Temporal Abstract Interpretation [1]

Interprétation abstraite temporelle

Patrick COUSOT

DI, École normale supérieure

45 rue d'Ulm, 75230 Paris cedex 05, France

mailto:Patrick.Cousot@ens.fr http://www.di.ens.fr/~cousot

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[1] Join work with R. Cousot, LIX.

1. Introductory motivations ...

Common believes

- The verification of a temporal specification for a transition system by model-checking is sound and complete [2]:

  A temporal property holds if (soundness) and only if (completeness) it can be model-checked.

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- The verification of a temporal specification for a program given by its small-step operational semantics by static analysis is sound and incomplete [3];
- so model-checking is to be preferred to program static analysis.

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Our point of view

Both for model-checking and program static analysis:

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- So (in)completeness is relative (to an implicit reference temporal semantics);
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Indeed both model-checking and program static analysis are abstract interpretations based on similar approximations.

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Yes, but abstract model checking [1]:

- is based on a state-to-state abstraction which is not general enough!

Reference

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Why bother? we already have "abstract model checking"!

Yes, but abstract model checking [1]:

- is based on a state-to-state abstraction which is not general enough!
- e.g. it cannot take into account polyhedral model checking of hybrid systems à la Halbwachs et al. [2].

Reference

[1] E. Clarke O. Grumberg & D. Long. Model checking and abstraction. *TOPLAS 16* 1994.

[2] N. Halbwachs, J.-É. Proy, & P. Raymond. Verification of linear hybrid systems by means of convex approximations. *SAS '94*, LNCS 864, 1994.

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- We would like to have to have a continuum of techniques ranging from model-checking to static program analysis;
- Abstract interpretation can help with this general point of view.

$\Longrightarrow$ We consider a very general temporal specification language;

$\Longrightarrow$ We study its abstractions.

2. Temporal specification language

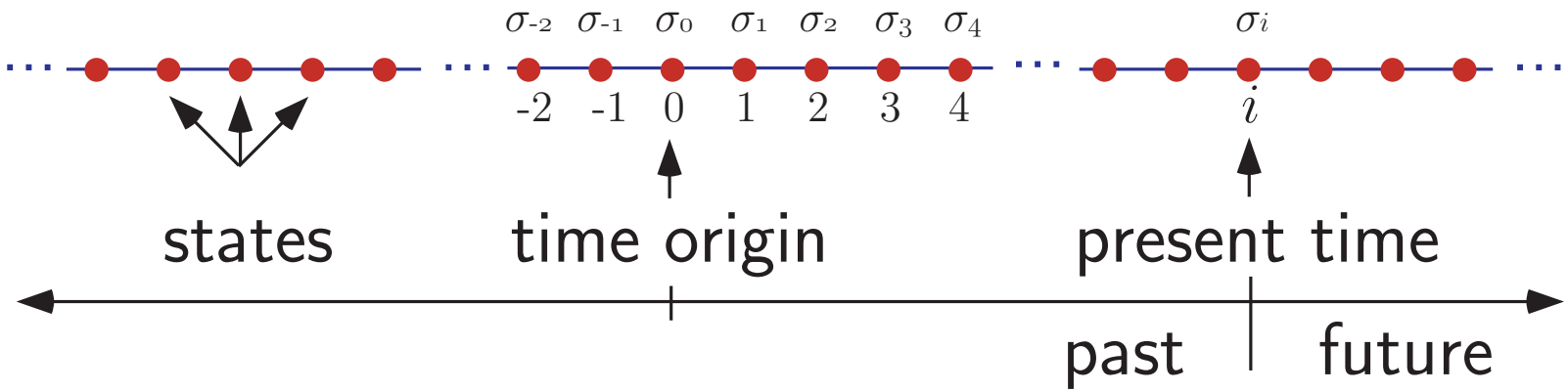
Temporal logics and calculi

- Temporal logics:
  - CTL*,
  - CTL;
- Temporal calculi:
  - propositional μ-calculus;

are all generalized by the reversible $\overset{\curvearrowleft}{\mu}^\star$-calculus.

Semantic domain for the reversible $\overleftrightarrow{\mu^\star}$-calculus

- The semantics of a formula of the reversible $\overleftrightarrow{\mu^\star}$-calculus is a model that is a set of infinite time-symmetric traces;
- An infinite time-symmetric trace $\langle i,\ \sigma \rangle$:

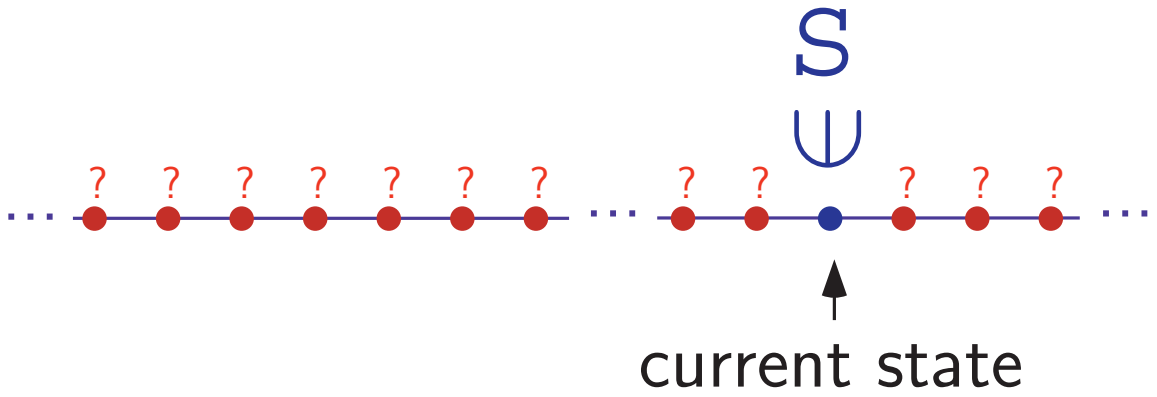


The reversible $\overset{\curvearrowleft}{\mu^\star}$-calculus

$\varphi ::=$	$\boldsymbol{\sigma}_S$	$S \in \wp(\mathbb{S})$	state predicate
$\mid$	$\boldsymbol{\pi}_t$	$t \in \wp(\mathbb{S} \times \mathbb{S})$	transition predicate
$\mid$	$\oplus\, \varphi_1$		next
$\mid$	$\varphi_1^{\curvearrowleft}$		reversal
$\mid$	$\varphi_1 \vee \varphi_2$		disjunction
$\mid$	$\neg\, \varphi_1$		negation
$\mid$	X	$X \in \mathbb{X}$	variable
$\mid$	$\boldsymbol{\mu}\, X \cdot \varphi_1$		least fixpoint
$\mid$	$\boldsymbol{\nu}\, X \cdot \varphi_1$		greatest fixpoint
$\mid$	$\forall\, \varphi_1 : \varphi_2$		universal state closure

