Program Verification by Abstract Interpretation

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CMU, Pittsburgh October 18, 2011

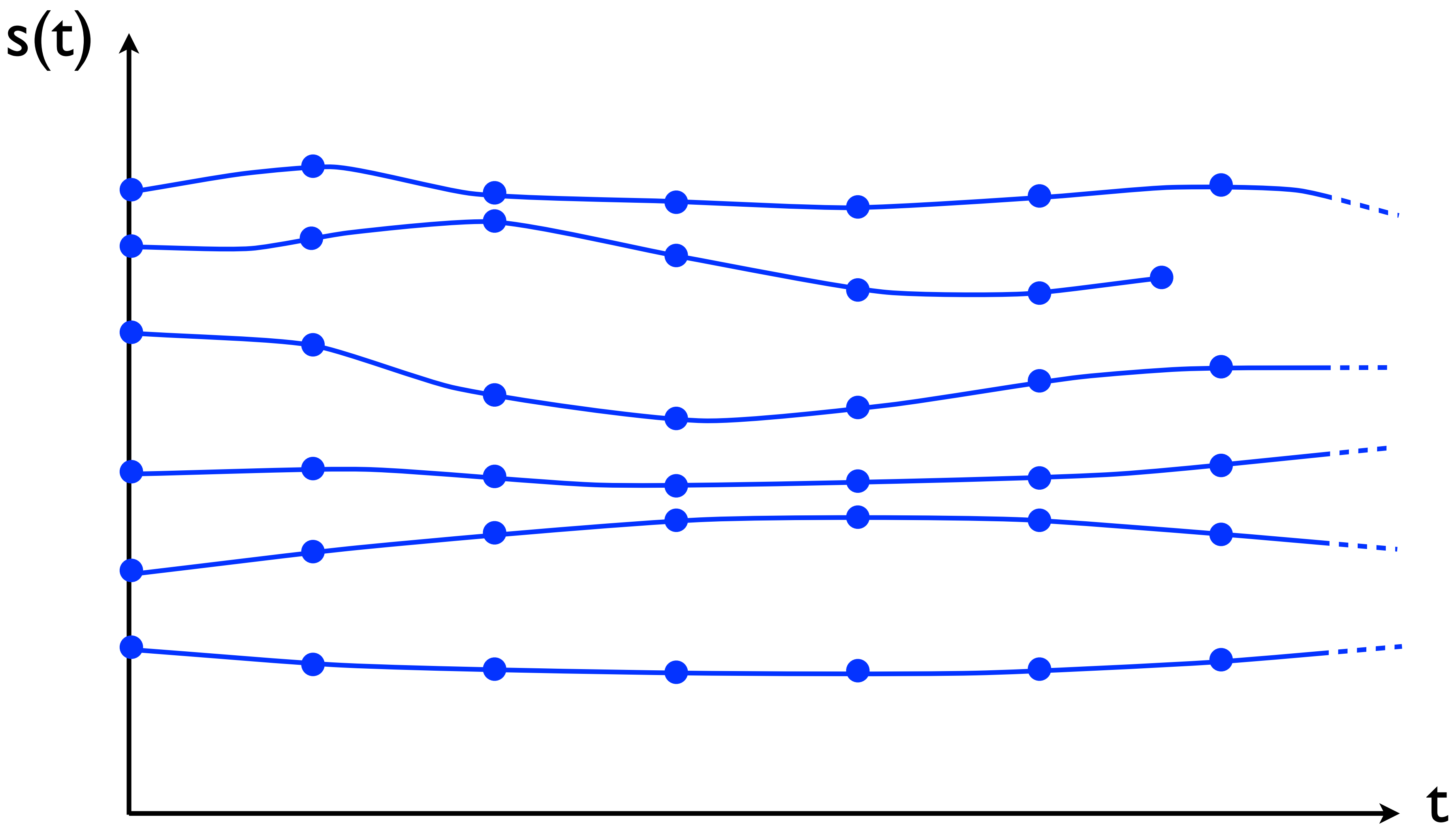
Content

- A lightweight informal introduction to Abstract Interpretation
- Application to the Verification of Embedded Control
- Commercial tools (ASTRÉE, CCCheck)
- Current and future research

