldsymbol{\lambda}\, \mathtt{X} \cdot \langle \min\{v_i \mid i \in \Delta_{c,\mathtt{X}}\},\ \max\{v_i \mid i \in \Delta_{c,\mathtt{X}}\}\rangle$$

α_1, α_2, α_3 and α_4, whence $\alpha_4 \circ \alpha_3 \circ \alpha_2 \circ \alpha_1$ are lower-adjoints of Galois connections

[6] assuming these values to be totally ordered.

Example 3: Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \subseteq \rangle \underset{\alpha_1}{\overset{\gamma_1}{\leftrightarrows}} \langle \mathcal{D}_1^\sharp, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \subseteq \rangle \underset{\alpha_2}{\overset{\gamma_2}{\leftrightarrows}} \langle \mathcal{D}_2^\sharp, \sqsubseteq_2 \rangle$$

the reduced product is

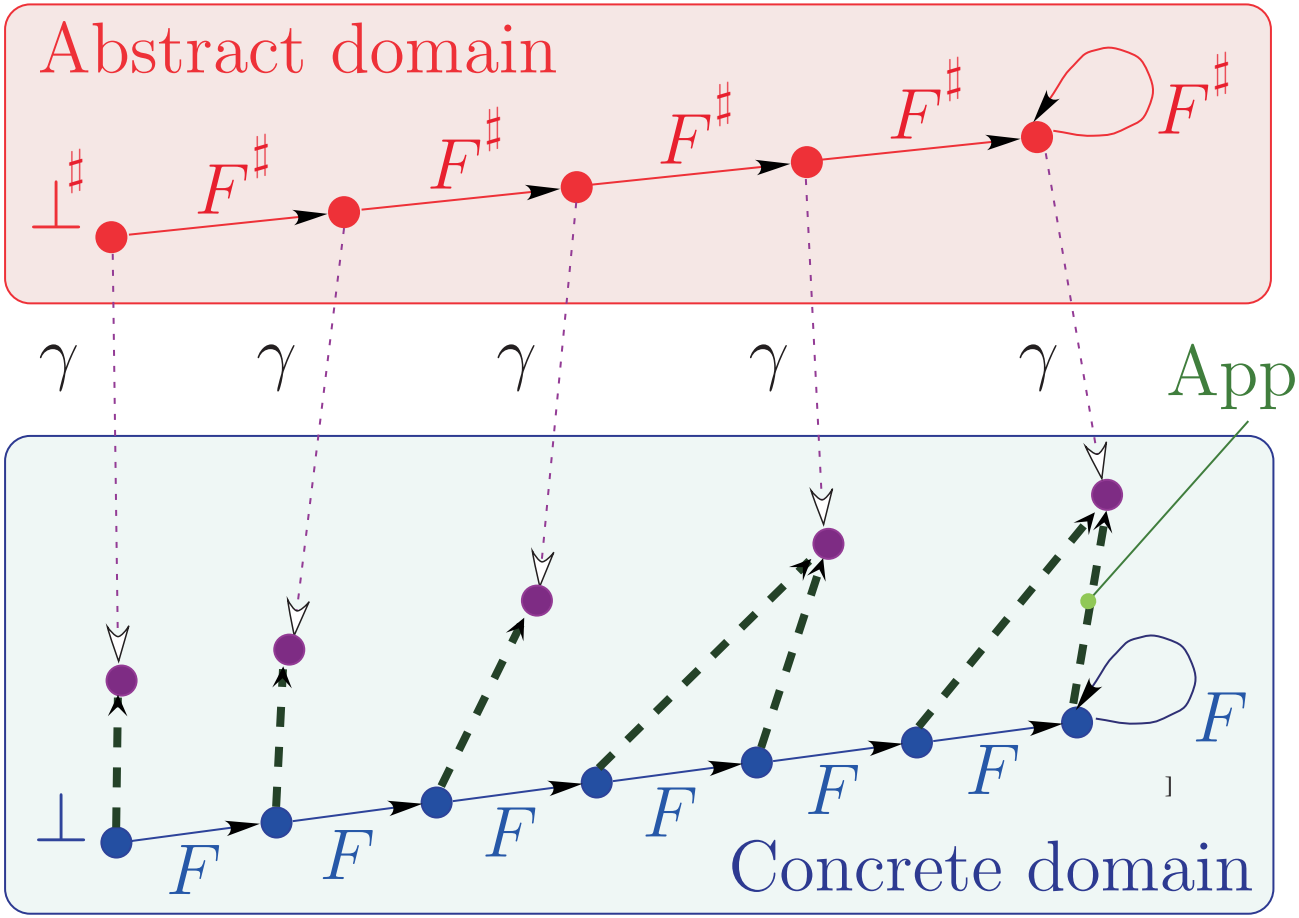
$$\alpha(X) \triangleq \sqcap \{ \langle x, y \rangle \mid X \subseteq \gamma_1(x) \wedge X \subseteq \gamma_2(y) \}$$