State predicates $\boldsymbol{\sigma}_S$

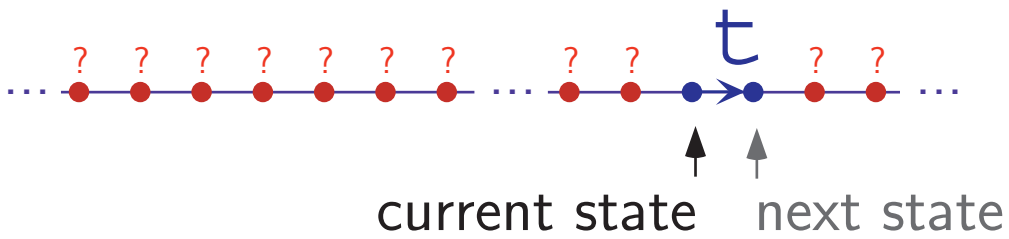
- The state predicate $\boldsymbol{\sigma}_S$ denotes all traces with current state in set S [4]:



[4] In this talk we identify a reversible $\overset{\curvearrowleft}{\mu^\star}$-calculus formula φ with its semantics/interrpetation $[\![\varphi]\!]$.

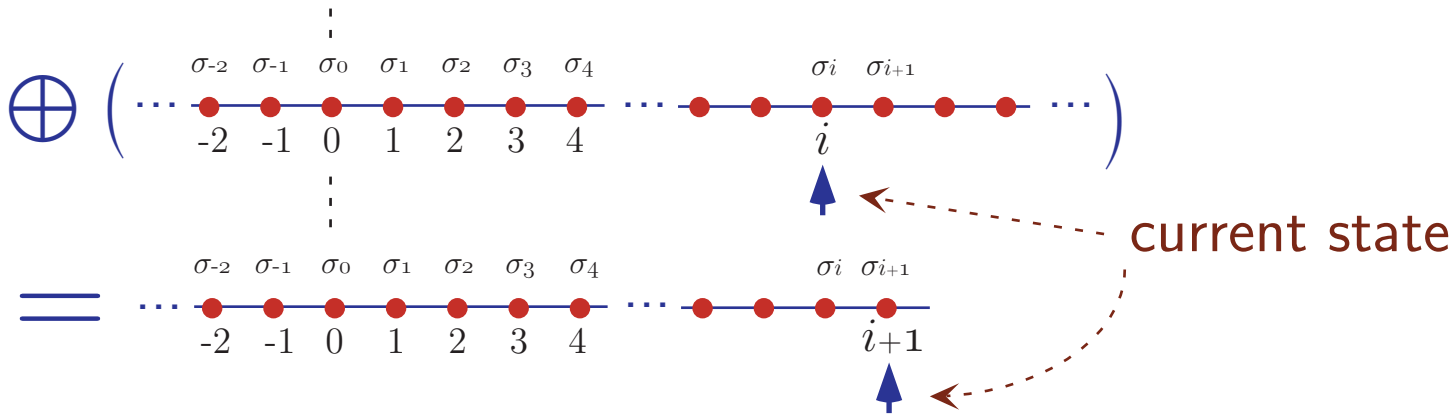
Transition predicates π_t

- The transition predicate π_t denotes all traces with a transition t from current to next state:



Next $\oplus$

- Trace next time:



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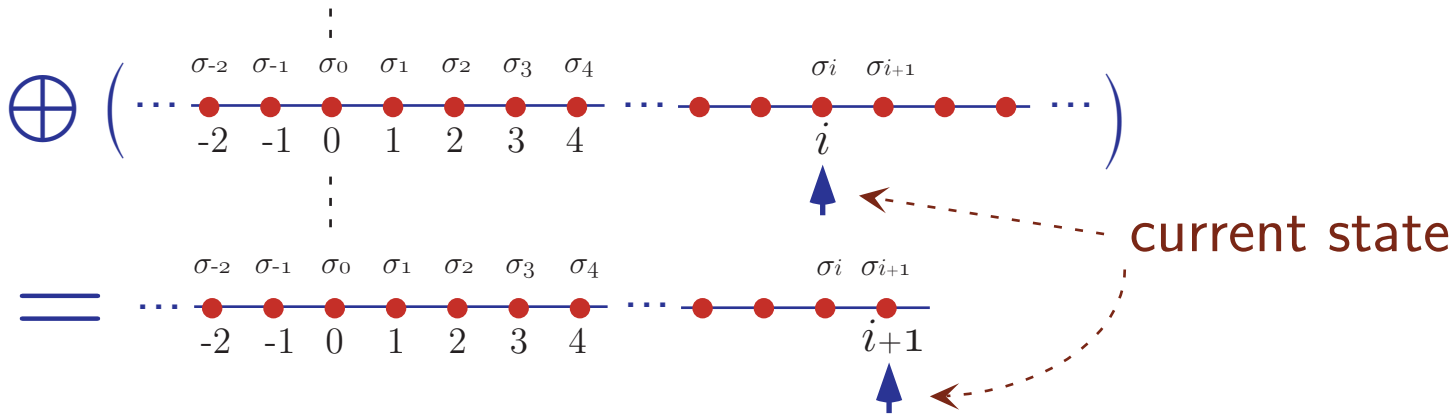


