Abstract Interpretation of Computations

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Talk Outline

A Few Elements of Abstract Interpretation

Reference

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In *6^{th} POPL*, pages 269–282, San Antonio, TX, 1979. ACM Press.

A Model of Computer Programs

- **Syntax :** a well-founded set of programs $\langle \mathbb{P}, \prec \rangle$ where $\prec$ is the "strict immediate subcomponent" relation ;
- **Semantics of $P \in \mathbb{P}$:**
  - **Semantic domain :** a complete lattice/cpo $\langle \mathcal{D}[\![P]\!], \sqsubseteq, \bot, \sqcup \rangle$
  - **Compositional Fixpoint Semantics :**

$$\mathcal{S}[\![P]\!] \stackrel{\text{def}}{=} \mathbf{lfp}_{\bot}^{\sqsubseteq} \mathcal{F}[\![P]\!] \left(\prod_{P' \prec P} \mathcal{S}[\![P']\!] \right)$$

$\mathbf{lfp}_{\bot}^{\sqsubseteq} f$ is the limit of $X^0 = \bot$, $X^{\delta+1} = f(X^\delta)$, $X^\lambda = \sqcup_{\beta<\lambda} X^\lambda$, λ limit ordinal, if any. Existence e.g. monotony (by Tarski).

Example: Syntax of Programs

$$
\begin{array}{lllll}
X & & & & \text{variables } X \in \mathbb{X} \\
T & & & & \text{types } T \in \mathbb{T} \\
E & & & & \text{arithmetic expressions } E \in \mathbb{E} \\
B & & & & \text{boolean expressions } B \in \mathbb{B} \\
D & ::= & T\ X\ \texttt{;} & & \text{declarations } D \in \mathbb{D}, \quad \text{vars}(D) = \{X\} \\
 & \mid & T\ X\ \texttt{;}\ D' & & X \notin \text{vars}(D'),\ \text{vars}(D) = \{X\} \cup \text{vars}(D') \\
C & ::= & X\ \texttt{=}\ E\texttt{;} & & \text{commands } C \in \mathbb{C} \quad (E \prec C) \\
 & \mid & \texttt{while}\ B\ C' & & \quad (B \prec C,\ C' \prec C) \\
 & \mid & \texttt{if}\ B\ C' & & \quad (B \prec C,\ C' \prec C) \\
 & \mid & \texttt{if}\ B\ C'\ \texttt{else}\ C'' & & \quad (B \prec C,\ C' \prec C,\ C'' \prec C) \\
 & \mid & \texttt{\{}\ \mathrm{C}_1\ \ldots\ \mathrm{C}_n\ \texttt{\}},\ (n \geq 0) & & \quad (C_1 \prec C, \ldots, C_n \prec C) \\
P & ::= & D\ C & & \text{program } P \in \mathbb{P} \quad (C \prec P)
\end{array}
$$

Example: Concrete Semantic Domain of Programs

Reachability properties:

$$\Sigma[\![D\ C]\!] \stackrel{\text{def}}{=} \Sigma[\![D]\!]$$
$$\Sigma[\![T\ X\,;]\!] \stackrel{\text{def}}{=} \{X\} \mapsto T$$
$$\Sigma[\![T\ X\,;\ D]\!] \stackrel{\text{def}}{=} (\{X\} \mapsto T) \cup \Sigma[\![D]\!]$$

states ρ ($\rho(X)$ is the value of X)

$$\mathcal{D}[\![P]\!] \stackrel{\text{def}}{=} \wp(\Sigma[\![P]\!])$$

sets of states

$$\sqsubseteq \stackrel{\text{def}}{=} \subseteq$$

implication

$$\perp \stackrel{\text{def}}{=} \emptyset$$

false

$$\sqcup \stackrel{\text{def}}{=} \cup$$

disjunction

Example: Concrete Semantics of Programs (Reachability)

$$\mathcal{S}[\![X = E;]\!]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \qquad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$\mathcal{S}[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{B}[\![\neg B]\!]R$$

$$\mathcal{B}[\![B]\!]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$

$$\mathcal{S}[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{S}[\![C'']\!](\mathcal{B}[\![\neg B]\!]R)$$

$$\mathcal{S}[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \mathbf{lfp}_{\emptyset}^{\subseteq} \lambda \mathcal{X} \cdot R \cup \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C_n]\!] \circ \ldots \circ \mathcal{S}[\![C_1]\!] \quad n > 0$$

$$\mathcal{S}[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C]\!](\Sigma[\![D]\!]) \quad \text{(uninitialized variables)}$$

Not computable (undecidability).

Abstraction

A reasoning/computation such that:

- only some properties can be used;
- the properties that can be used are called "*abstract*";
- so, the (other concrete) properties must be approximated by the abstract ones;

Abstract Properties

- Abstract Properties: a set $\overline{\mathcal{A}} \subsetneq \wp(\Sigma)$ of properties of interest (the only one which can be used to approximate others).

Direction of Approximation

- Approximation from above: approximate P by $\overline{P}$ such that $P \subseteq \overline{P}$;
- Approximation from below: approximate P by $\underline{P}$ such that $\underline{P} \subseteq P$ (dual).

Best Abstraction

- We require that all concrete property $P \in \wp(\Sigma)$ have a best abstraction $\overline{P} \in \overline{\mathcal{A}}$:

$$P \subseteq \overline{P}$$
$$\forall \overline{P'} \in \overline{\mathcal{A}} : (P \subseteq \overline{P'}) \Longrightarrow (\overline{P} \subseteq \overline{P'})$$

- So, by definition of the greatest lower bound/meet $\cap$:

$$\overline{P} = \cap\{\overline{P'} \in \overline{\mathcal{A}} \mid P \subseteq \overline{P'}\} \in \overline{\mathcal{A}}$$

(Otherwise see [JLC '92].)

Reference

[JLC '92] P. Cousot & R. Cousot. Abstract interpretation frameworks. *J. Logic and Comp.*, 2(4):511–547, 1992.