An informal introduction to Abstract Interpretation

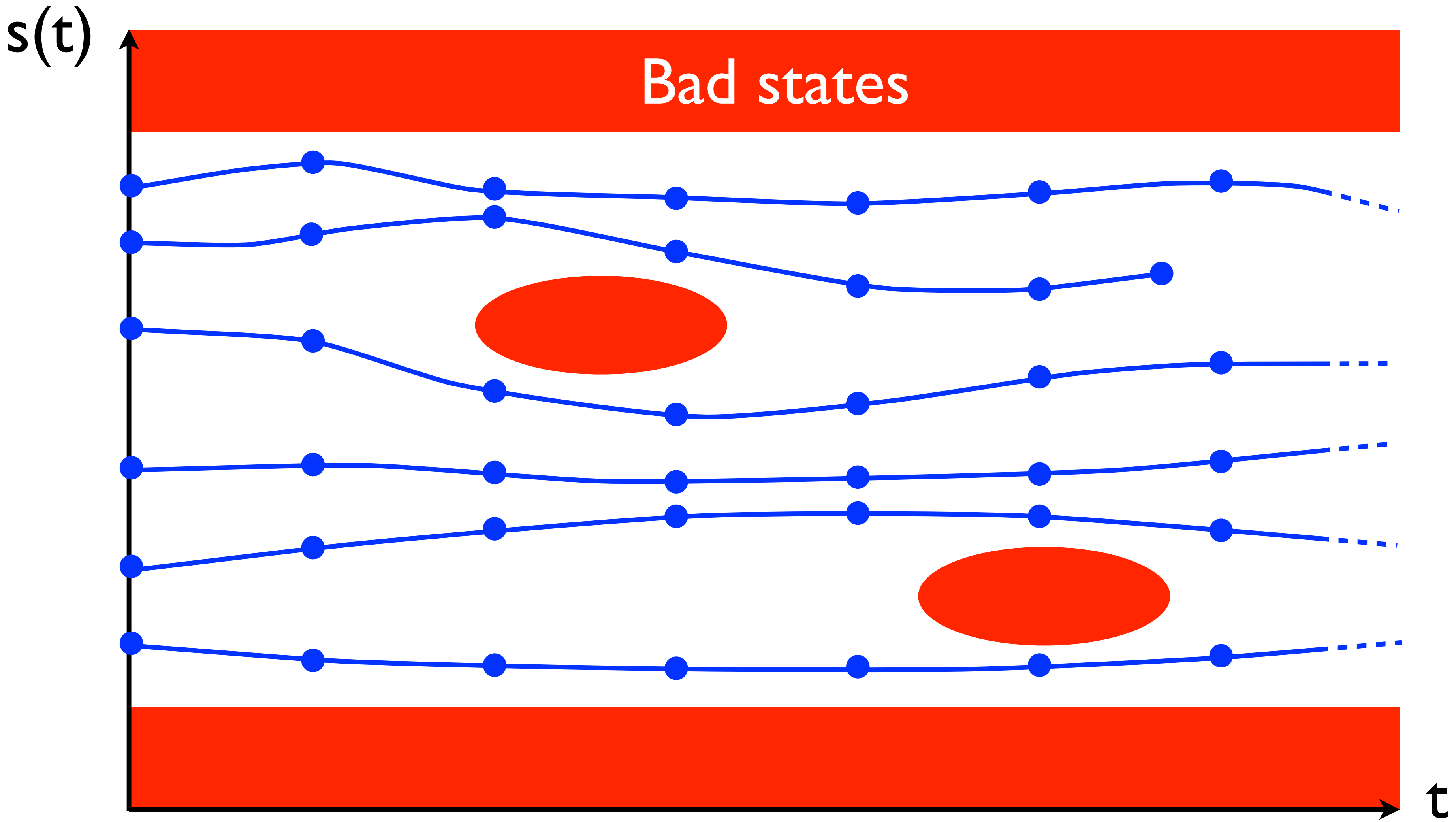
Program concrete models/semantics

- Program executions are modelled by the language formal semantics (observed at discrete times)