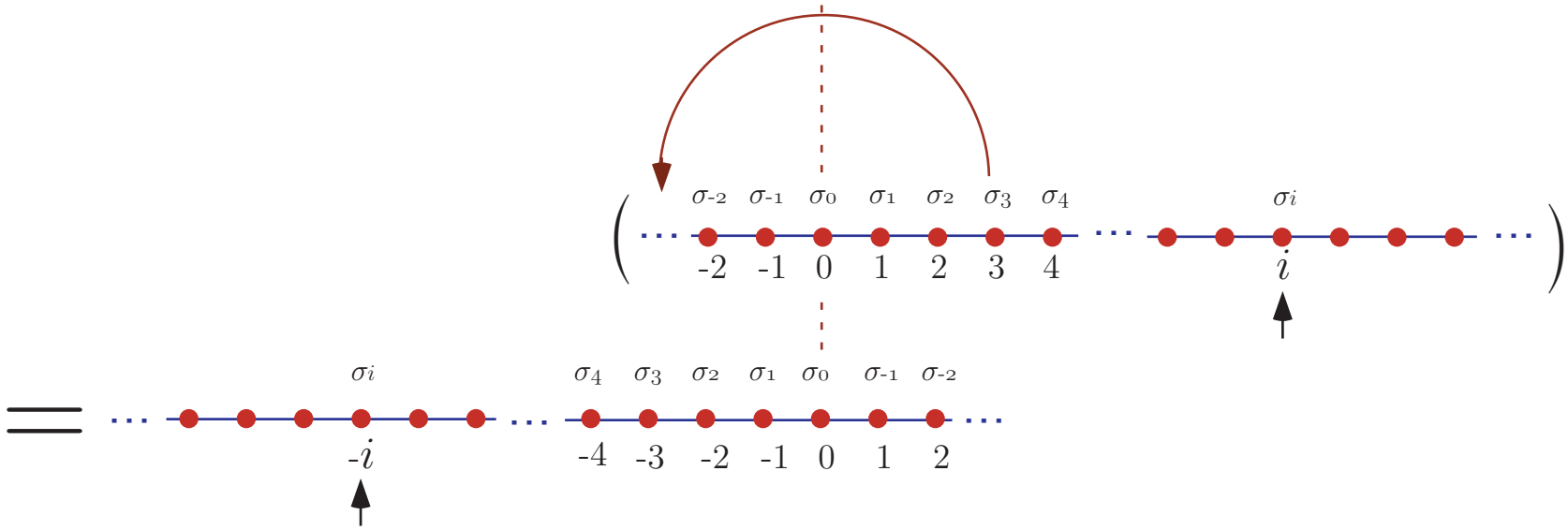
- Model next time:

$$\oplus M \stackrel{\triangle}{=} \{\langle i,\, \sigma\rangle \mid \oplus\langle i,\, \sigma\rangle \in M\}$$

A trace of $\oplus M$ will, at next time, be a trace of M.

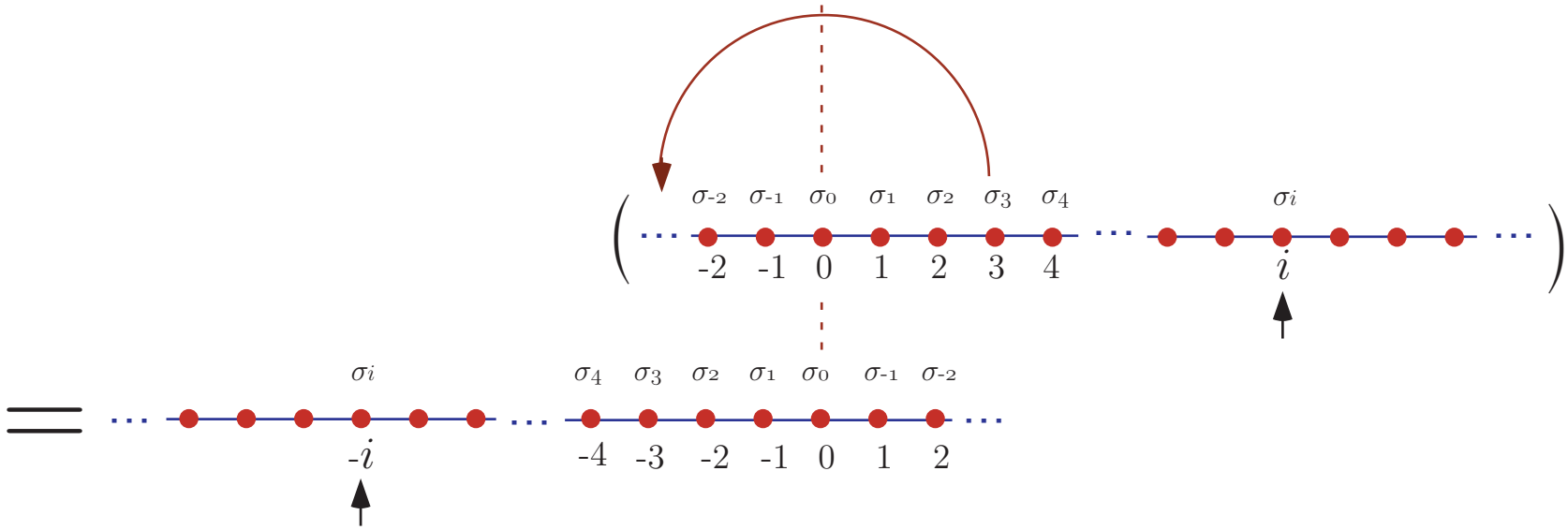
Reversal ↶

- Trace reversal:



Reversal $\overset{\curvearrowleft}{}$

- Trace reversal:



- Model reversal:

$$M^{\curvearrowleft} \triangleq \{\langle i,\, \sigma\rangle \mid \langle i,\, \sigma\rangle^{\curvearrowleft} \in M\}$$

Universal $\forall$ and existential $\exists$ state closures

- The universal state closure $\forall\, \varphi_1 \,:\, \varphi_2$ is the set of traces $\langle i,\, \sigma \rangle$ of φ_1 such that all traces in φ_1 with the same current state σ_i belong to φ_2;

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- The existential state closure $\exists\, \varphi_1 : \varphi_2 = \neg(\forall\, \varphi_1 : \neg\, \varphi_2)$ is the set of traces $\langle i, \sigma \rangle$ of φ_1 such that some trace in φ_1 with the same current state σ_i belongs to φ_2.