Moore Family

- This hypothesis that any concrete property $P \in \wp(\Sigma)$ has a best abstraction $\overline{P} \in \overline{\mathcal{A}}$ implies that:

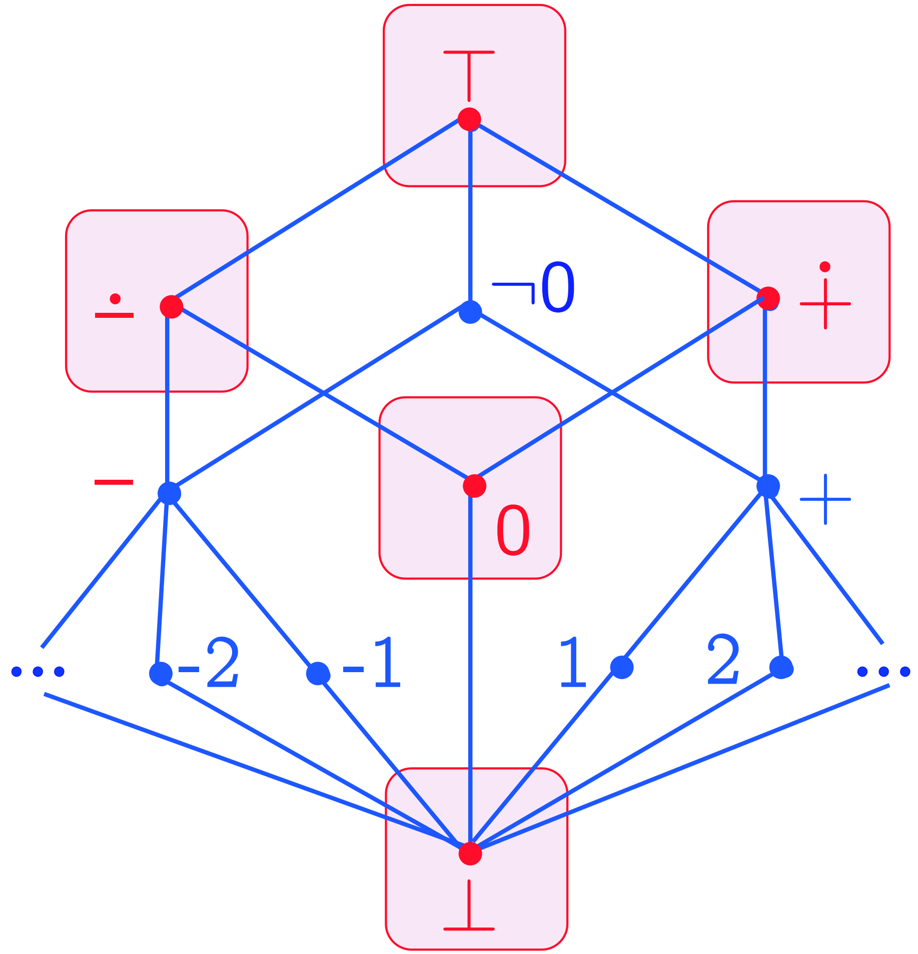
  $$\overline{\mathcal{A}} \text{ is a Moore family}$$

  i.e. it is closed under intersection $\cap$:

  $$\forall S \subseteq \overline{\mathcal{A}} : \cap S \in \overline{\mathcal{A}}$$

- In particular $\cap\emptyset = \Sigma \in \overline{\mathcal{A}}$ is "I don't know".

Example of Moore Family-Based Abstraction



Closure Operator Induced by an Abstraction

The map $\rho_{\overline{\mathcal{A}}}$ mapping a concrete property $P \in \wp(\Sigma)$ to its best abstraction $\rho_{\overline{\mathcal{A}}}(P)$ in $\overline{\mathcal{A}}$:

$$\rho_{\overline{\mathcal{A}}}(P) = \cap\{\overline{P} \in \overline{\mathcal{A}} \mid P \subseteq \overline{P}\}$$

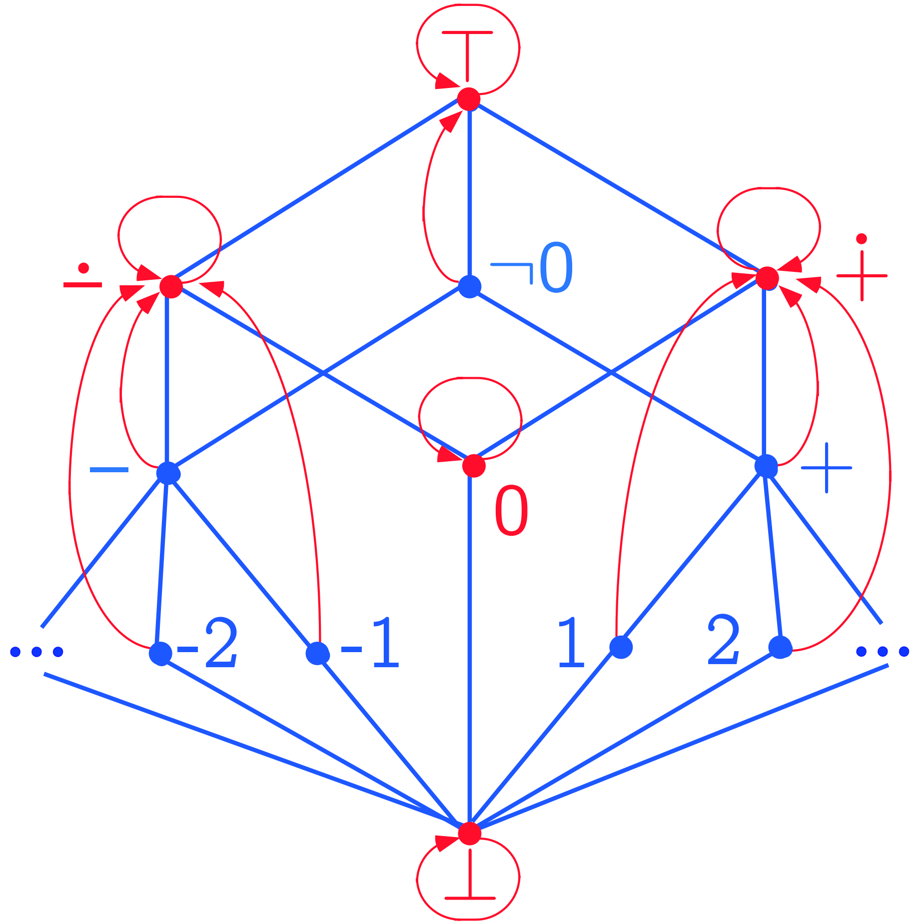
is a closure operator:

- extensive,
- idempotent,
- isotone/monotonic;

such that $P \in \bar{A} \iff P = \rho_{\overline{\mathcal{A}}}(P)$

hence $\overline{\mathcal{A}} = \rho_{\overline{\mathcal{A}}}(\wp(\Sigma))$.