such that $\sqsubseteq \triangleq \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \subseteq \rangle \underset{\alpha}{\overset{\gamma_1 \times \gamma_2}{\leftrightarrows\!\!\!\!\!\!\rightarrow}} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $\mathrm{x} \in [1, 9] \wedge \mathrm{x} \bmod 2 = 0$ reduces to $\mathrm{x} \in [2, 8] \wedge \mathrm{x} \bmod 2 = 0$

Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\sharp \Rightarrow \mathsf{lfp}\, F \sqsubseteq \gamma(\mathsf{lfp}\, F^\sharp)$$

Abstract Reachability Semantics of Programs

$$\mathcal{S}^\sharp[\![X = E;]\!]R \triangleq \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \mathrm{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B \ C']\!]R \triangleq \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \triangleq \alpha(\{\rho \in \gamma(R) \cap \mathrm{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B \ C' \ \texttt{else} \ C'']\!]R \triangleq \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

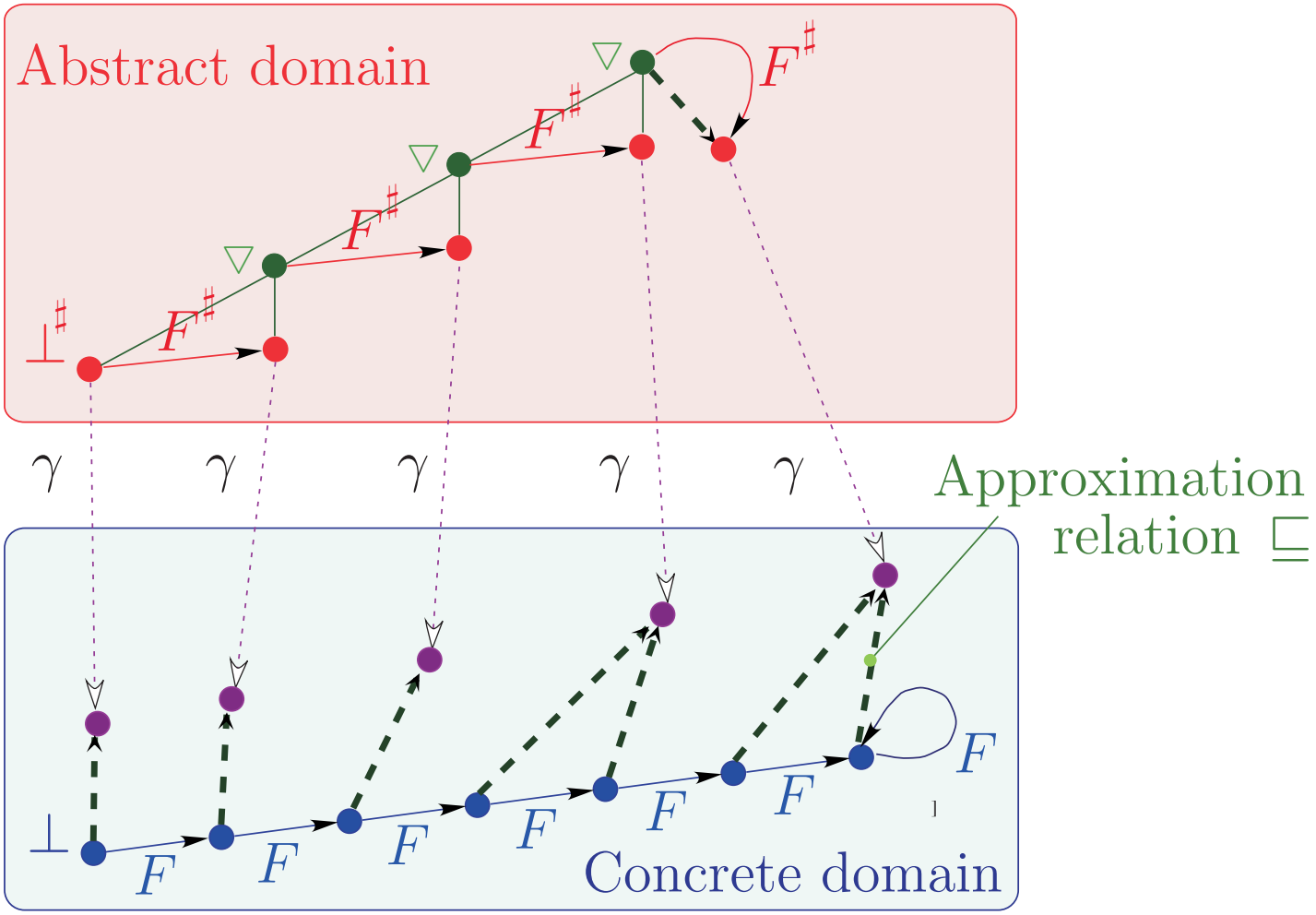
$$\mathcal{S}^\sharp[\![\texttt{while } B \ C']\!]R \triangleq \text{let } \mathcal{W} = \mathbf{lfp}^{\sqsubseteq}_{\perp} \ \boldsymbol{\lambda}\, \mathcal{X} \cdot R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \triangleq R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \triangleq \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0$$