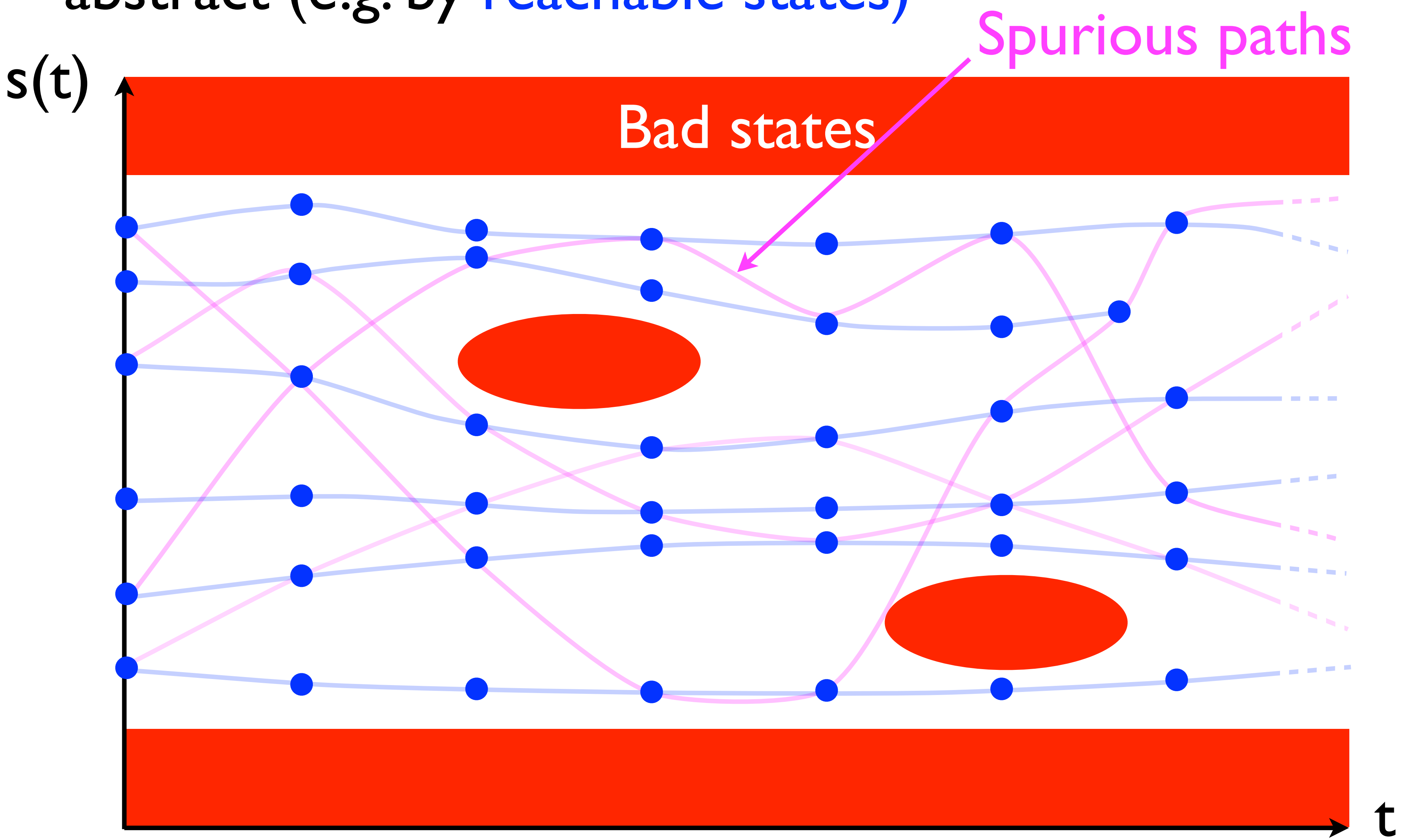
Verification of safety properties

- Program executions cannot reach a state in which computations can go wrong



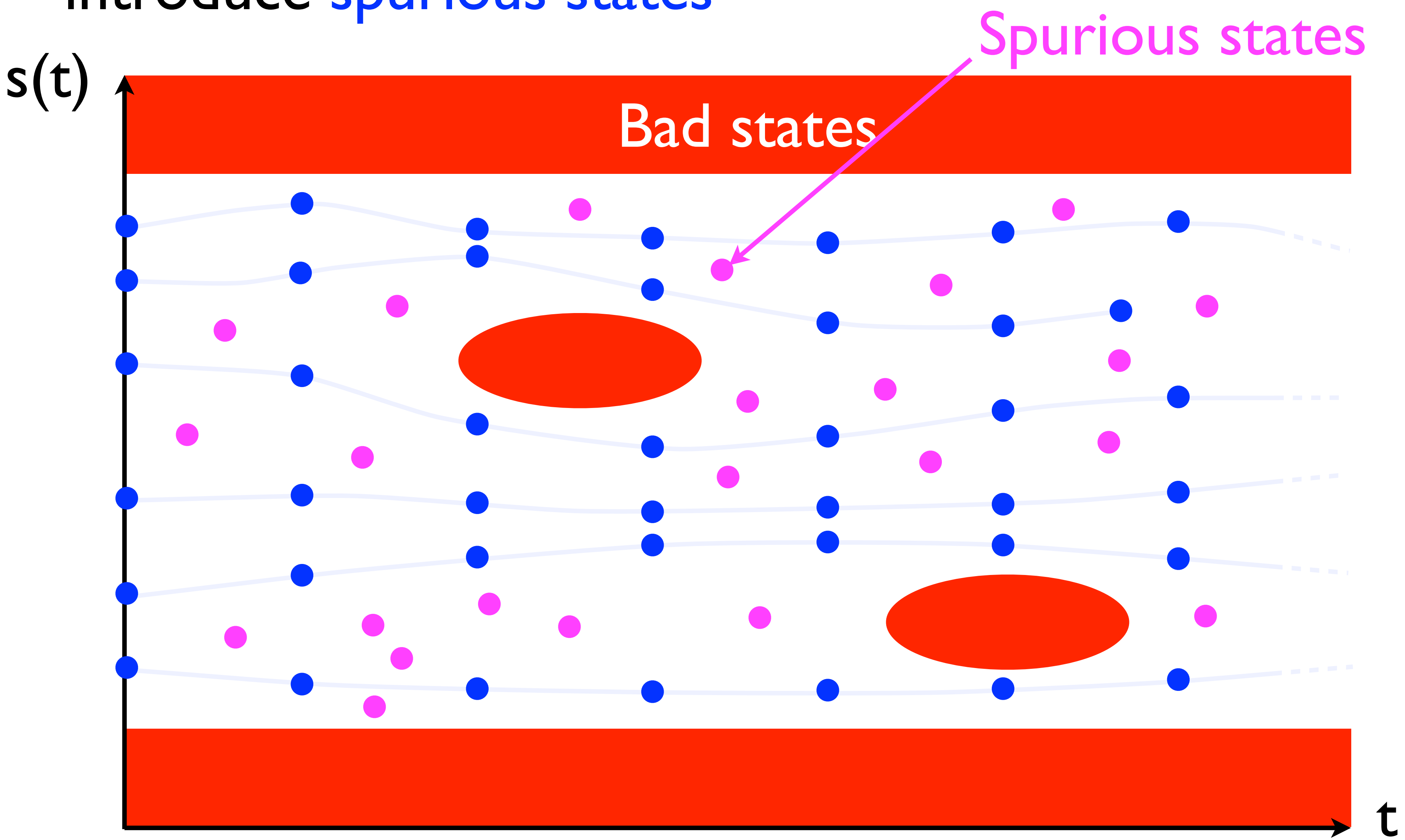
Abstraction

- The computations are over-approximated in the abstract (e.g. by reachable states)