Example of Closure Operator-Based Abstraction



The Lattice of Abstract Interpretations

- The set of all possible abstractions that is of all upper closure operators on the complete lattice

$$\langle \mathcal{D}[\![P]\!],\ \sqsubseteq,\ \bot,\ \top,\ \sqcup,\ \sqcap \rangle$$

is a complete lattice

$$\langle \mathrm{uco}(\mathcal{D}[\![P]\!] \mapsto \mathcal{D}[\![P]\!]),\ \dot{\sqsubseteq},\ \lambda x \cdot x,\ \lambda x \cdot \top,\ \lambda R \cdot \mathrm{uco}(\dot{\sqcup} R),\ \dot{\sqcap} \rangle$$

- The meet of abstractions called the reduced product ($\dot{\bigsqcap}_{i \in \Delta} \rho_i$ is that most abstract abstraction more precise than all ρ_i, $i \in \Delta$)

Galois Connection Between Concrete and Abstract Properties

- For closure operators ρ, we have:

$$\rho(P) \subseteq \rho(P') \;\Leftrightarrow\; P \subseteq \rho(P')$$

written:

$$\langle \wp(\Sigma),\ \subseteq\rangle \underset{\rho}{\overset{1}{\leftrightarrows}} \langle \rho(\wp(\Sigma)),\ \subseteq\rangle$$

where 1 is the identity and:

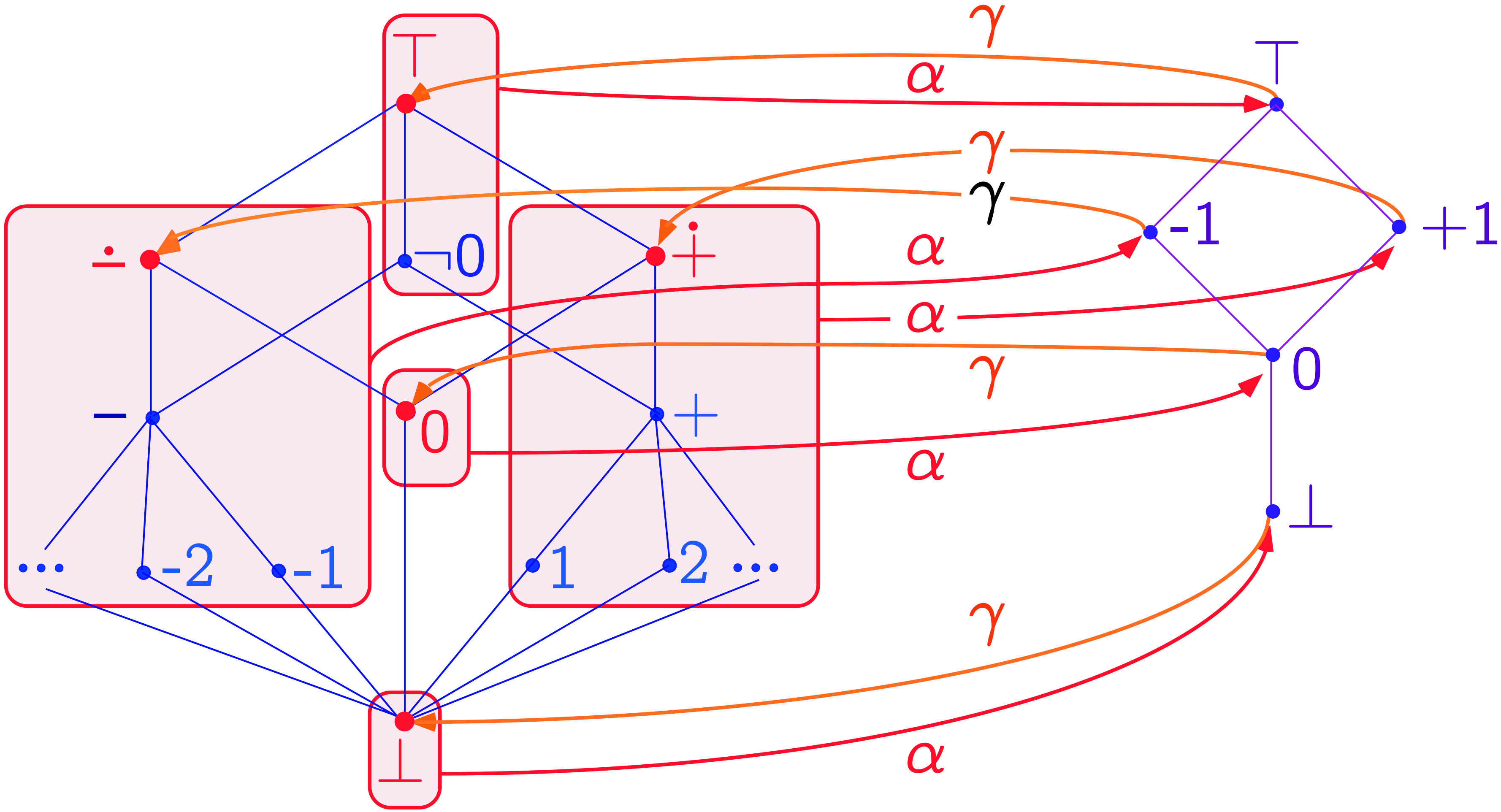
$$\langle \wp(\Sigma),\ \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \overline{\mathcal{D}},\ \sqsubseteq\rangle$$

means that $\langle \alpha,\ \gamma\rangle$ is a Galois connection:

$$\forall P \in \wp(\Sigma), \overline{P} \in \overline{\mathcal{D}} : \alpha(P) \sqsubseteq \overline{P} \;\Leftrightarrow\; P \subseteq \gamma(\overline{P});$$

- A Galois connection defines a closure operator $\rho = \alpha \circ \gamma$, hence a best abstraction.

Example of Galois Connection-Based Abstraction



Example: abstract semantic domain of programs

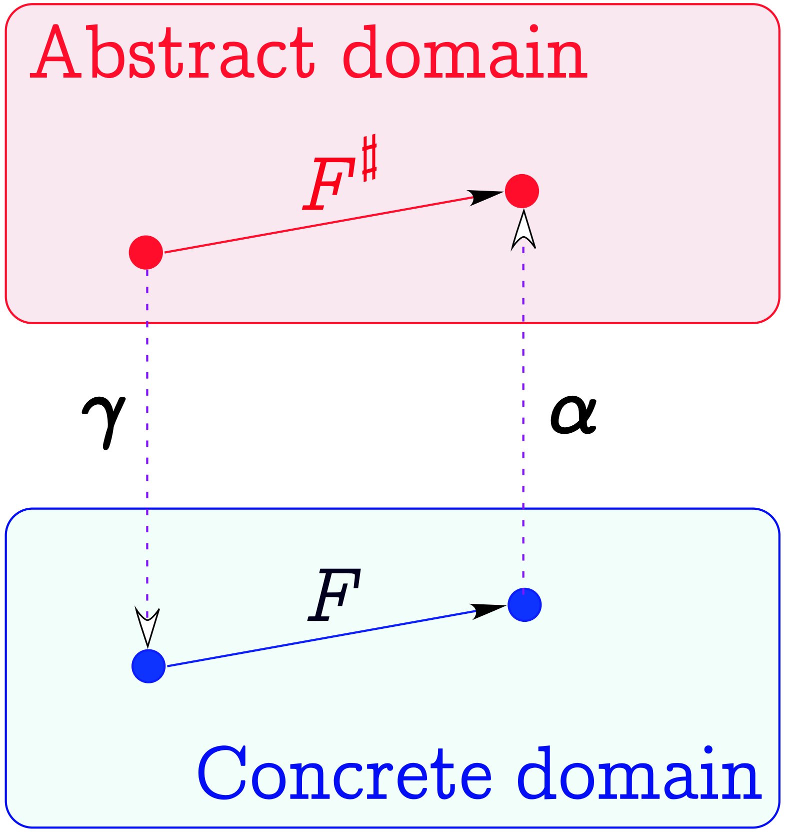
$$\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup \rangle$$

such that:

$$\langle \mathcal{D},\ \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq \rangle$$

hence $\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \bot,\ \sqcup \rangle$ is a complete lattice such that $\bot = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup\, \gamma(X))$