$$\mathcal{S}^\sharp[\![D \ C]\!]R \triangleq \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

Convergence Acceleration with Widening



Abstract Semantics with Convergence Acceleration [7]

$$\mathcal{S}^\sharp[\![X = E;]\!]R \triangleq \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \mathrm{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B \texttt{ } C']\!]R \triangleq \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \triangleq \alpha(\{\rho \in \gamma(R) \cap \mathrm{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B \texttt{ } C' \texttt{ else } C'']\!]R \triangleq \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

$$\mathcal{S}^\sharp[\![\texttt{while } B \texttt{ } C']\!]R \triangleq \text{let } \mathcal{F}^\sharp = \boldsymbol{\lambda}\,\mathcal{X} \cdot \text{let } \mathcal{Y} = R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X})$$
$$\text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y}$$
$$\text{and } \mathcal{W} = \mathbf{lfp}^{\sqsubseteq}_{\perp} \mathcal{F}^\sharp \qquad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \triangleq R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \triangleq \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0$$

$$\mathcal{S}^\sharp[\![D \texttt{ } C]\!]R \triangleq \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

[7] Note: $\mathcal{F}^\sharp$ not monotonic!

5. Application to the Astrée Static Analyzer

Project Members

Patrick Cousot

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Jérôme Feret

Laurent Mauborgne

Antoine Miné

Xavier Rival

Bruno Blanchet (Nov. 2001 — Nov. 2003)

David Monniaux (Nov. 2001 — Aug. 2007).

Programs

Programs Analysed by Astrée

– Application Domain: large safety critical embedded synchronous software (for real-time non-linear control of very complex control/command systems).

– C programs:

  - with

    · basic numeric datatypes, structures and arrays

    · pointers (including on functions),

    · floating point computations

    · tests, loops and function calls

    · limited branching (forward `goto`, `break`, `continue`)

– with (cont'd)
  - `union`
  - pointer arithmetics & casts
– without
  - dynamic memory allocation
  - recursive function calls
  - unstructured/backward branching
  - conflicting side effects
  - C libraries, system calls (parallelism)

Such limitations are quite common for embedded safety-critical software.

The Class of Considered Periodic Synchronous Programs

declare volatile input, state and output variables;
initialize state and output variables;
loop forever
- read volatile input variables,
- compute output and state variables,
- write to output variables;

__ASTREE_wait_for_clock ();
end loop

Task scheduling is static:

– Requirements: the only interrupts are clock ticks;

– Execution time of loop body less than a clock tick, as verified by the aiT WCET Analyzers [FHL+01].

Concrete Semantics

Concrete Trace Semantics

- International norm of C (ISO/IEC 9899:1999)
- *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific user requirements (e.g. `assert`)

The Semantics of C is Hard (Ex. 1: Floats)

"*Put* $\mathtt{x}$ *in* $[\mathtt{m}, \mathtt{M}]$ *modulo* $(\mathtt{M} - \mathtt{m})$":

```
x' = x - (int) ((x-m)/(M-m))*(M-m);
```

- The programmer thinks $\mathtt{x'} \in [\mathtt{m}, \mathtt{M}]$
- But with $\mathtt{M} = 4095$, $\mathtt{m} = -\mathtt{M}$, IEEE double precision, and $\mathtt{x}$ is the greatest float strictly less than $\mathtt{M}$, then $\mathtt{x'} = \mathtt{m} - \epsilon$ (ϵ very small).

Floats are not real.