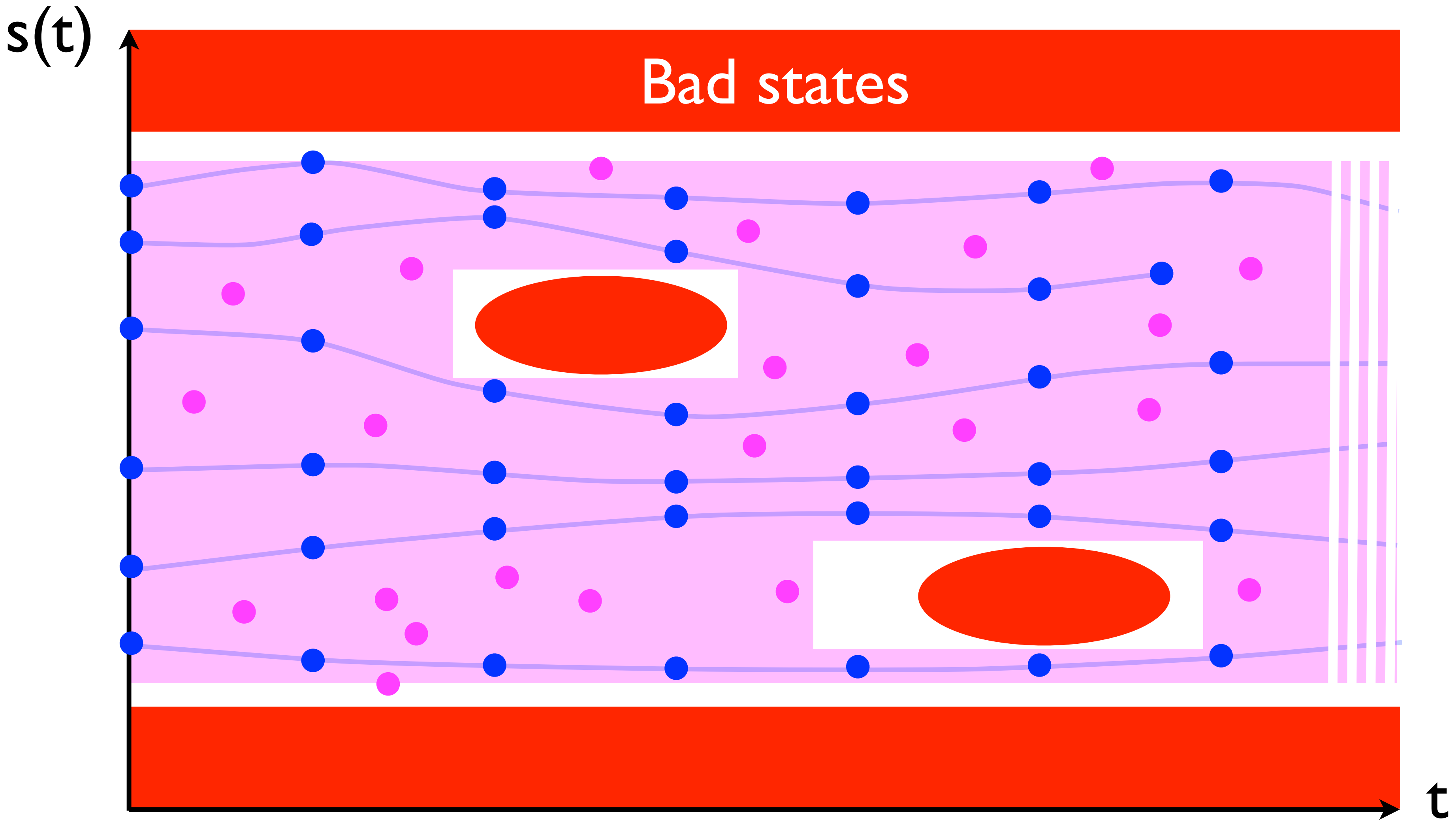
Abstraction over-approximation

- Further approximations of the reachable states may introduce spurious states



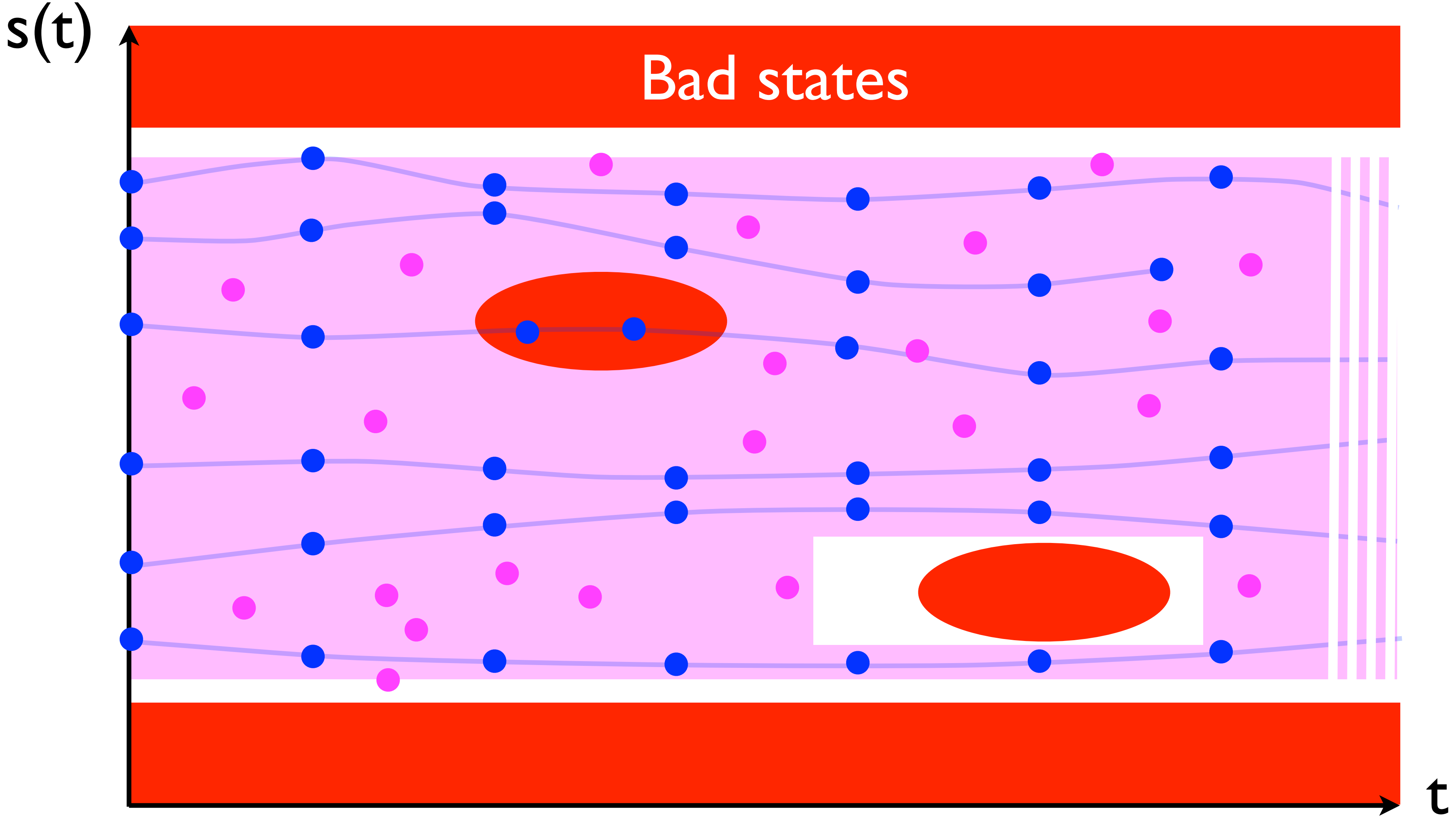
Machine-computable abstractions