Function Abstraction



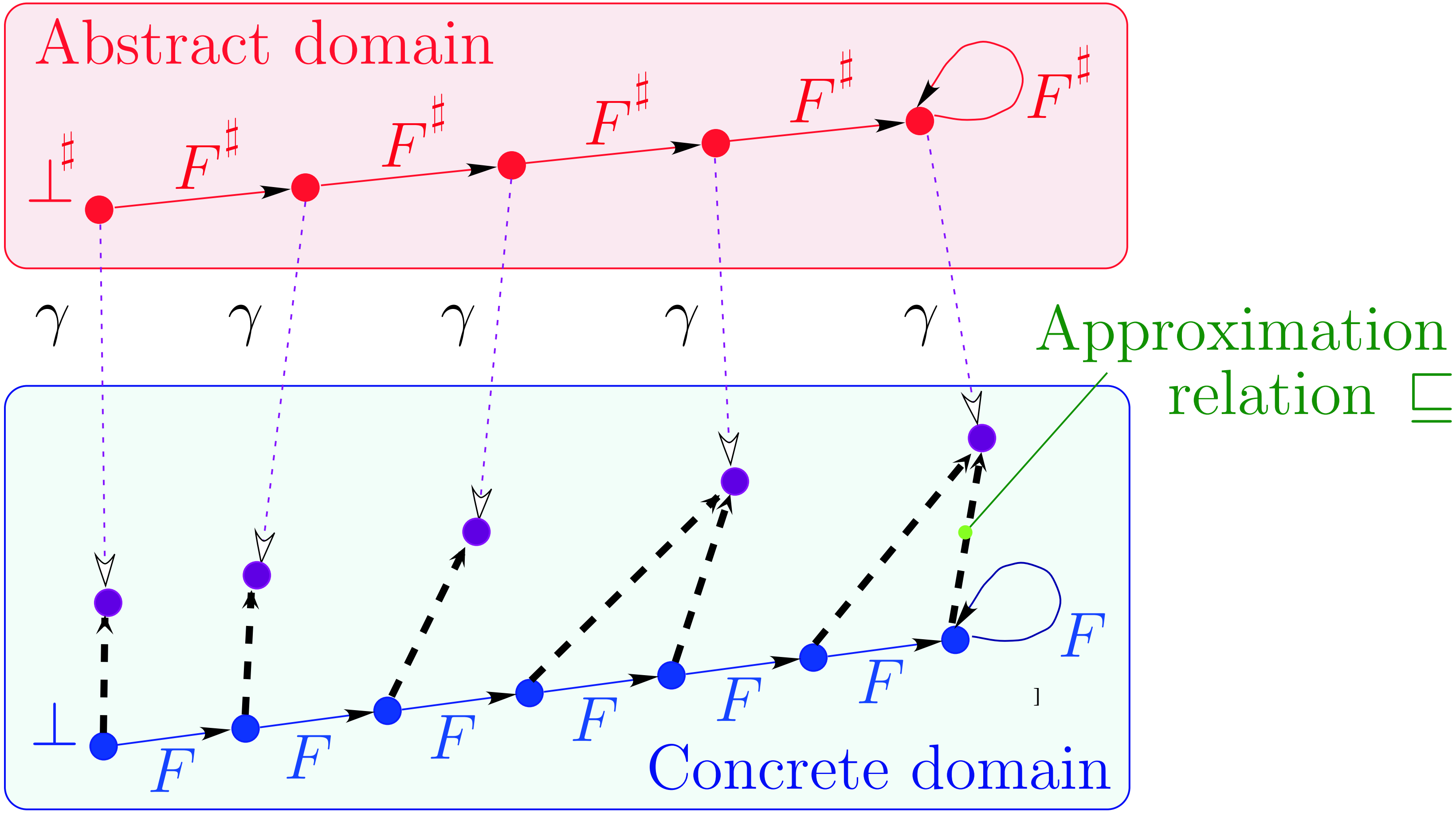
$$F^\sharp = \alpha \circ F \circ \gamma$$

i.e. $F^\sharp = \rho \circ F$

$$\langle P,\ \subseteq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle Q,\ \sqsubseteq\rangle \Rightarrow$$

$$\langle P \overset{\text{mon}}{\longmapsto} P,\ \dot{\subseteq}\rangle \underset{\lambda F \cdot \alpha \circ F \circ \gamma}{\overset{\lambda F^\sharp \cdot \gamma \circ F^\sharp \circ \alpha}{\leftrightarrows}} \langle Q \overset{\text{mon}}{\longmapsto} Q,\ \dot{\sqsubseteq}\rangle$$

Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\sharp \Rightarrow \mathbf{lfp}\, F \sqsubseteq \gamma(\mathbf{lfp}\, F^\sharp)$$

Example: abstract semantics of programs (reachability)

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \mathrm{dom}(E)\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$

$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \mathrm{dom}(B) \mid B \text{ holds in } \rho\})$$