The Semantics of C is Hard (Ex. 2: Runtime Errors)

What is the effect of out-of-bounds array indexing?

```
% cat unpredictable.c
#include <stdio.h>
int main () { int n, T[1];
 n = 2147483647;
 printf("n = %i, T[n] = %i\n", n, T[n]);
}
```

Yields different results on different machines:

`n = 2147483647, T[n] = 2147483647`	Macintosh PPC
`n = 2147483647, T[n] = -1208492044`	Macintosh Intel
`n = 2147483647, T[n] = -135294988`	PC Intel 32 bits
`Bus error`	PC Intel 64 bits

Execution stops after a runtime error with unpredictable results [8].

[8] Equivalent semantics if no alarm.

Specification

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds, division by zero)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767, NaN)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Example: Dichotomy Search I

```
% cat dichotomy.c
int main () {
   int R[100], X; short lwb, upb, m;
   lwb = 0; upb = 99;
   while (lwb <= upb) {
      m = upb + lwb;
      m = m » 1;
      if (X == R[m]) { upb = m; lwb = m+1; }
      else if (X < R[m]) { upb = m - 1; }
      else { lwb = m + 1; }
   }
   __ASTREE_log_vars((m));
}
% astree -exec-fn main dichotomy.c |& egrep "(WARN)|(m in)"
direct = <integers (intv+cong+bitfield+set): m in [0, 99] /\ Top >
%
```

Example: Dichotomy Search II

```
% diff dichotomy.c dichotomy-bug.c
2,3c2,3
<     int R[100], X; short lwb, upb, m;
<     lwb = 0; upb = 99;
--
>     int R[30000], X; short lwb, upb, m;
>     lwb = 0; upb = 29999;
%
% astree -exec-fn main dichotomy-bug.c |& egrep "WARN" | head -n2
dichotomy-bug.c:5.6-19::[call#main@1:loop@4=2:]: WARN: implicit signed int->signed
short conversion range [14998, 44999] not included in [-32768, 32767]
dichotomy-bug.c:7.15-19::[call#main@1:loop@4=2:]: WARN: invalid dereference:
dereferencing 4 byte(s) at offset(s) [0;4294967295] may overflow the variable R of
byte-size 120000 or mis-aligned pointer (1Z+0) may not a multiple of 4
%
```

ASTRÉE finds bugs in programs based on algorithms which have been formally proved correct.

Specification Can Be Tricky

- What is known about the execution environment?
- Warn on integer arithmetic overflows? Including left shifts (to extract bit fields)? Including in initializers?
- Warn on implicit cast/conversion? When they overflow [9]?
- What is an incorrect access to a `union` field?
- ...

ASTRÉE proposes "reasonable default choices" (with variants through analysis parameters)

[9] undefined except for unsigned to unsigned.

Different Classes of Run-time Errors

1. Errors terminating the execution [10]. ASTRÉE warns and continues by taking into account only the executions that did not trigger the error.
2. Errors not terminating the execution with predictable outcome [11]. ASTRÉE warns and continues with worst-case assumptions.
3. Errors not terminating the execution with unpredictable outcome [12]. ASTRÉE warns and continues by taking into account only the executions that did not trigger the error.

$\Rightarrow$ ASTRÉE is sound with respect to C standard, unsound with respect to C implementation, unless no false alarm.

[10] floating-point exceptions e.g. (invalid operations, overflows, etc.) when traps are activated

[11] e.g. overflows over signed integers resulting in some signed integer.

[12] e.g. memory corruptionss.