- To scale up, machine computable abstraction must be very efficient and precise enough



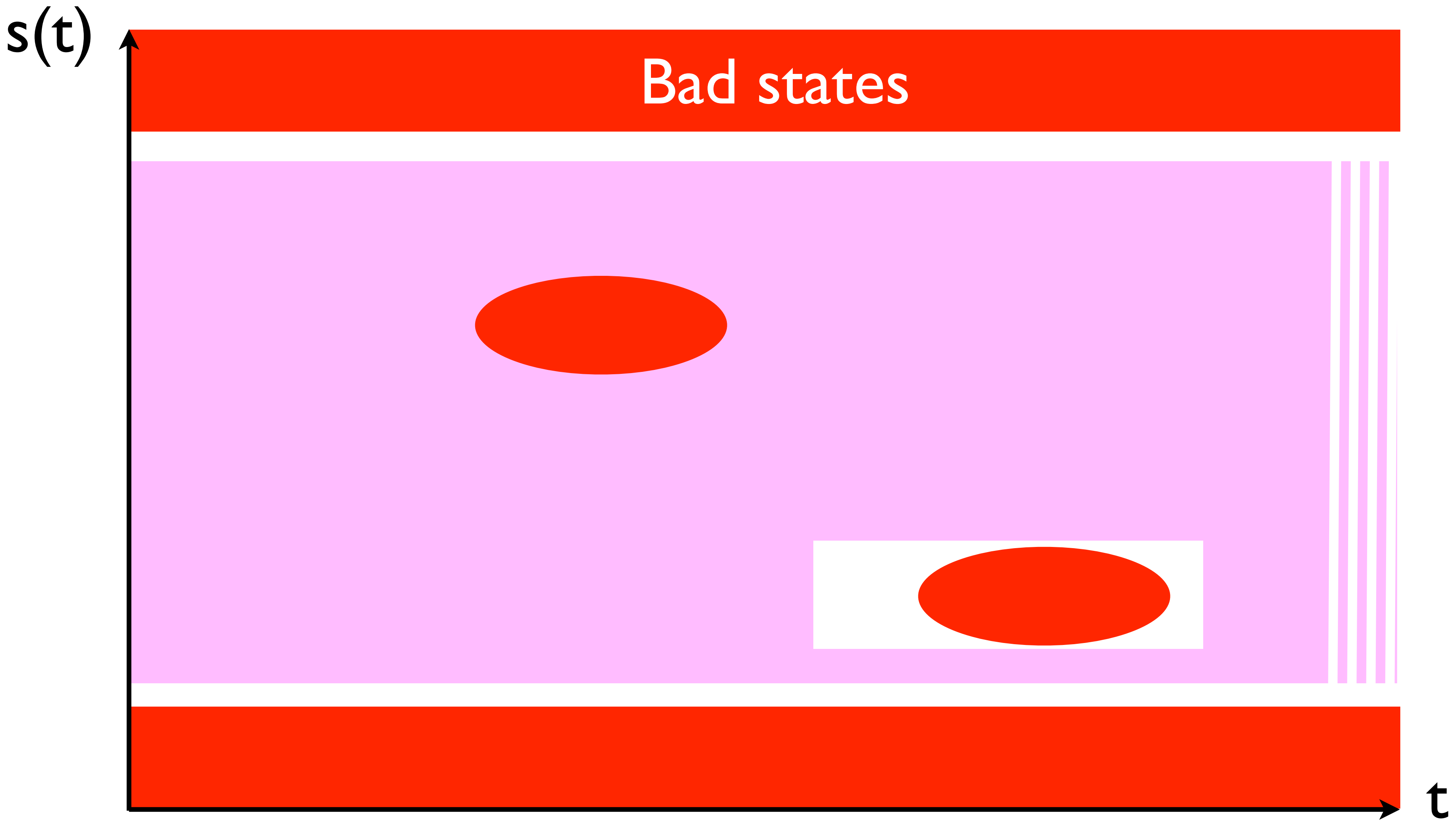
Soundness

- No definite error is *ever* omitted (counter-examples: Coverity, Klocwork, etc)