Abbreviations (examples)

$$\varphi_1 \,\mathbf{U}\, \varphi_2 \stackrel{\triangle}{=} \boldsymbol{\mu}\, X \cdot (\varphi_2 \vee (\varphi_1 \wedge \oplus X)) \qquad \text{until}$$

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$$\varphi_1 \,\mathbf{S}\, \varphi_2 \stackrel{\triangle}{=} (\varphi_1{}^{\curvearrowleft}\, \mathbf{U} \varphi_2{}^{\curvearrowleft})^{\curvearrowleft} \qquad \text{since}$$

Subcalculi
(example: Kozen's propositional μ-calculus)

$$\varphi ::= \boldsymbol{\sigma}_S \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \neg \varphi_1 \mid \Box \varphi_1 \mid \Diamond \varphi_1 \mid$$
$$X \mid \boldsymbol{\mu} X \cdot \varphi_1 \mid \boldsymbol{\nu} X \cdot \varphi_1$$

where:

τ : transition relation (program SOS semantics);

$\Box \varphi_1 \stackrel{\triangle}{=} \forall \boldsymbol{\pi}_\tau : \oplus \varphi_1$ always (after next step);

$\Diamond \varphi_1 \stackrel{\triangle}{=} \exists \boldsymbol{\pi}_\tau : \oplus \varphi_1$ sometime (after next step).

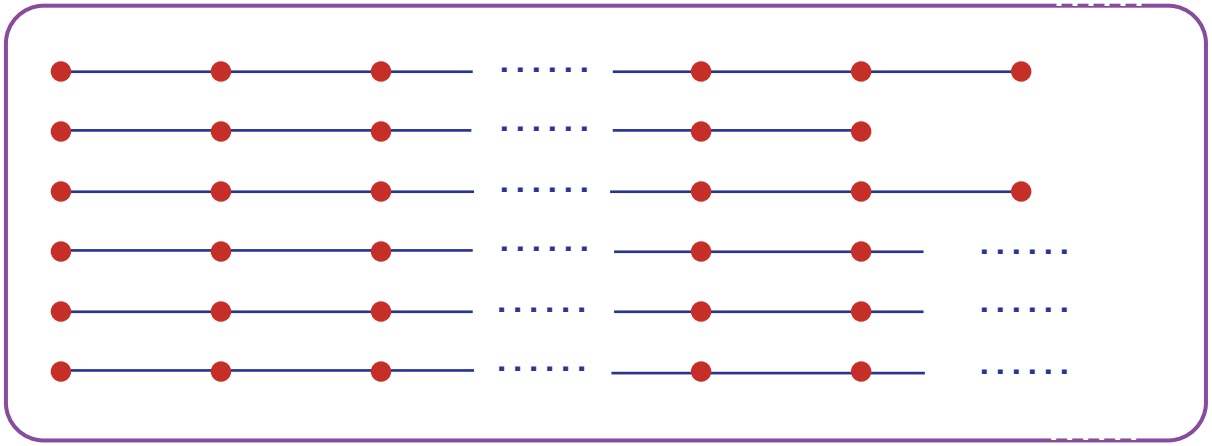
On the reversible $\overset{\curvearrowleft}{\mu}^{\star}$-calculus

- Generalization of previous temporal logics and calculi;
- Contrary to previous propositions:
  - Every logical statement is explicit (e.g. no implicit underlying Kripke structure),
  - A single temporal operator $\curvearrowleft$ to handle past and future,
  - Completely time-symmetric.

3. An intuitive example of abstraction/ approximation

An example of approximation:

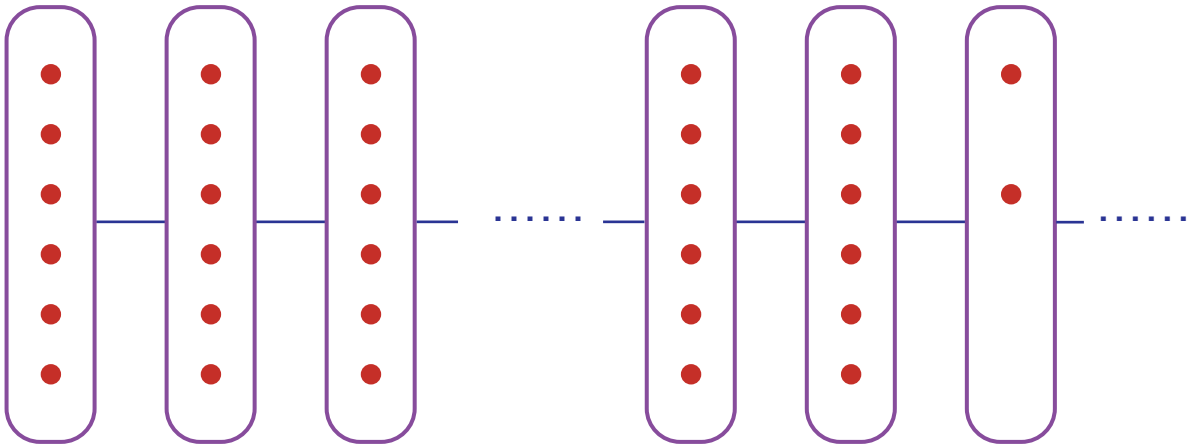
a set of sequences of states



can be approximated/abstracted by .../...