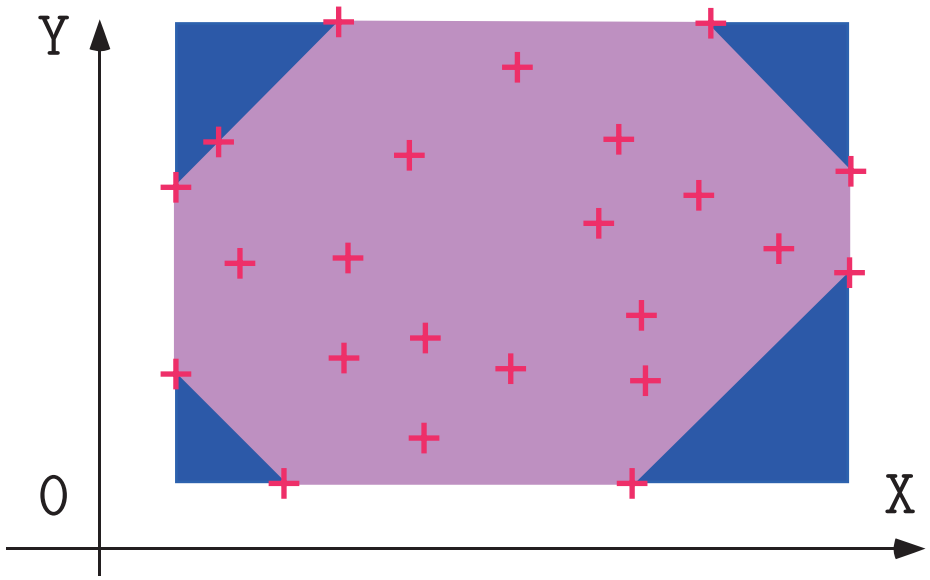
Abstraction

Abstraction is Extremely Hard

– The analysis must be automatic (no user interaction)
– The abstraction must
  - ensure termination (and efficiency) of the analysis
  - be sound (ASTRÉE is a verifier, not a bug-finder)
  - scale up (100.000 to 1.000.000 LOCs)
  - be precise (no false alarm)

A grand challenge

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [Min01]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [CC77, Min01, Min04a]

Termination

SLAM uses CEGAR and does not terminate [13] on

```
% cat slam.c
int main() { int x, y;
  x = 0; y = 0;
  while (x < 2147483647)
     { x = x + 1; y = y + 1; }
  __ASTREE_assert((x == y));
}
```

whereas ASTRÉE uses widening/narrowing-based extrapolation techniques to prove the assertion

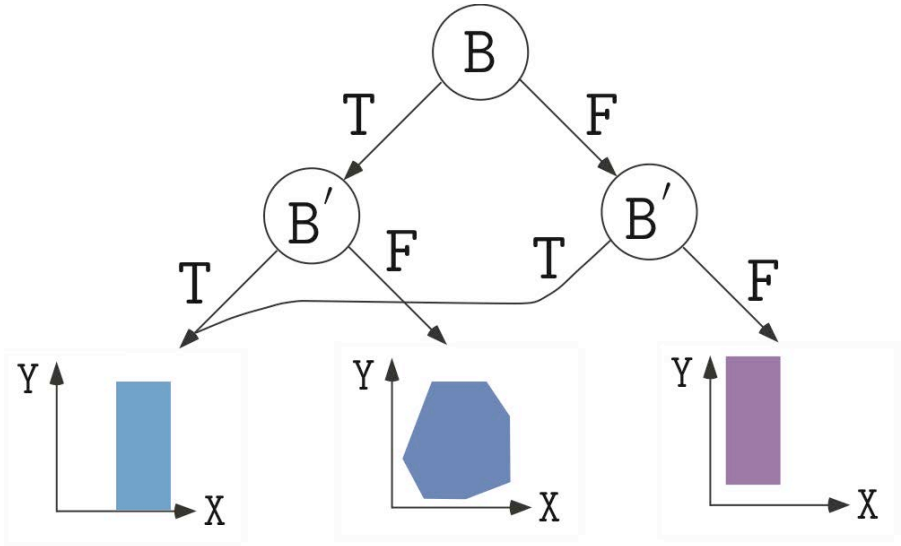
```
% astree -exec-fn main slam.c |& egrep "WARN"
%
```

[13] CEGAR cannot generate the invariant `y = x - 1` so produces all counter examples $x = i + 1 \wedge y = i$, $i = 0, 1, 2, 3, \ldots$

Boolean Relations for Boolean Control

– Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Trace Partitioning [MR05]

Principle:

- Semantic equivalence:

```
if (B) { C1 } else { C2 }; C3
```

```
if (B) { C1; C3 } else { C2; C3 };
```

- More precise in the abstract: concrete execution paths are merged later.

Application:

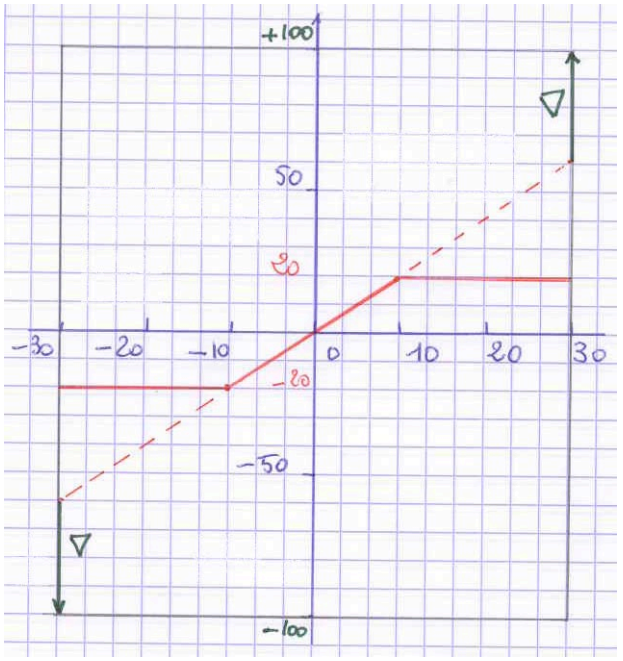
```
if (B)
  { X=0; Y=1; }
else
  { X=1; Y=0; }
R = 1 / (X-Y);
```

cannot result in a division by zero

Case analysis with loop unrolling

– Code Sample:

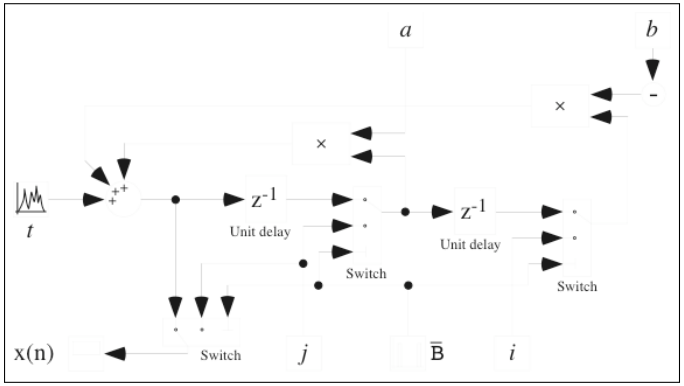
```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;
  __ASTREE_known_fact(((-30.0 <= x) && (x <= 30.0)));
  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
  __ASTREE_log_vars((r));
}
```



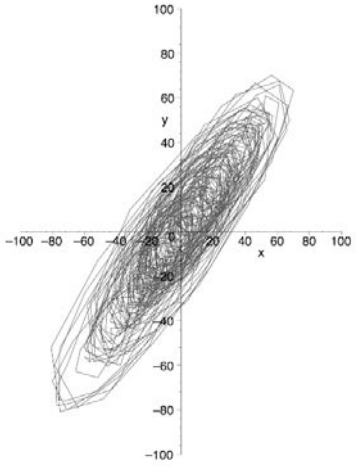

```
% astree -exec-fn main -no-trace -no-relational trace-partitioning.c |& egrep "(WARN)|(r in)"
direct = <float-interval: r in [-20, 20] >
%
% astree -exec-fn main -no-partition -no-trace -no-relational trace-partitioning.c \
  |& egrep "(WARN)|(r in)"
direct = <float-interval: r in [-100, 100] >
%
```

Ellipsoid Abstract Domain for Filters

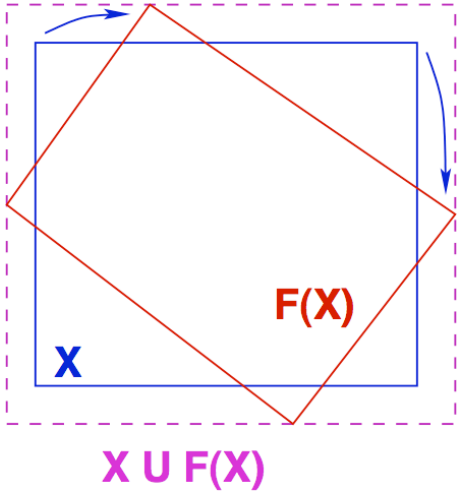
2[d] Order Digital Filter:



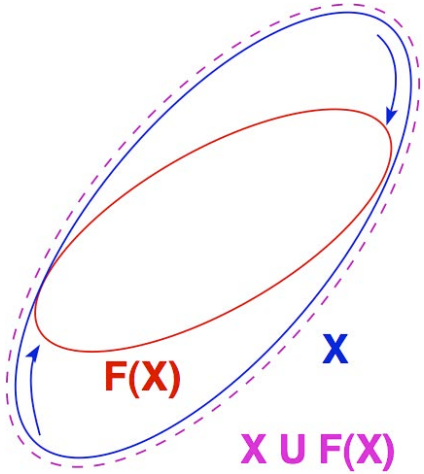
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is bounded, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an ellipsoid.