$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$

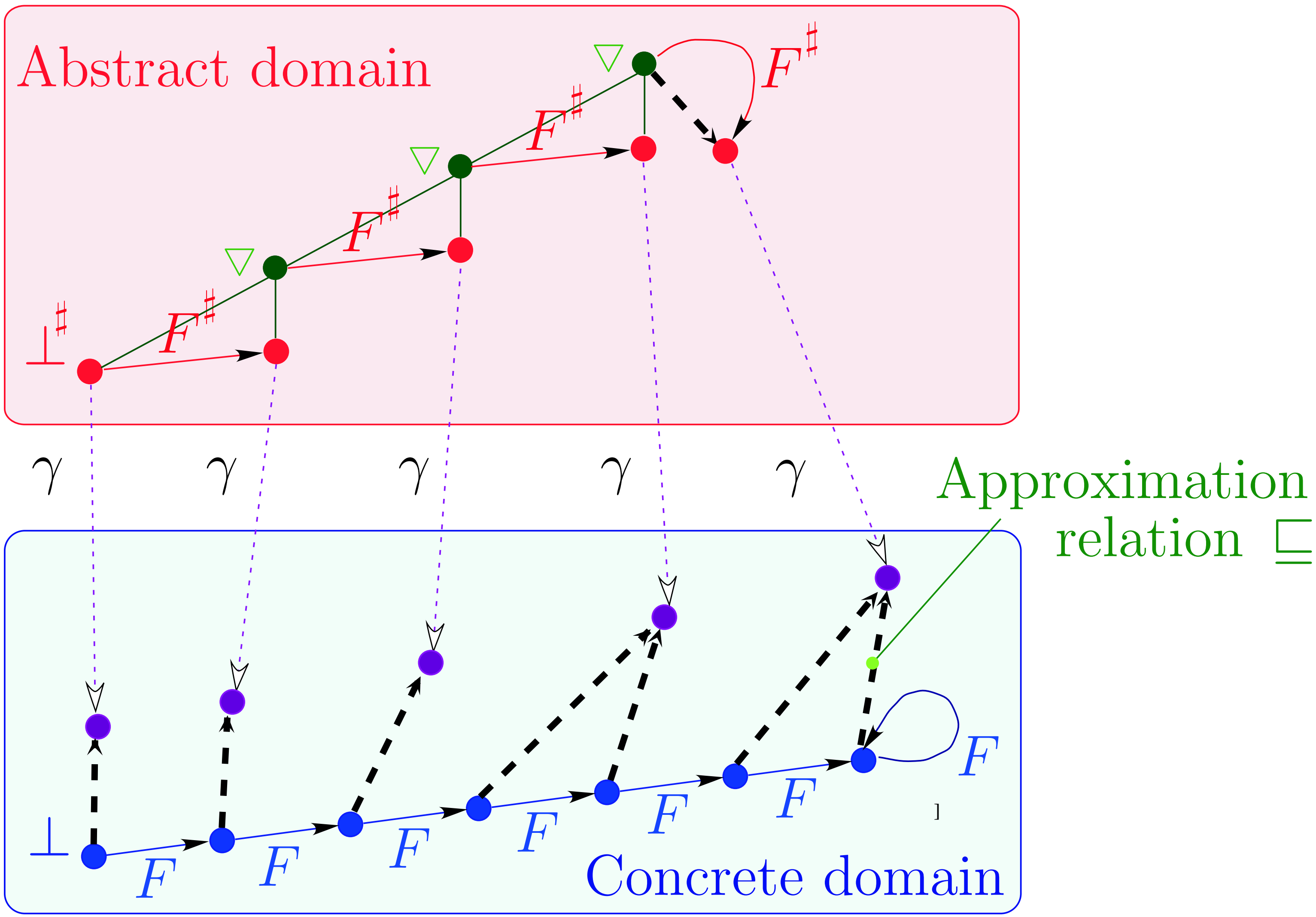
$$\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \mathbf{lfp}^{\sqsubseteq}_{\bot}\, \lambda\mathcal{X} \cdot R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$

$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$

$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0$$

$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

Convergence Acceleration with Widening



Widening Operator

A widening operator $\nabla \in \overline{L} \times \overline{L} \mapsto \overline{L}$ is such that:

- Correctness:
  - $\forall x, y \in \overline{L} : \gamma(x) \sqsubseteq \gamma(x \nabla y)$
  - $\forall x, y \in \overline{L} : \gamma(y) \sqsubseteq \gamma(x \nabla y)$
- Convergence:
  - for all increasing chains $x^0 \sqsubseteq x^1 \sqsubseteq \ldots$, the increasing chain defined by $y^0 = x^0, \ldots, y^{i+1} = y^i \nabla x^{i+1}, \ldots$ is not strictly increasing.

Fixpoint Approximation with Widening

Concergence Theorem:

The upward iteration sequence with widening:

- $X^0 = \bot$ (infimum)
- $X^{i+1} = X^i \qquad \text{if } F^\sharp(X^i) \sqsubseteq X^i$
  $\qquad = X^i \nabla F(X^i) \qquad \text{otherwise}$

is ultimately stationary and its limit A is a sound upper approximation of $\mathbf{lfp}_\bot^\sqsubseteq F^\sharp$:

$$\mathbf{lfp}_\bot^\sqsubseteq F^\sharp \sqsubseteq A$$

Example: Abstract Semantics with Convergence Acceleration [1]

$$
\begin{array}{rcl}
\mathcal{S}^\sharp[\![X = E;]\!]R & \stackrel{\text{def}}{=} & \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\}) \\
\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R & \stackrel{\text{def}}{=} & \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R \\
\mathcal{B}^\sharp[\![B]\!]R & \stackrel{\text{def}}{=} & \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\}) \\
\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R & \stackrel{\text{def}}{=} & \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R) \\
\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R & \stackrel{\text{def}}{=} & \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X} \cdot \text{let } \mathcal{Y} = R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X}) \\
 & & \qquad \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y} \\
 & & \text{and } \mathcal{W} = \mathbf{lfp}_\perp^\sqsubseteq \mathcal{F}^\sharp \text{ in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W}) \\
\mathcal{S}^\sharp[\![\{\}]\!]R & \stackrel{\text{def}}{=} & R \\
\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R & \stackrel{\text{def}}{=} & \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0 \\
\mathcal{S}^\sharp[\![D\ C]\!]R & \stackrel{\text{def}}{=} & \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}
\end{array}
$$

[1] Note: $\mathcal{F}^\sharp$ not monotonic!

Extrapolation by Widening is Essentially Not Monotone

Proof by contradiction:

- Let ∇ be a widening operator
- Define $x \mathbin{\nabla'} y = \text{if } y \sqsubseteq x \text{ then } x \text{ else } x \mathbin{\nabla} y$
- Assume $x \sqsubset y = F(x)$ (during iteration)

  then:
$$\begin{array}{ccccccc} x & \nabla' & y & = & x \mathbin{\nabla} y & \sqsupseteq & y \\ \sqsubseteq & & \sqsubseteq & & \sqsubseteq & & \\ y & \nabla' & y & = & y & & \end{array}$$
  (*soundness*)
  (*monotony hypothesis*)
  (*termination*)

$\Rightarrow$ $x \mathbin{\nabla} y = y$, by antisymmetry!

$\Rightarrow$ $x \mathbin{\nabla} F(x) = F(x)$ during iteration $\Rightarrow$ convergence cannot be enforced with monotone widening (so widening by finite abstraction is less powerful!)

Soundness Theorem

- Convergence by extensivity (no longer monotone)
- Improvement by narrowing [POPL '77]
- *Soundness Corollary:* any abstract safety proof is valid in the concrete in that:

$$\mathcal{S}^{\sharp}[\![P]\!] \sqsubseteq Q \Longrightarrow \mathcal{S}[\![P]\!] \subseteq \gamma(Q)$$

- Example: $\gamma(Q)$ expresses the absence of run-time errors.

Reference

[POPL '77] P. Cousot & R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In 4^{th} *POPL*, pages 238–252, Los Angeles, CA, 1977. ACM Press.