Incompleteness: false alarms

- Spurious errors are possible (e.g. PolySpace) and may be eliminated by refining the abstraction (e.g. Astrée)

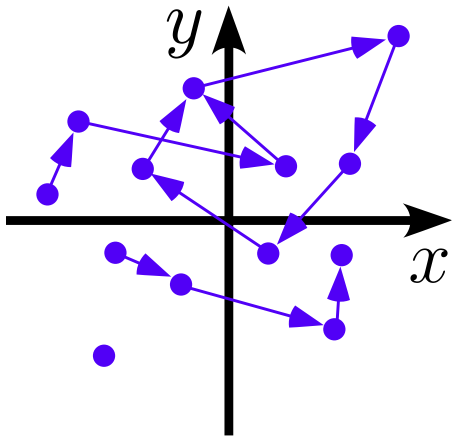


Application to the Verification of Embedded Control Systems

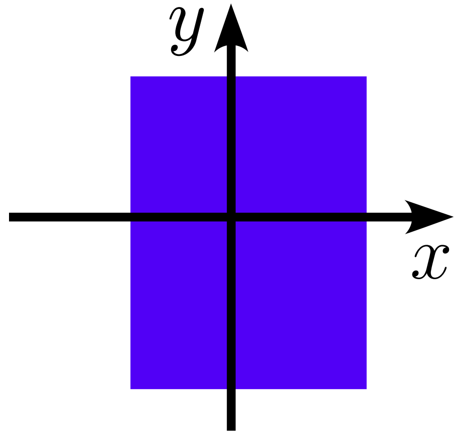
Applications

- Verification of absence of runtime errors (arithmetic overflows, divisions by zero, buffer overruns, null and dangling pointers, user assertion violations, unreachability, ...) so specification is *fully automatic*
- Avionics, Spatial, Automotive, Medical, Systems on Chip (SoC), etc
- Use general abstractions for programming languages (integers, floats, arrays, structures, pointers, ...)
- Use domain-specific abstractions incorporating knowledge on control systems (filters, quaternions, integrators, etc)

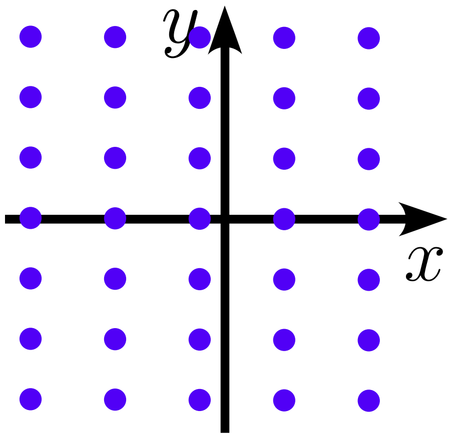
Abstractions



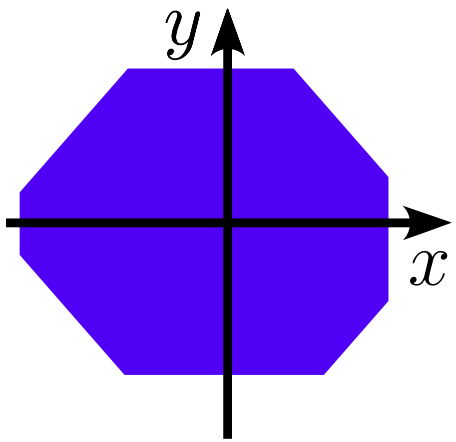
Collecting semantics:
partial traces



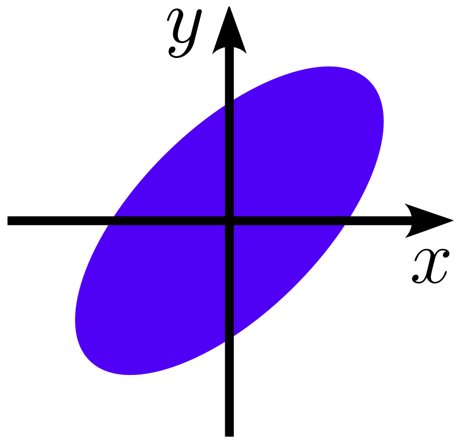
Intervals:
$\mathrm{x} \in [a, b]$



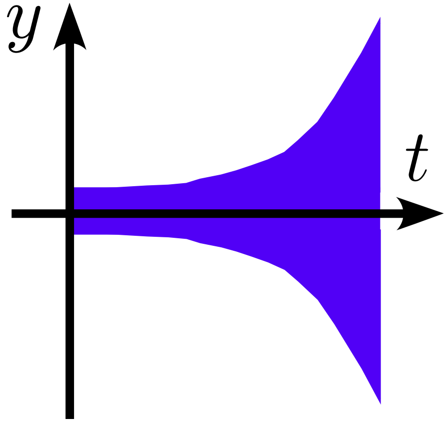
Simple congruences:
$\mathrm{x} \equiv a[b]$



Octagons:
$\pm\mathrm{x} \pm \mathrm{y} \leqslant a$



Ellipses:
$\mathrm{x}^2 + b\mathrm{y}^2 - a\mathrm{xy} \leqslant d$



Exponentials:
$-a^{bt} \leqslant \mathrm{y}(t) \leqslant a^{bt}$

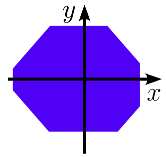
Example of general purpose abstraction: octagons