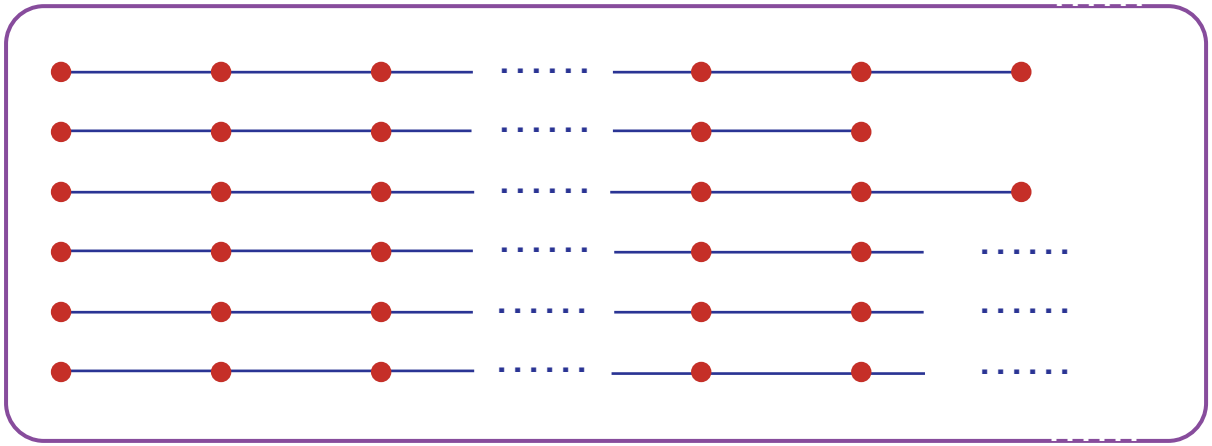
An example of approximation (cont'd)

a sequence of sets of states



Back to a set of sequences of states

- The concretization contains all original traces:



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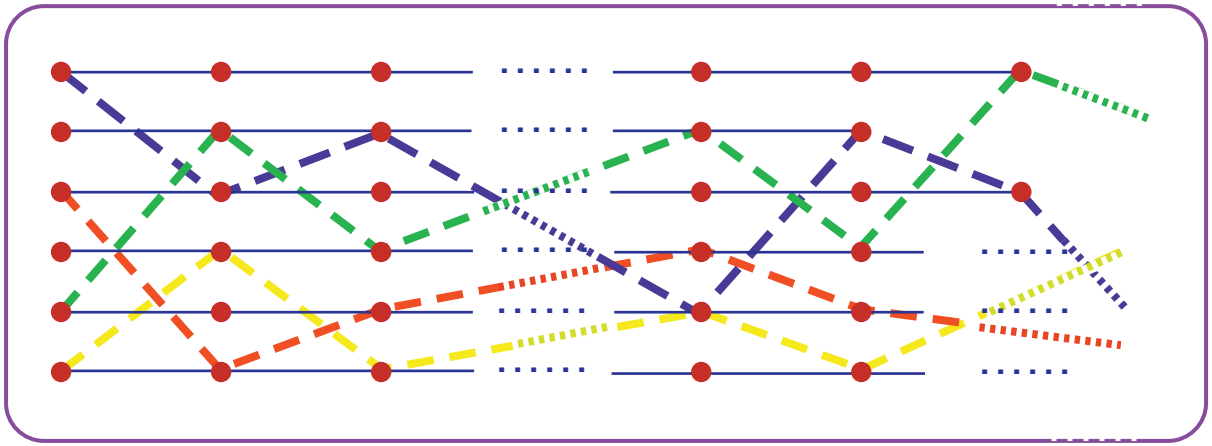

plus (unrealistic) additional ones (, , , , ...);

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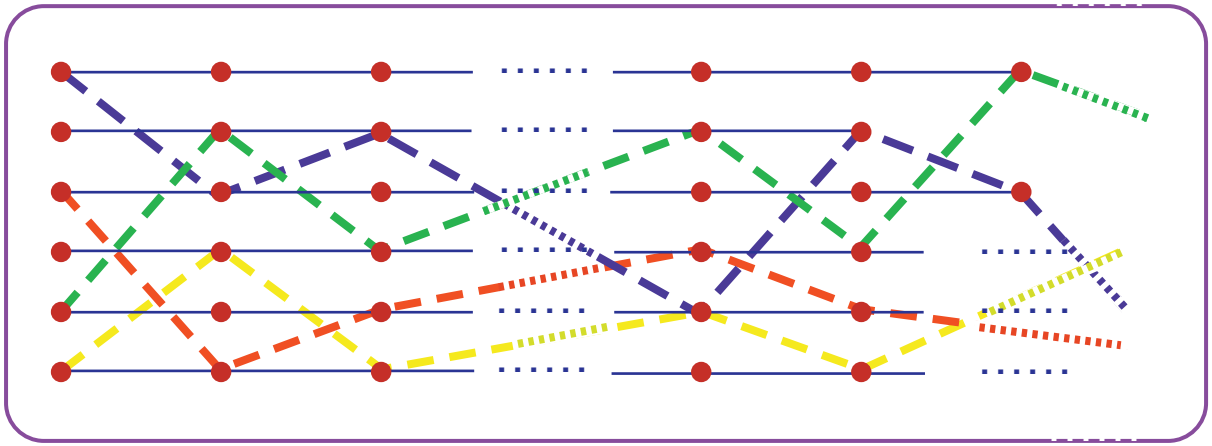

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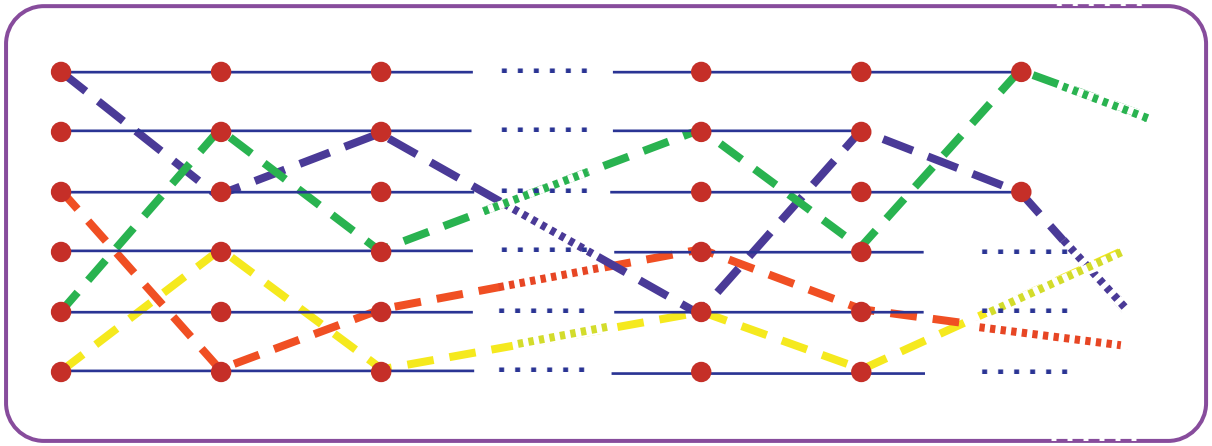