Applications of Abstract Interpretation

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03]

- **Syntax Analysis** [TCS 290(1) 2002]

- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1–2) 2002]

- **Typing** [TCS 277(1–2) 2002]

Applications of Abstract Interpretation (Cont'd)

- **(Abstract) Model Checking** [POPL '00]
- **Program Transformation** [POPL '02]
- **Software Watermarking** [POPL '04]
- **Bisimulations** [RT-ESOP '04]

All these techniques involve sound approximations that can be formalized by abstract interpretation

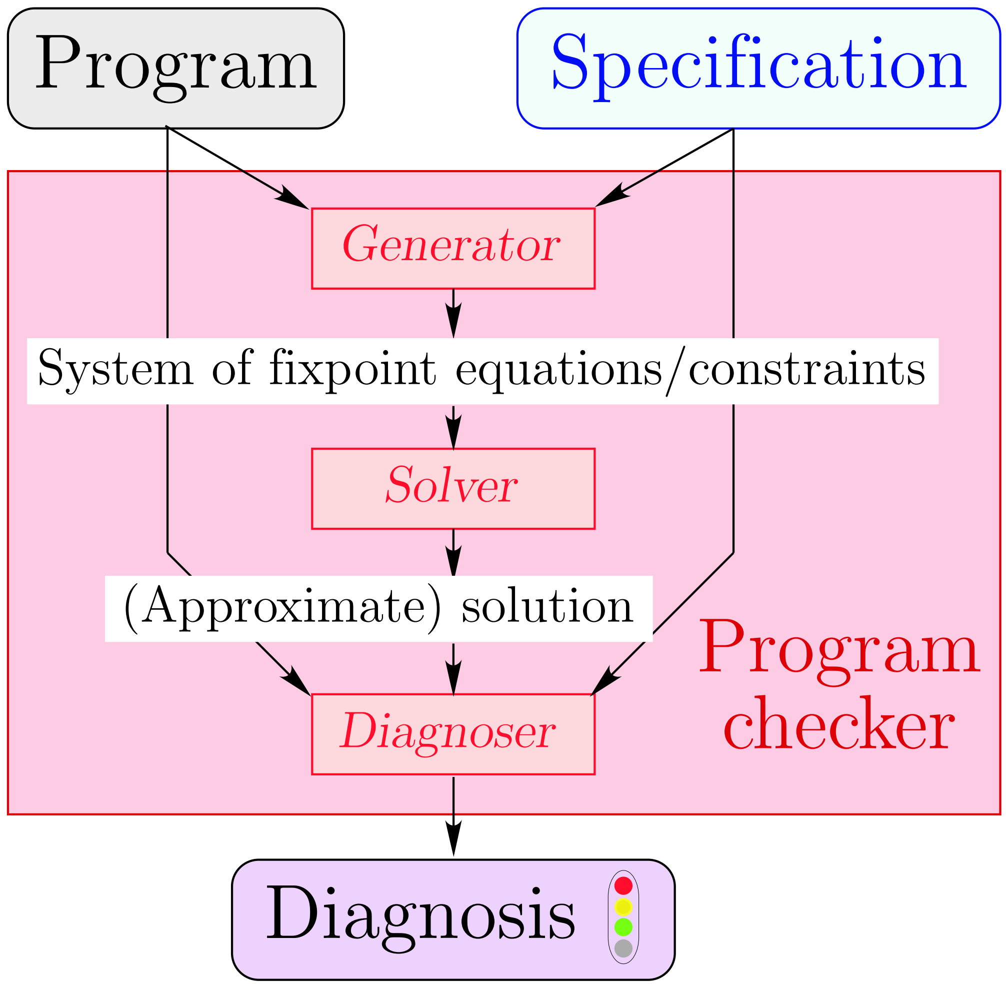
A Practical Application of Abstract Interpretation to the Verification of Safety Critical Embedded Software

Reference

[1] B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, and X. Rival. Design and implementation of a special-purpose static program analyzer for safety-critical real-time embedded software. *The Essence of Computation: Complexity, Analysis, Transformation. Essays Dedicated to Neil D. Jones*, LNCS 2566, pages 85–108. Springer, 2002.

[2] B. Blanchet, P. Cousot, R. Cousot, J. Feret, L. Mauborgne, A. Miné, D. Monniaux, and X. Rival. A static analyzer for large safety-critical software. PLDI'03, San Diego, June 7–14, ACM Press, 2003.

Static Program Analysis



ASTRÉE: A Sound, Automatic, Specializable, Domain-Aware, Parametric, Modular, Efficient and Precise Static Program Analyzer

www.astree.ens.fr

- C programs:
  - structured C programs;
  - no dynamic memory allocation;
  - no recursion.
- Application Domain: safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.

Concrete Operational Semantics

- International norm of C (ISO/IEC 9899:1999)
- *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by* program specific user requirements (e.g. `assert`)

Abstract Semantics

- Reachable states for the concrete operational semantics
- Volatile environment is specified by a *trusted* configuration file.

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Example application

- Primary flight control software of the Airbus A340/A380 fly-by-wire system

- C program, automatically generated from a proprietary high-level specification
- A340: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays.

The Class of Considered Periodic Synchronous Programs

declare `volatile input, state and output variables;`
`initialize state and output variables;`
loop forever
  `- read volatile input variables,`
  `- compute output and state variables,`
  `- write to volatile output variables;`
  wait_for_clock ();
end loop

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick [3].

Reference

[3] C. Ferdinand, R. Heckmann, M. Langenbach, F. Martin, M. Schmidt, H. Theiling, S. Thesing, and R. Wilhelm. Reliable and precise WCET determination for a real-life processor. *ESOP (2001)*, LNCS 2211, 469–485.

Characteristics of the ASTRÉE Analyzer

Static: compile time analysis ($\neq$ run time analysis Rational Purify, Parasoft Insure++)

Program Analyzer: analyzes programs not micromodels of programs ($\neq$ PROMELA in SPIN or Alloy in the Alloy Analyzer)

Automatic: no end-user intervention needed ($\neq$ ESC Java, ESC Java 2)

Sound: covers the whole state space ($\neq$ MAGIC, CBMC) so never omit potential errors ($\neq$ UNO, CMC from coverity.com) or sort most probable ones ($\neq$ Splint)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains ($\neq$ symbolic constraints in Bane)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing ($\neq$ model checking based analyzers such as VeriSoft, Bandera, Java PathFinder)

Efficient: always terminate ($\neq$ counterexample-driven automatic abstraction refinement BLAST, SLAM)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) ($\neq$ general-purpose analyzers PolySpace Verifier)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in C Global Surveyor)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