execution trace



unstable interval

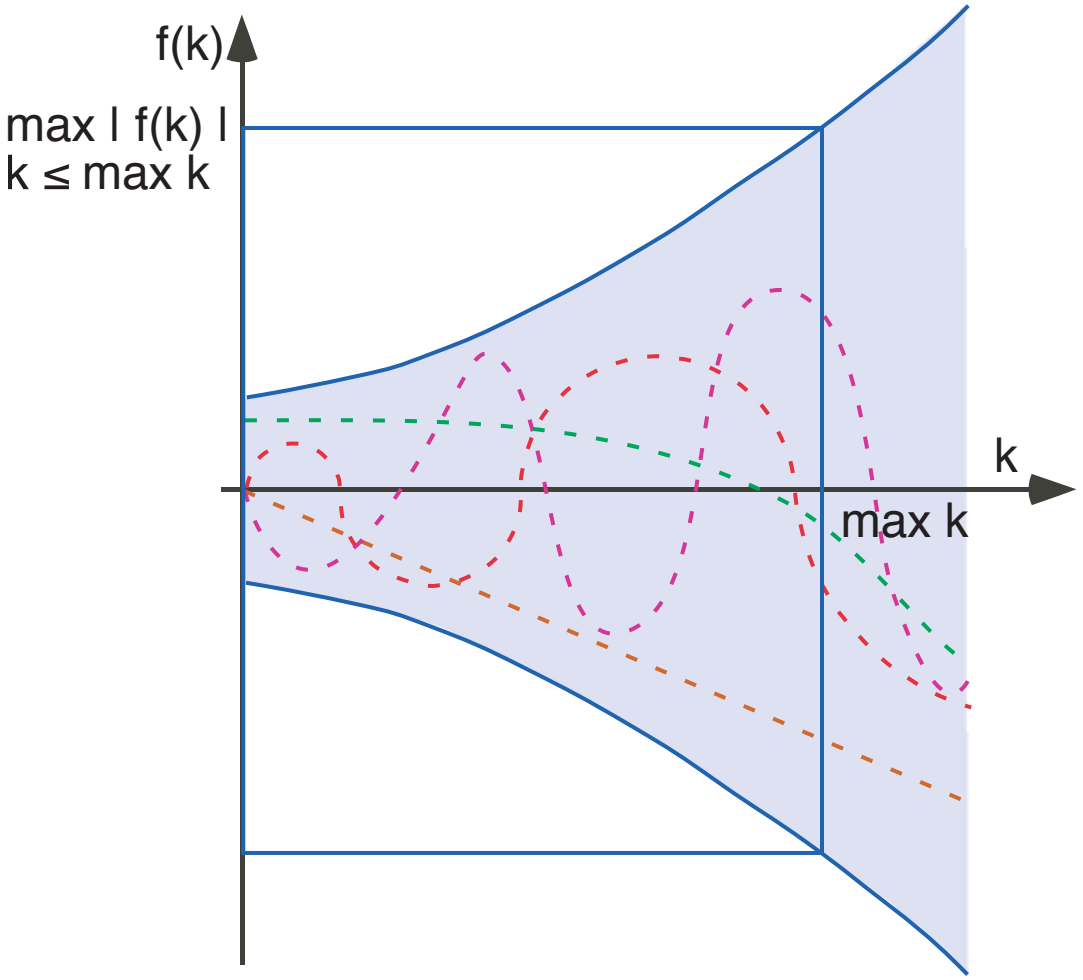


stable ellipsoid

Filter Example [Fer04]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
             + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```

Overapproximation with an Arithmetic-Geometric Progression



Arithmetic-geometric progressions [14] [Fer05]

– Abstract domain: $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \longmapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') =$$
$$\{f \mid \forall k \in \mathbb{N} : |f(k)| \leq \left(\boldsymbol{\lambda}\, x \cdot ax + b \circ (\boldsymbol{\lambda}\, x \cdot a'x + b')^k\right)(M)\}$$

i.e. any function bounded by the arithmetic-geometric progression [15].

[14] here in $\mathbb{R}$, in practice in floats, so rounding must be taken into account [].

[15] Note that exhaustive enumeration would be simply hopeless.

Example 1: Bounding Increments [Fer05]

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
  R = 0;
  while (TRUE) {
    __ASTREE_log_vars((R));
    if (I) { R = R + 1; }          ← potential overflow!
    else { R = 0; }
    T = (R >= 100);
    __ASTREE_wait_for_clock(());
  }}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