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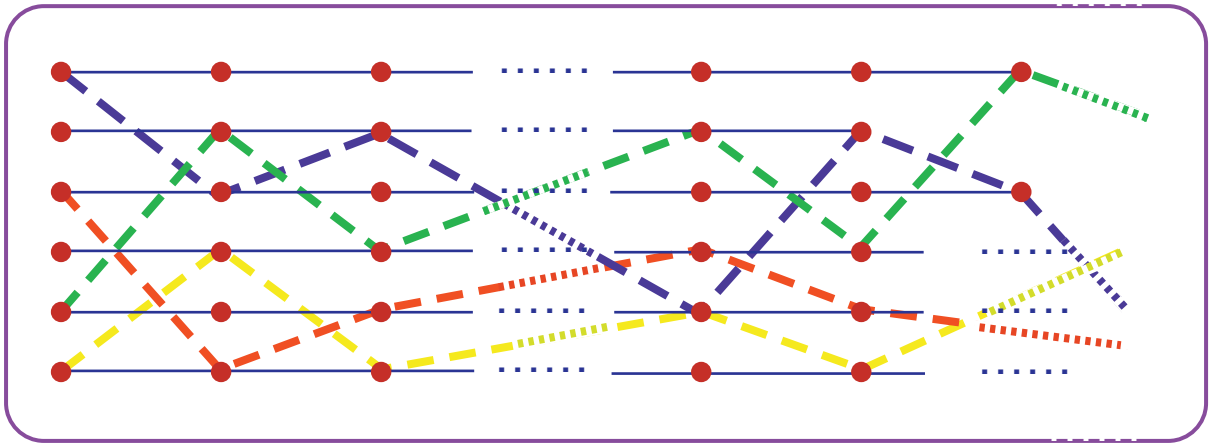
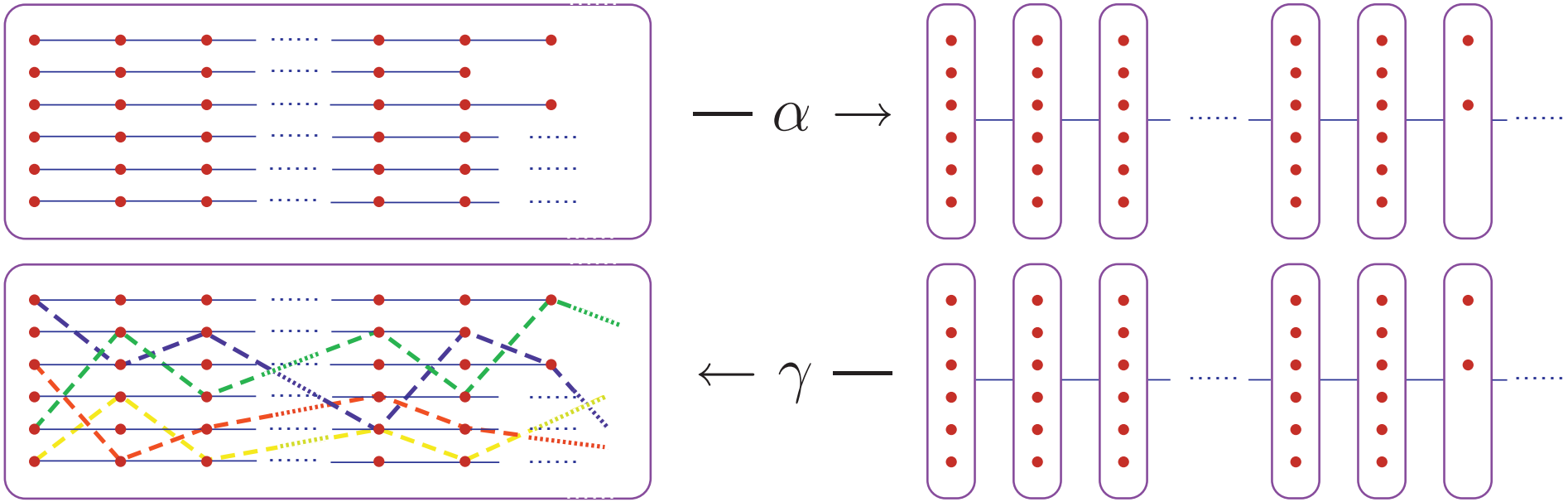


  plus (unrealistic) additional ones (, , , , ...);

- This particular approximation is therefore from above;
- It contains more traces than possible. These additional traces would yield the same abstraction anyway!

Set-based abstraction

Let us call this abstraction the set-based abstraction:



4. Introduction to abstraction soundness/completeness

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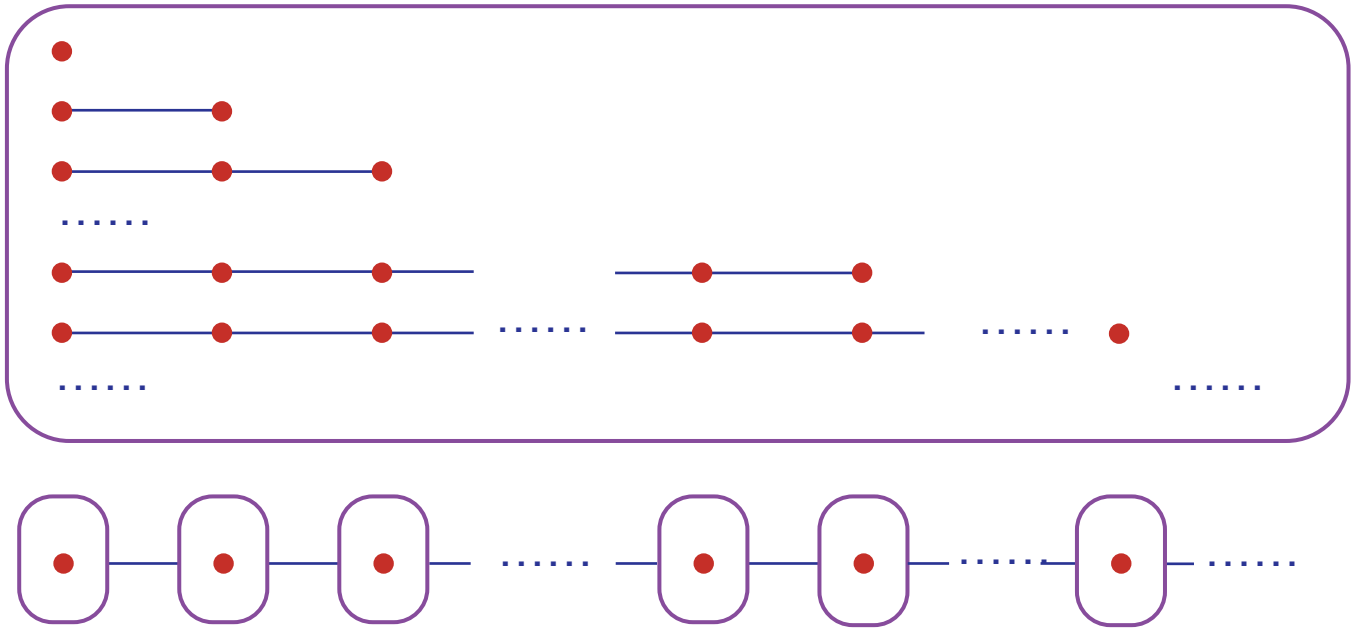
Example for unsoundness



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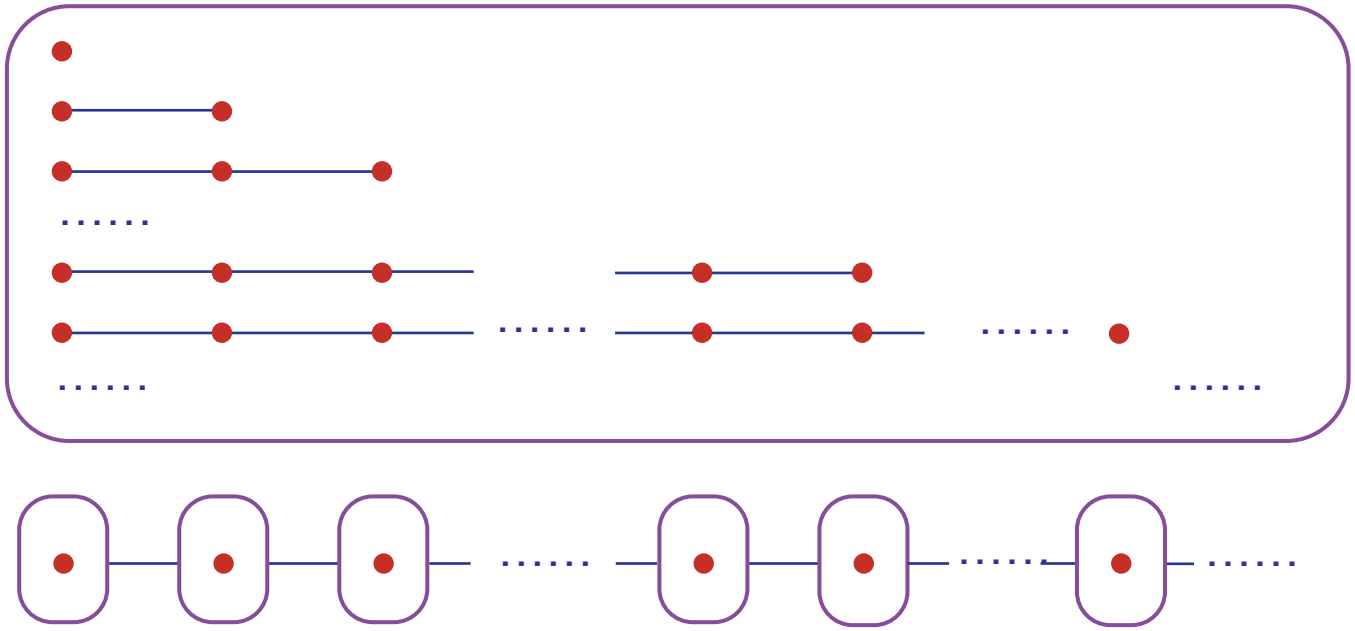
  - Counter-example: "all traces are finite";

Example for uncompleteness



All concrete traces are finite but not the abstract ones!

4.1 Classical temporal-logics/calculi involve implicit abstractions

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- In general, temporal-logic/calculi cannot express all properties of models, but only specific ones (e.g. [3]);

Reference

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- Some concrete properties of the model can only be approximated in the abstract temporal-logic/calculus;
- The semantics of the temporal-logic/calculus can be understood as an abstraction of the concrete semantics (of the models).

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Abstraction closedness

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- For example Kozen's propositional μ-calculus is closed for the set-based abstraction.

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- The concrete properties P such that $\alpha_{\mathcal{T}}(P) \neq P$ cannot be expressed by the temporal logic/calculus $\mathcal{T}$;
- So e.g. the propositional μ-calculus is not expressive enough to capture all concrete models (as expressible e.g. by the reversible $\overset{\curvearrowleft}{\mu}^{\star}$-calculus);

5. Is model checking a complete temporal abstract interpretation?

Current state abstraction

- The model-checking algorithms for the propositional μ-calculus can be derived by classical abstract interpretation techniques [5] with the *current state abstraction* [6]:

$$\alpha^{\bullet}(M) \stackrel{\triangle}{=} \{\sigma_i \mid \langle i, \sigma \rangle \in M\}$$

$$\gamma^{\bullet}(S) \stackrel{\triangle}{=} \{\langle i, \sigma \rangle \mid \sigma_i \in S\} = \boldsymbol{\sigma}_S$$

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$\Longrightarrow$ Model-checking is a sound and complete abstract interpretation.