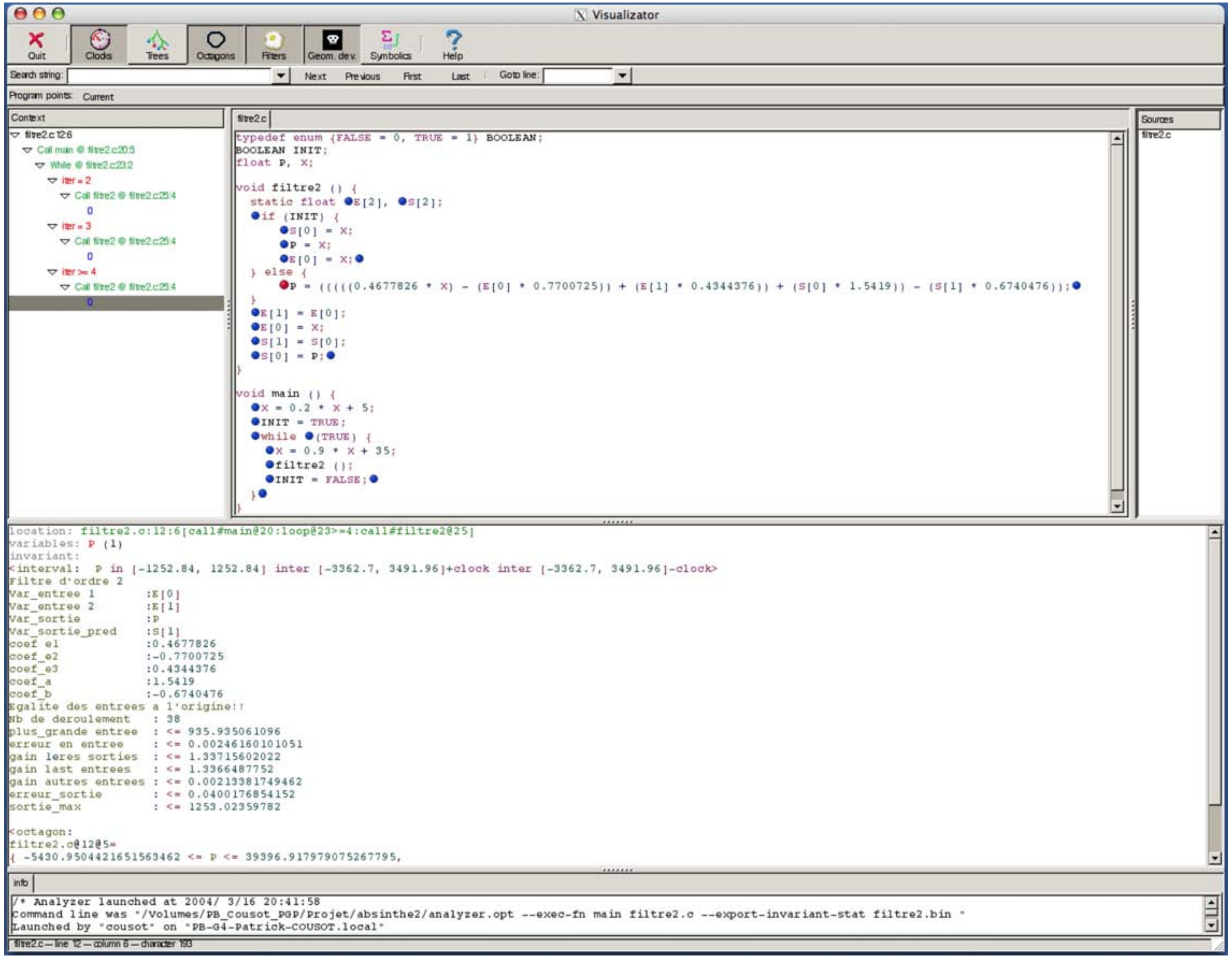
Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Modular: an analyzer instance is built by selection of O-CAML modules from a collection each implementing an abstract domain

Precise: few or no false alarm when adapted to an application domain $\longrightarrow$ VERIFIER!

Example of Analysis Session



Benchmarks for the Primary Flight Control Software of the Airbus A340

- Comparative results (commercial software):

  4,200 (false?) alarms,

  3.5 days;

- Our results:

  0 alarm,

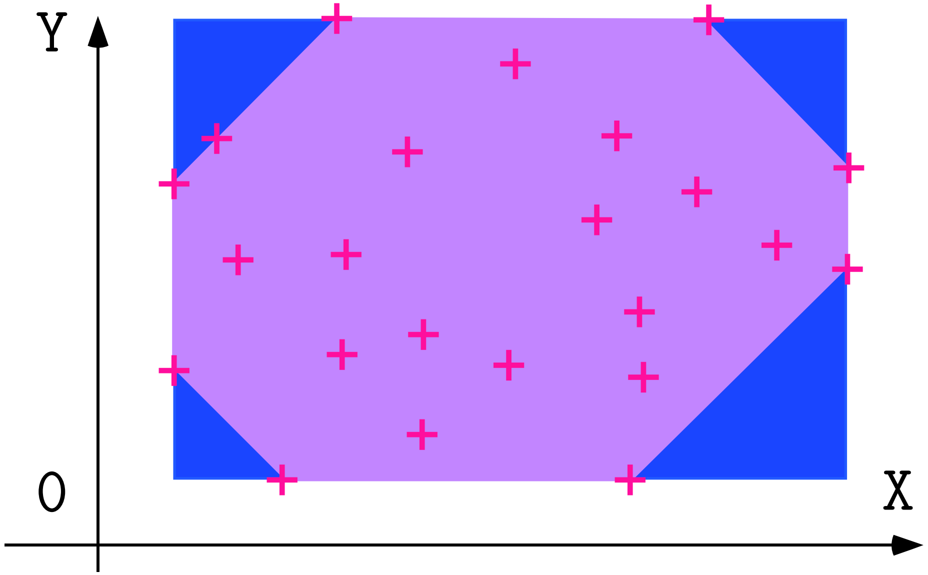
  1h20 on 2.8 GHz PC,

  300 Megabytes

  $\longrightarrow$ A world première!

Examples of Abstractions

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:
$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [4]:
$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 78 \\ 1 \leq y \leq 20 \\ x - y \leq 03 \end{cases}$$

Difficulties: many global variables, IEEE 754 floating-point arithmetic (in program <u>and</u> analyzer)

Reference

[4] A. Miné. A New Numerical Abstract Domain Based on Difference-Bound Matrices. In *PADO'2001*, LNCS 2053, Springer, 2001, pp. 155–172.

[5] A. Miné. Relational abstract domains for the detection of floating-point run-time errors. In *ESOP'04*, Barcelona, LNCS , Springer, 2004 (to appear).