- Invariants of the form $\pm\, \mathtt{x} \pm \mathtt{y} \leq \mathtt{c}$, with $\mathcal{O}(\mathbf{N}^2)$ memory and $\mathcal{O}(\mathbf{N}^3)$ time cost.
- Example:

```
while (1) {
  R = A-Z;
  L = A;
  if (R>V)
    { ★ L = Z+V; }
  ★
}
```

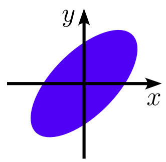
- At ★, the interval domain gives $\mathtt{L} \leq \max(\max \mathtt{A}, (\max \mathtt{Z})+(\max \mathtt{V}))$.
- In fact, we have $\mathtt{L} \leq \mathtt{A}$.
- To discover this, we must know at ★ that $\mathtt{R} = \mathtt{A}\text{-}\mathtt{Z}$ and $\mathtt{R} > \mathtt{V}$.

- Here, $\mathtt{R} = \mathtt{A}\text{-}\mathtt{Z}$ cannot be discovered, but we get $\mathtt{L}\text{-}\mathtt{Z} \leq \max \mathtt{R}$ which is sufficient.
- We use many octagons on **small packs** of variables instead of a large one using all variables to cut costs.



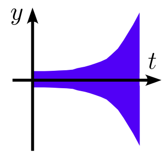
Example of domain-specific abstraction: ellipses

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
              + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```