Example 2: Accumulation of Small Rounding Errors

```
% cat -n rounding-c.c
 1  #include <stdio.h>
 2  int main () {
 3   int i; double x; x = 0.0;
 4   for (i=1; i<=1000000000; i++) {
 5    x = x + 1.0/10.0;
 6   }
 7  printf("x = %f\n", x);
 8  }
% gcc rounding-c.c
% ./a.out
x = 99999998.745418
%
```

since $(0.1)_{10} = (0.0001100110011001100\ldots)_2$

Static Analysis with ASTRÉE

```
% cat -n rounding.c
     1  int main () {
     2   double x; x = 0.0;
     3   while (1) {
     4    x = x + 1.0/10.0;
     5    __ASTREE_log_vars((x));
     6    __ASTREE_wait_for_clock(());
     7   }
     8  }
% cat rounding.config
 __ASTREE_max_clock((1000000000));
% astree -exec-fn main -config-sem rounding.config -unroll 0 rounding.c\
 |& egrep "(x in)|(\|x\|)|(WARN)" | tail -2
direct = <float-interval: x in [0.1, 200000040.938] >
  |x| <= 1.*((0. + 0.1/(1.-1))*(1.)^clock - 0.1/(1.-1)) + 0.1
      <= 200000040.938
```

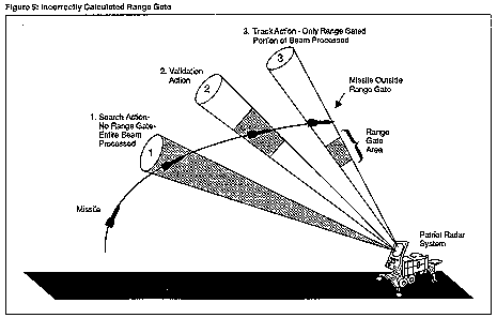
The Patriot missile failure

– "On February 25th, 1991, a Patriot missile ... failed to track and intercept an incoming Scud (*)."

– The software failure was due to accumulated rounding error (†)

(*) This Scud subsequently hit an Army barracks, killing 28 Americans.

(†)
– "Time is kept continuously by the system's internal clock in tenths of seconds"
  – "The system had been in operation for over 100 consecutive hours"
  – "Because the system had been on so long, the resulting inaccuracy in the time calculation caused the range gate to shift so much that the system could not track the incoming Scud"



Example 3: Time Dependent Deviations [Fer05]

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P =  (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
```

```
|P| <= (15.  + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <= 23.0393526881
```

Incompleteness

ASTRÉE does not know that

$$\forall x, y \in \mathbb{Z} : 7y^2 - 1 \neq x^2$$

so on the following program

```
void main() { int x, y;
  if ((-4681 < y) && (y < 4681) && (x < 32767) && (-32767 < x) && ((7*y*y - 1) == x*x))
     { y = 1 / x; };
}
```

it produces a false alarm

```
% astree -exec-fn main false-alarm.c |& egrep "WARN"
false-alarm.c:5.9-14::[call#main@1:]: WARN: integer division by zero ([-32766, 32766]
and {1} / Z)
%
```

Zero False Alarm Objective

Industrial constraints require ASTRÉE to be extremely precise:

- ASTRÉE is designed for a well-identified family of programs
- The analysis can be tuned using
  - parameters
  - analysis directives (which insertion can be automated)
  - extensions of the analyzer (by the tool designers)

Example of directive

```
% cat repeat1.c
typedef enum {FALSE=0,TRUE=1} BOOL;
int main () {
  int x = 100; BOOL b = TRUE;

  while (b) {
    x = x - 1;
    b = (x > 0);
  }
}
% astree -exec-fn main repeat1.c |& egrep "WARN"
repeat1.c:5.8-13::[call#main@2:loop@4>=4:]: WARN: signed int arithmetic
range [-2147483649, 2147483646] not included in [-2147483648, 2147483647]
%
```

Example of directive (Cont'd)

```
% cat repeat2.c
typedef enum {FALSE=0,TRUE=1} BOOL;
int main () {
  int x = 100; BOOL b = TRUE;
  __ASTREE_boolean_pack((b,x));
  while (b) {
    x = x - 1;
    b = (x > 0);
  }
}
% astree -exec-fn main repeat2.c |& egrep "WARN"
%
```

The insertion of this directive could have been automated in Astrée.

Industrial Application

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Application to Avionics Software

– Primary flight control software [16]

– C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)

– A340/600: 200,000 lines [17], A380: $\times$ 5

No false alarm, a world première!

[16] "Flight Control and Guidance Unit" (FCGU) running on the "Flight Control Primary Computers" (FCPC). The A340 electrical flight control system is placed between the pilot's controls (sidesticks, rudder pedals) and the control surfaces of the aircraft, whose movement they control and monitor.

[17] 6 hours on a 2.6 GHz, 16 Gb RAM PC

6. Conclusion

Abstract Interpretation

– Abstract interpretation is
  - a theory
  - with effective applications
  - and unprecedented industrial accomplishments.
– Further investigations of the theory are needed (while its scope of application broaden)
– The demand for applications is quasi-illimited

THE END, THANK YOU

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