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- The completeness is relative to the abstract world not to the concrete world!
- This set-based abstraction is itself incomplete (e.g. for the the reversible $\overset{\curvearrowleft}{\mu}^{\star}$-calculus);
- Intuition: with general temporal specifications, model-checking algorithms could not only deal with sets of states only and would have to handle sets of traces (which would be too costly).

6. Model checking is an incomplete temporal abstract interpretation!

Universal checking abstraction

- State projection:

$$M_{\downarrow s} \stackrel{\triangle}{=} \{\langle i,\, \sigma\rangle \in M \mid \sigma_i = s\}$$

- Universal checking abstraction:

$$\alpha^{\forall}_{M}(\phi) \stackrel{\triangle}{=} \{s \mid M_{\downarrow s} \subseteq \phi\}$$

- Universal checking concretization:

$$\gamma^{\forall}_{M}(S) \stackrel{\triangle}{=} \{\langle i,\, \sigma\rangle \mid \langle i,\, \sigma\rangle \in M \wedge \sigma_i \in S\}$$

- Galois connection:

$$\langle \mathbb{M},\, \supseteq\rangle \underset{\alpha^{\forall}_{M}}{\overset{\gamma^{\forall}_{M}}{\leftrightarrows}} \langle \wp(\mathbb{S}),\, \supseteq\rangle$$

Existential checking abstraction

- Dual existential checking abstraction:

$$\begin{aligned}\alpha^{\exists}_{M}(\phi) &\stackrel{\triangle}{=} \neg\alpha^{\forall}_{M}(\neg\phi)\\ &= \{s \mid (M_{\downarrow s} \cap \phi) \neq \emptyset\}\end{aligned}$$

- Existential checking concretization:

$$\begin{aligned}\gamma^{\exists}_{M}(\phi) &\stackrel{\triangle}{=} \neg\gamma^{\forall}_{M}(\neg\phi)\\ &= \{\langle i,\, \sigma\rangle \mid (\langle i,\, \sigma\rangle \in M) \Longrightarrow (\sigma_i \in S)\}\end{aligned}$$

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Model checking (algorithms)

- Fix the model M to be generated by a transition relation (Kripke structure) [7]:

$$M = \boxplus \boldsymbol{\pi}_t$$

[7] may be with fairness conditions also expressible with the reversible $\overset{\curvearrowleft}{\mu^\star}$-calculus.

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- Define the abstraction by structural inductively on formulae:
  - Basic temporal operators are defined by universal/existential abstraction of concrete ones ;
  - Inductive combination with abstraction closed operations (e.g. join, meet, complement, fixpoints, etc.).

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Example: propositional μ-calculus

- $\alpha^{\forall}_{\boldsymbol{\pi}_t}(\boldsymbol{\sigma}\, S) = \alpha^{\exists}_{\boldsymbol{\pi}_t}(\boldsymbol{\sigma}\, S) = S$;

- If φ_1 and φ_2 are $\alpha^{\forall}_{\boldsymbol{\pi}_t}$ or $\alpha^{\exists}_{\boldsymbol{\pi}_t}$ abstraction closed, i.e.:

$$\alpha_{\boldsymbol{\pi}_t}(\varphi_i) = \varphi_i\,,\ i = 1, \ldots, 2$$

  then so are:

$$\varphi_1 \vee \varphi_2\,,\ \varphi_1 \wedge \varphi_2\,,\ \neg\, \varphi_1\,,\ \Box\, \varphi_1\,,\ \Diamond\, \varphi_1\,,\ \boldsymbol{\mu}\, X \cdot \varphi_1\,,\ \boldsymbol{\nu}\, X \cdot \varphi_1.$$

7. Abstraction completeness for sub-logics and calculi

Model checking algorithms incompleteness

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Model checking algorithms incompleteness

- The classical model-checking algorithms are set of states based (but not set of traces based);
- In general, this abstraction is incomplete (e.g. for the full complete reversible $\overset{\curvearrowleft}{\mu}^{\star}$-calculus);
- We can identify sub-calculi (whence logics) for which the model-checking abstractions are complete;

Example: $\mu_{+}^{\forall}$-calculus

$$\psi ::= \boldsymbol{\sigma}\, S \mid \psi_1 \vee \psi_2 \mid \psi_1 \wedge \psi_2 \mid \neg\, \psi_1 \mid \forall\, \varphi \qquad \text{state formulae}$$

$$\varphi ::= \psi \mid \boldsymbol{\pi}\, t \mid \oplus\, \varphi_1 \mid \varphi_1 \wedge \varphi_2 \mid \qquad \text{path formulae}$$

$$\psi_1 \vee \varphi_2 \mid \varphi_1 \vee \psi_2 \mid X \mid \boldsymbol{\mu}\, X \cdot \varphi_1 \mid \boldsymbol{\nu}\, X \cdot \varphi_1$$

(with $\forall\, \varphi \stackrel{\triangle}{=} \forall\, \boxplus \boldsymbol{\pi}\, t : \varphi$) is such that:

$$[\![\varphi]\!]^{\forall} = \vec{\alpha}^{\forall}_{\mathcal{M}_\tau}([\![\varphi]\!])$$

- This covers $\forall$CTL (but not $\forall$CTL*);
- Same for $\exists$CTL (but not $\exists$CTL*) and inductive combination with joins, ... to get completeness for CTL (but not CTL*).

8. Conclusion

More in the forthcoming POPL'00 paper ...

- Compositional abstract interpretation of generic μ-calculi (independently of a particular semantics, including for non-monotone operators);
- Study of the model-checking abstractions;
- Study of (sufficient) abstraction completeness conditions;
- Applications to:
  - Abstract model checking;
  - Dataflow analysis (and the soundness of live variables).

Perspectives

- Anyway, model-checking is an incomplete abstract interpretation;
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- Anyway, model-checking is an incomplete abstract interpretation;
- So for infinite state systems:
  - other abstractions can be used (e.g. as in abstract testing);
  - because of incompleteness, other algorithms should be used [4] (the common model-checking algorithms are not the most precise ones).

Reference

[4] P. Cousot and R. Cousot. Abstract interpretation and application to logic programs. *Journal of Logic Programming*, 13(2–3):103–179, 1992.