Example of domain-specific abstraction: exponentials

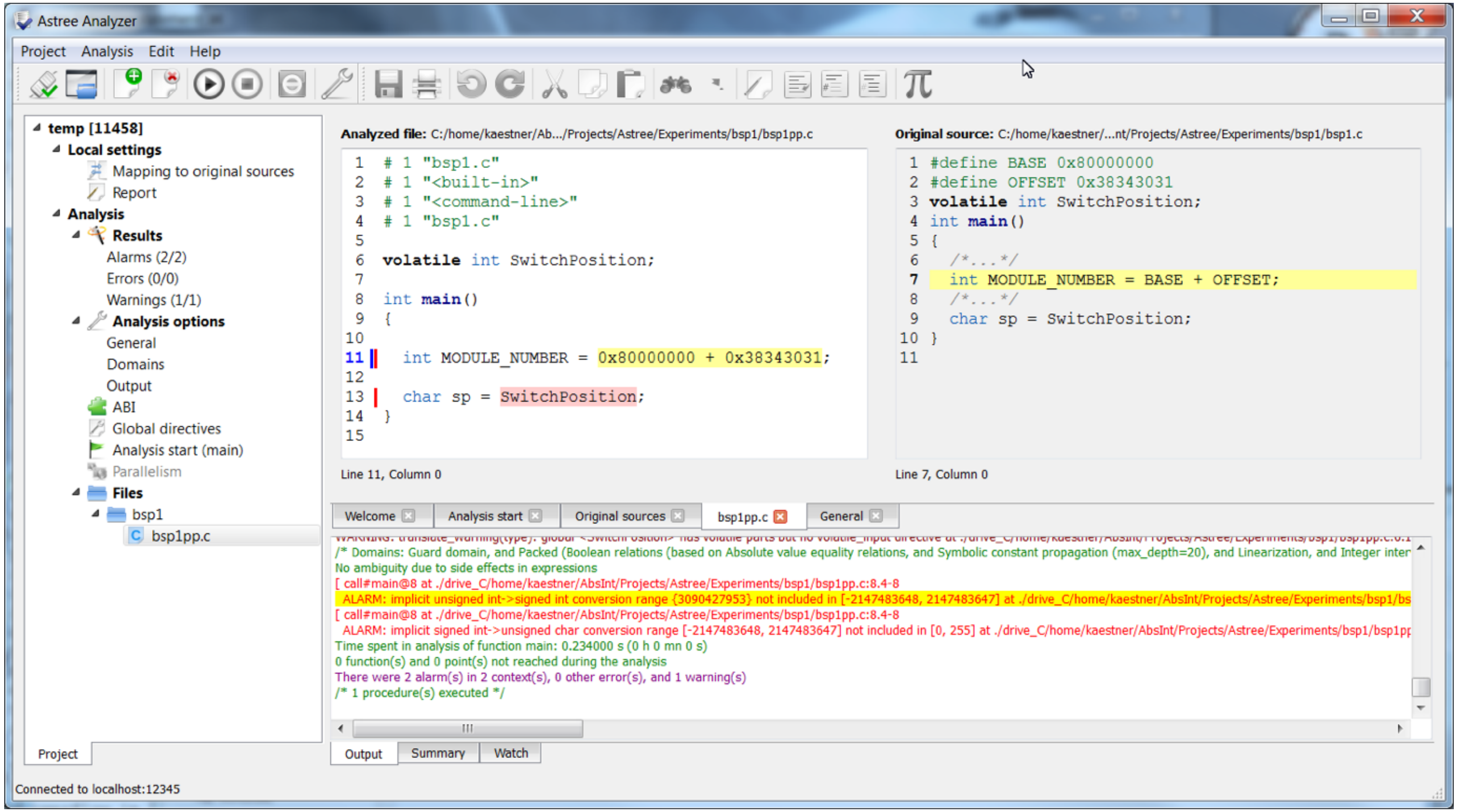
```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
  R = 0;
  while (TRUE) {
    __ASTREE_log_vars((R));
    if (I) { R = R + 1; }          ← potential overflow!
    else { R = 0; }
    T = (R >= 100);
    __ASTREE_wait_for_clock(());
  }}
% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```



Commercial Tools

Commercialization

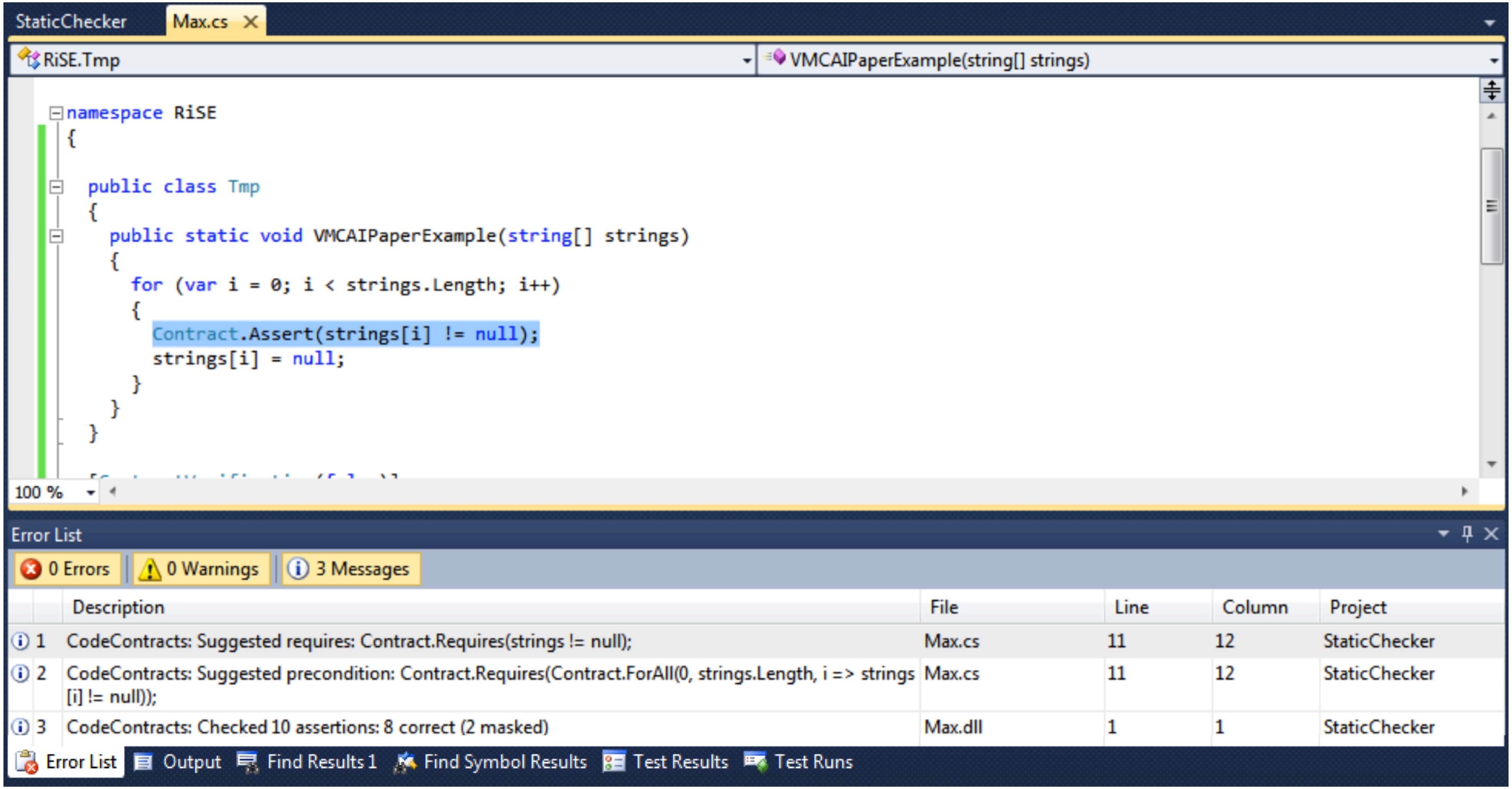
- AbsInt (www.absint.de)
- Astrée (run-time error analysis)



- Other abstract-interpretation-based tools: WCET, stack usage, memory safety analysis

Clousot/CCcheck in Visual Studio

- Modular code contract verification (and inference)



- see online, www.rise4fun.com

Research Challenges

CMACS achievements

- Static analysis of array content (POPL 2011)
- Necessary precondition inference for code contracts (VMCAI 2011)
- Abstract interpretation-based theory to combine abstract interpretation, model-checking and verifiers /SMT solvers (FOSSACS 2011)
- Termination analysis (POPL 2012)
- Probabilistic Abstract Interpretation

Research challenges

- Complex data structures
- Liveness, Closing the loop, ...
- Parallelism, Fairness, Scheduling, ... (AstréeA, www.astreea.ens.fr/)
- Security (AstréeS)

Other application domains:

Security

- Information flow analysis

Biology

- Cellular signaling networks
- Formal rule-based model reduction

Conclusion

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- Does scale up (to > 10^6 LOCS) !
- Find bugs not found by simulation, testing, enumerative bug finding methods
- Can prove the absence of well-defined categories of bugs
- Covers new requirements on formal methods (e.g. DO 178 C)
- Mandatory in all embedded control systems of an European plane manufacturer
- Unfortunately not so well-known and well-used in the US

The End