Floating-Point Computations

- Code Sample:

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
} % gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

Clock Abstract Domain for Counters

- Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

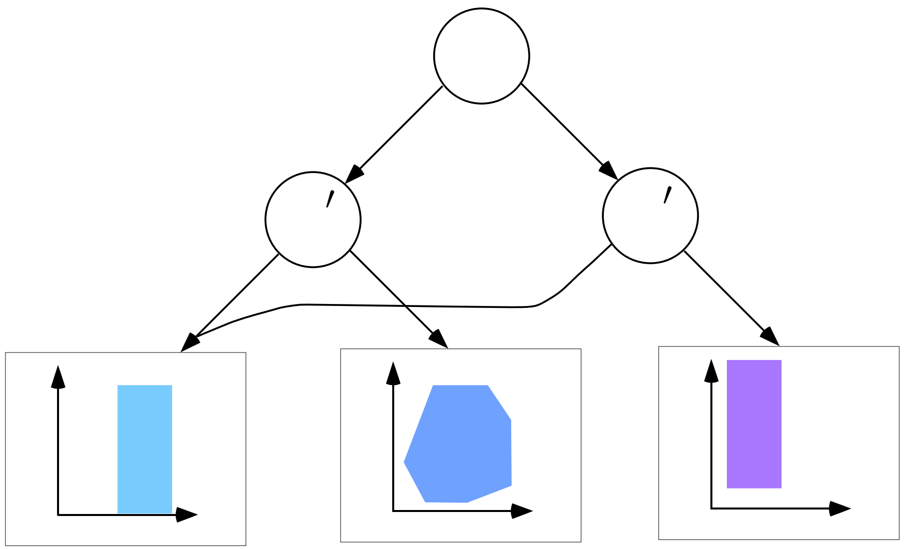
  - Output `T` is true iff the volatile input `I` has been true for the last `n` clock ticks.
  - The clock ticks every `s` seconds for at most `h` hours, thus `R` is bounded.
  - To prove that `R` cannot overflow, we must prove that `R` cannot exceed the elapsed clock ticks *(impossible using only intervals)*.

- Solution:
  - We add a phantom variable `clock` in the concrete user semantics to track elapsed clock ticks.
  - For each variable `X`, we abstract three intervals: `X`, `X+clock`, and `X-clock`.
  - If `X+clock` or `X-clock` is bounded, so is `X`.

Boolean Relations for Boolean Control

- Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



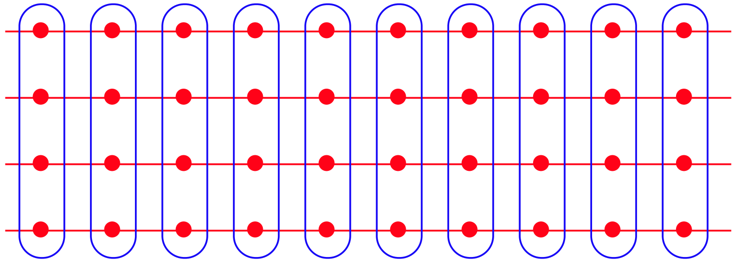
The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Control Partitionning for Case Analysis

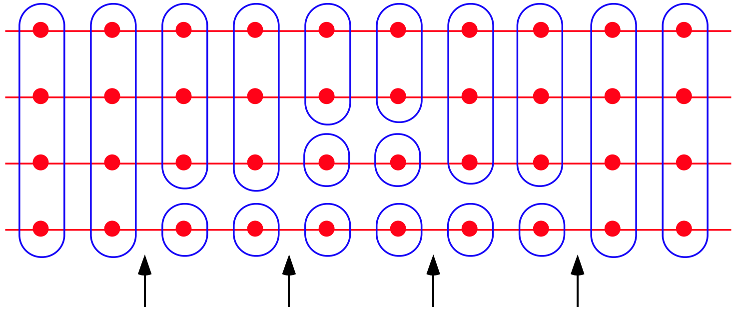
- Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant -100 ≤ x ≤ 100 ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:

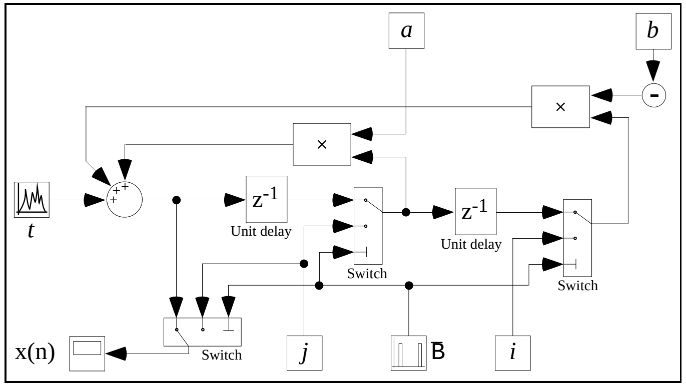


Trace partitionning:

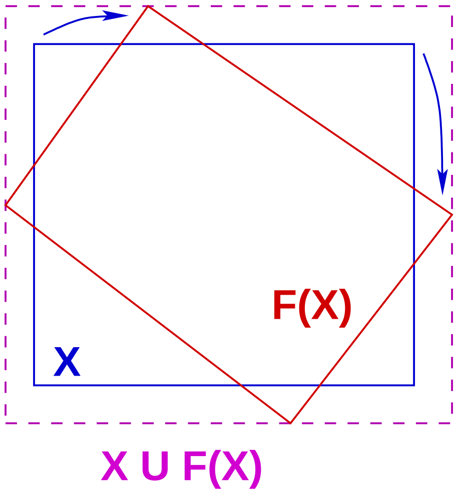


Ellipsoid Abstract Domain for Filters

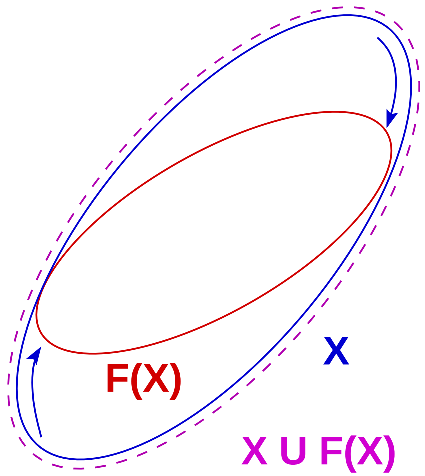
2[d] Order Digital Filter:



- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is bounded, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an ellipsoid.



unstable interval



stable ellipsoid

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The main loop invariant

A textual file over 4.5 Mb with

- 6,900 boolean interval assertions ($x \in [0;1]$)
- 9,600 interval assertions ($x \in [a;b]$)
- 25,400 clock assertions ($x + \mathrm{clk} \in [a;b] \wedge x - \mathrm{clk} \in [a;b]$)
- 19,100 additive octagonal assertions ($a \leq x + y \leq b$)
- 19,200 subtractive octagonal assertions ($a \leq x - y \leq b$)
- 100 decision trees
- 60 ellipse invariants, etc …

involving over 16,000 floating point constants (only 550 appearing in the program text) × 75,000 LOCs.

Conclusion

Conclusion

- Most applications of abstract interpretation tolerate a small rate (typically 5 to 15%) of false alarms:
  - Program transformation → do not optimize,
  - Typing → reject some correct programs, etc,
  - WCET analysis → overestimate;
- Some applications require no false alarm at all:
  - Program verification.
- Theoretically possible [SARA '00], practically feasible [PLDI '03]

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THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.

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